Lösungen

1

```
Remove["Global`*"]

m[1] = 1 / 2;
s[1] = m[1];
```

Modell

Momentenbedingung oder Hebelgesetz:

```
gleichung = ( Gewicht * (k - 1) * mDist[k - 1] +
     Gewicht * 1 * (mDist[k - 1] + 1 / 2) == Gewicht * k * mDist[k] );
Solve[gleichung, {mDist[k] }] // Flatten // Simplify // TraditionalForm
```

$\left\{\text{mDist}(k) \quad \text{mDist}(k \quad 1) \quad \frac{1}{2\,k}\right\}$

Erstetze mDist durch m .

```
(m[k - 1] * (k - 1) + (m[k - 1] + 1 / 2) * 1) / k   // Simplify // TraditionalForm
```

$m(k \quad 1) \quad \frac{1}{2\,k}$

```
m[k_] := (m[k - 1] * (k - 1) + (m[k - 1] + 1 / 2) * 1) / k;

s[k_] := m[k] - m[k - 1];
```

a

```
Table[s[k], {k, 1, 8}]
```

$\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{14}, \frac{1}{16}\right\}$

```
% // N
```

{0.5, 0.25, 0.166667, 0.125, 0.1, 0.0833333, 0.0714286, 0.0625}

```
Table[m[k], {k, 1, 8}]
```

$\left\{\frac{1}{2}, \frac{3}{4}, \frac{11}{12}, \frac{25}{24}, \frac{137}{120}, \frac{49}{40}, \frac{363}{280}, \frac{761}{560}\right\}$

```
% // N
```

{0.5, 0.75, 0.916667, 1.04167, 1.14167, 1.225, 1.29643, 1.35893}

b Summe = unendlich (harmonische Reihe)

```
summe = 2 Sum[1 / k, {k, 1, Infinity}]
```

Sum::div : Sum does not converge. Mehr…

$$2 \sum_{k=1}^{\infty} \frac{1}{k}$$

```
summe == Infinity
```

$$2 \sum_{k=1}^{\infty} \frac{1}{k} == \infty$$

2

```
Remove["Global`*"]
```

```
f[x_] := x Sin[x] + x / Sin[x] ; f[x]
```

x Csc[x] + x Sin[x]

a

```
D[x Sin[x] + x / Sin[x], x] // Simplify
```

x Cos[x] + Csc[x] − x Cot[x] Csc[x] + Sin[x]

```
x Cos[x] + Csc[x] - x Cot[x] Csc[x] + Sin[x] /. {Sin[x] → sin[x], Cos[x] → cos[x],
  Tan[x] → sin[x] / cos[x], Cot[x] → cos[x] / sin[x], Csc[x] → 1 / sin[x]}
```

$$x \cos[x] - \frac{x \cos[x]}{\sin[x]^2} + \frac{1}{\sin[x]} + \sin[x]$$

```
% // Simplify
```

$$\cos[x] \left(x - \frac{x}{\sin[x]^2}\right) + \frac{1}{\sin[x]} + \sin[x]$$

```
%% // Together
```

$$\frac{-x \cos[x] + \sin[x] + x \cos[x] \sin[x]^2 + \sin[x]^3}{\sin[x]^2}$$

```
(D[x Sin[x] + x / Sin[x], x] // Simplify) /. x → Pi / 2
```

2

b

```
Remove["Global`*"]
```

```
ln[x_] := Log[x];
```

```
f[x_] := ln[x] / x ; f[x]
```

$$\frac{\text{Log}[x]}{x}$$

Methoden: Substitution oder partielle Integration.

```
Integrate[f[x], {x, 1, E}]
```

$$\frac{1}{2}$$

c

```
Remove["Global`*"]
```

```
f[x_] := x^(E); f[x]
```

x^{e}

i

```
Integrate[f[x], {x, 1, t}, GenerateConditions → False]
```

$$\frac{-1 + t^{1+e}}{1 + e}$$

ii

```
D[Evaluate[Integrate[f[x], {x, 1, t}, GenerateConditions → False]], {t}]
```

t^{e}

d

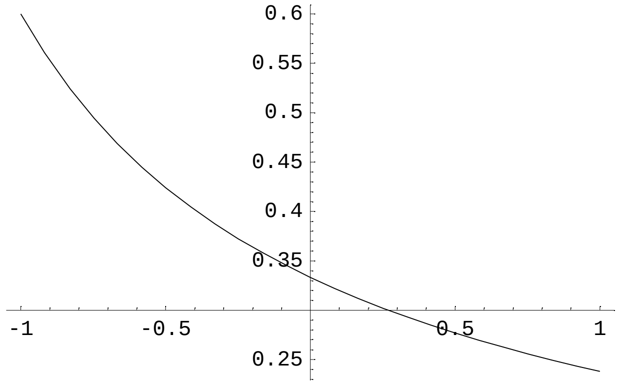
```
Remove["Global`*"]
```

```
f[x_] := (x + 4) / ((x + 2) (x + 6)); f[x]
```

$$\frac{4 + x}{(2 + x)\ (6 + x)}$$

i

```
Plot[f[x], {x, -1, 1}];
```



ii

```
Apart[f[x]]
```

$$\frac{1}{2\,(2+x)}+\frac{1}{2\,(6+x)}$$

iii

```
Apart[f'[x]]
```

$$-\frac{1}{2\,(2+x)^2}-\frac{1}{2\,(6+x)^2}$$

```
Apart[f'[x]] /. x → 0
```

$$-\frac{5}{36}$$

iv

```
% // ArcTan // N
```

-0.138006

```
% / Degree
```

-7.90716

v

```
Integrate[Evaluate[Apart[f[x]]], x]
```

$$\frac{1}{2}\,\mathrm{Log}[12+8\,x+x^2]$$

```
Integrate[Evaluate[Apart[f[x]]], {x, -1, w}, GenerateConditions → False]
```

$$\frac{1}{2}\,\left(\mathrm{Log}[2+w]+\mathrm{Log}\left[\frac{6+w}{5}\right]\right)$$

3

```
Remove["Global`*"]

f[x_, a2_, a1_, a0_] := a2 x^2 + a1 x + a0

g1 = (f[1, a2, a1, a0] == 6)
```

$a0 + a1 + a2 == 6$

```
g2 = (f[6, a2, a1, a0] == 9)
```

$a0 + 6\, a1 + 36\, a2 == 9$

```
g3 = (Evaluate[D[f[x, a2, a1, a0], x] /. x → 6] == Tan[60 Degree])
```

$a1 + 12\, a2 == \sqrt{3}$

```
solv = Solve[{g1, g2, g3}, {a0, a1, a2}] // Flatten
```

$\left\{a0 \to \frac{3}{25}\left(39 + 10\sqrt{3}\right),\ a1 \to \frac{1}{25}\left(36 - 35\sqrt{3}\right),\ a2 \to \frac{1}{25}\left(-3 + 5\sqrt{3}\right)\right\}$

```
f[x_] := a2 x^2 + a1 x + a0 /. solv

f[x]
```

$\frac{3}{25}\left(39 + 10\sqrt{3}\right) + \frac{1}{25}\left(36 - 35\sqrt{3}\right) x + \frac{1}{25}\left(-3 + 5\sqrt{3}\right) x^2$

```
f[x] // N
```

$6.75846 - 0.984871\, x + 0.22641\, x^2$

```
solv1 = Solve[Evaluate[D[f[x], x] == 0], {x}] // Flatten
```

$\left\{x \to \frac{-36 + 35\sqrt{3}}{2\left(-3 + 5\sqrt{3}\right)}\right\}$

```
x3 = x /. solv1
```

$\frac{-36 + 35\sqrt{3}}{2\left(-3 + 5\sqrt{3}\right)}$

```
% // N
```

2.17497

```
f[x3]
```

$\frac{3}{25}\left(39 + 10\sqrt{3}\right) + \frac{\left(36 - 35\sqrt{3}\right)\left(-36 + 35\sqrt{3}\right)}{50\left(-3 + 5\sqrt{3}\right)} + \frac{\left(-36 + 35\sqrt{3}\right)^2}{100\left(-3 + 5\sqrt{3}\right)}$

```
f[x3] // N
```

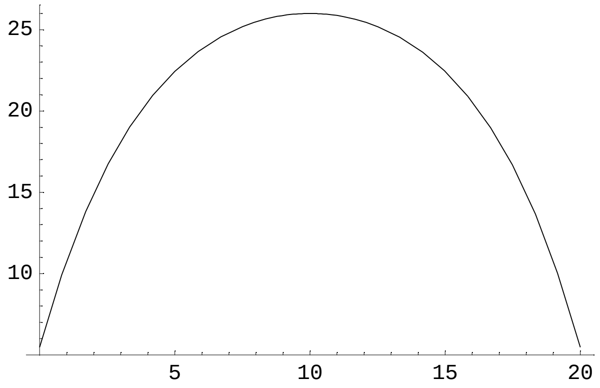
5.68743

4

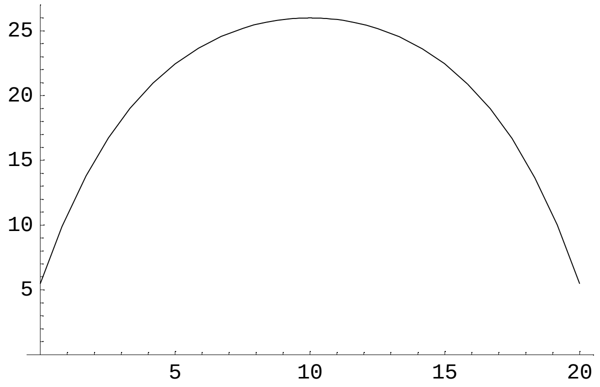
```
Remove["Global`*"]
```

a

```
f[x_] := c Cosh[(x - x0) / (c)] + y0 /. {c → -4, x0 → 10, y0 → 30};
Plot[f[x], {x, 0, 20}];
```



```
f[x_] := c Cosh[(x - x0) / (c)] + y0 /. {c → -4, x0 → 10, y0 → 30};
Plot[f[x], {x, 0, 20}, PlotRange → {0, 27}];
```



b

```
m[a_] := (20 - 2 a) f[a] + (20 - 2 a) (f[10] - f[a]) / 2; m[a]
```

$$(20 - 2\,a)\left(30 - 4\,\mathrm{Cosh}\left[\frac{10-a}{4}\right]\right) + \frac{1}{2}\,(20 - 2\,a)\left(-4 + 4\,\mathrm{Cosh}\left[\frac{10-a}{4}\right]\right)$$

```
m'[a] == 0
```

$$4 - 2\left(30 - 4\,\mathrm{Cosh}\left[\frac{10-a}{4}\right]\right) - 4\,\mathrm{Cosh}\left[\frac{10-a}{4}\right] + \frac{1}{2}\,(20 - 2\,a)\,\mathrm{Sinh}\left[\frac{10-a}{4}\right] == 0$$

```
fRoot = FindRoot[Evaluate[m'[a] == 0], {a, 2}]
```

{a → 1.27977}

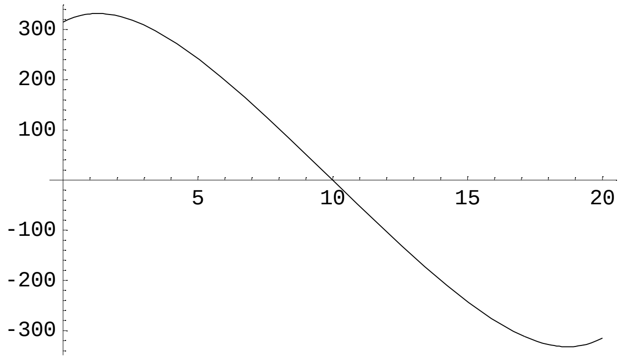
```
aa = a /. fRoot
```

1.27977

```
m[aa]
```

332.069

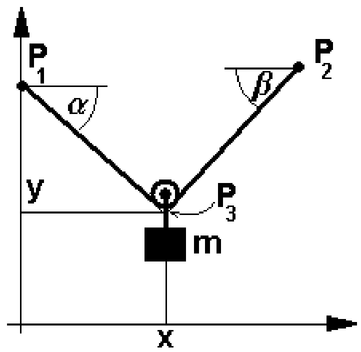
```
Plot[Evaluate[m[a]], {a, 0, 20}];
```



5

```
Remove["Global`*"]
```

a



Gegeben ist diese Graphik

P1 = P1 (x1, y1); P2 = P2 (x2, y2); etc. Dann gilt (Pythagoras):

```
g = (Sqrt[(10 - y)^2 + x^2] + Sqrt[(12 - y)^2 + (8 - x)^2] == 11)
```

$\sqrt{x^2 + (10-y)^2} + \sqrt{(8-x)^2 + (12-y)^2} == 11$

```
solv = Solve[g, {y}] // Flatten
```

$\{y \to \frac{1}{234}\left(2446 + 32x - 11\sqrt{53}\sqrt{53 + 32x - 4x^2}\right),$
$y \to \frac{1}{234}\left(2446 + 32x + 11\sqrt{53}\sqrt{53 + 32x - 4x^2}\right)\}$

```
y1[x_] := y /. solv[[1]];
y2[x_] := y /. solv[[2]];
Evaluate[y1[x]] /. x → u
```

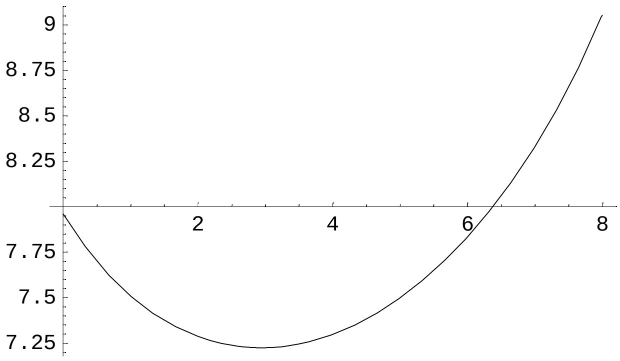
$$\frac{1}{234}\left(2446+32\,u-11\sqrt{53}\,\sqrt{53+32\,u-4\,u^2}\right)$$

```
y[u_] := Evaluate[y1[x]] /. x → u
```

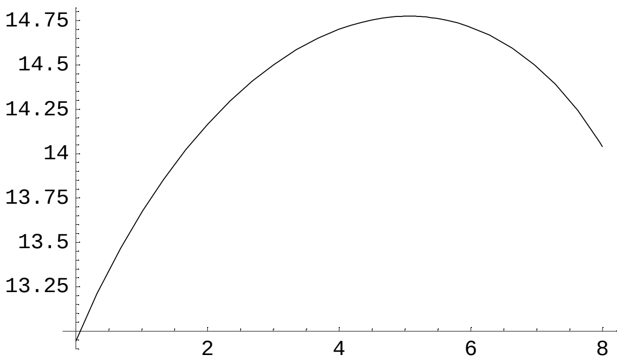
```
y[u]
```

$$\frac{1}{234}\left(2446+32\,u-11\sqrt{53}\,\sqrt{53+32\,u-4\,u^2}\right)$$

```
Plot[Evaluate[y[u]], {u, 0, 8}];
```




```
Plot[Evaluate[y2[x]], {x, 0, 8}];
```



y1 hat die erwarteten Masse.

b

```
solv1 = Solve[Evaluate[y'[u] == 0] /. u → x, {x}] // Flatten
```

$$\left\{x\to\frac{4}{57}\left(57-2\sqrt{57}\right)\right\}$$

```
x0 = x /. solv1
```

$$\frac{4}{57}\left(57-2\sqrt{57}\right)$$

```
N[%]
```

2.94037

```
y[u]
```

$$\frac{1}{234}\left(2446+32\,u-11\sqrt{53}\,\sqrt{53+32\,u-4\,u^2}\right)$$

```
y0 = y[u] /. u → x0
```

$$\frac{1}{234}\left(2446+\frac{128}{57}\left(57-2\sqrt{57}\right)-11\sqrt{53\left(53+\frac{128}{57}\left(57-2\sqrt{57}\right)-\frac{64\left(57-2\sqrt{57}\right)^2}{3249}\right)}\right)$$

```
N[%]
```

7.22508

c

```
Tan[(10 - y0) / x0] // N
```

1.38001

```
Tan[(12 - y0) / (8 - x0)] // N
```

1.38001

```
(Tan[(10 - y0) / x0] // N) == (Tan[(12 - y0) / (8 - x0)] // N)
```

True

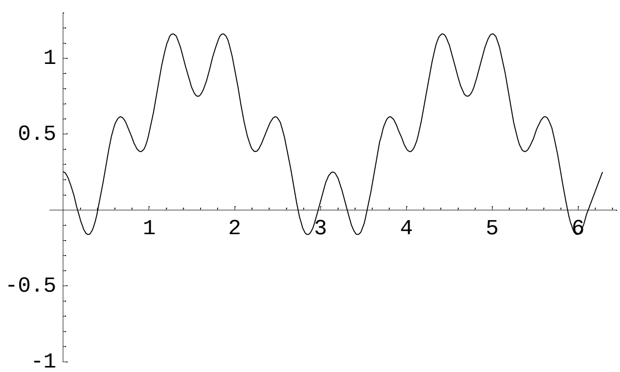
6

```
Remove["Global`*"]
```

a

```
f[x_] := Sin[x]^2 + Cos[10 x] / 4;
```

```
Plot[f[x], {x, 0, 2 Pi}, PlotRange → {-1, 1.3}];
```



Hinweis: Dei y-Koord. und damit die Radien r (x) = y (x) können negativ sein.
Daher muss bei der Integration allenfalls mit dem Betrag von y(x) gerechnet werden!!!!!!

b

```
(* Laenge = Integrate[Evaluate[Sqrt[1+(f'[x])^2]],{x,0,2 Pi}] *)
```

```
Laenge = NIntegrate[Evaluate[Sqrt[1 + (f '[x]) ^2]], {x, 0, 2 Pi}]
```

12.6438

c

```
Inhalt = Pi Integrate[f[x] ^2, {x, 0, 2 Pi}]
```

$$\frac{13\,\pi^2}{16}$$

```
Inhalt = Pi Integrate[f[x] ^2, {x, 0, 2 Pi}] // N
```

8.01905

d

```
(* Oberflaeche = 2 Pi Integrate[Evaluate[f[x]  Sqrt[1+(f'[x])^2]],{x,0,2 Pi}] *)
```

```
OberflaecheFalsch =
 2 Pi NIntegrate[Evaluate[f[x] Sqrt[1 + (f '[x]) ^2]], {x, 0, 2 Pi}] // N
```

39.7217

Betrag von f[x] verwenden!

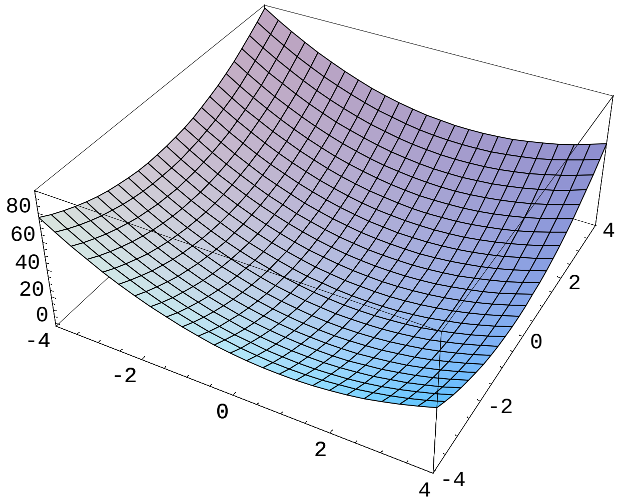
```
OberflaecheRichtig =
 2 Pi NIntegrate[Evaluate[Abs[f[x]] Sqrt[1 + (f '[x]) ^2]], {x, 0, 2 Pi}] // N
```

41.6113

7

```
Remove["Global`*"]
```

```
f[x_, y_] := x^2 + y^2 + (x - 2)^2 + (y + 1)^2;
Plot3D[f[x, y], {x, -4, 4}, {y, -4, 4}];
```




```
f[x, y] // Expand
```

$5 - 4x + 2x^2 + 2y + 2y^2$

a

```
solv = Solve[Evaluate[{D[f[x, y], x] == 0, D[f[x, y], y] == 0}], {x, y}] // Flatten
```

$\{x \to 1,\ y \to -\frac{1}{2}\}$

```
{xx, yy} = {x, y} /. solv;
z = f[xx, yy]
```

$\frac{5}{2}$

```
% // N
```

2.5

b

```
grad[h_] := {D[h[x, y], x], D[h[x, y], y]};  grad[h]
```

$\{h^{(1,0)}[x, y],\ h^{(0,1)}[x, y]\}$

```
grad[f]
```

$\{2(-2 + x) + 2x,\ 2y + 2(1 + y)\}$

```
g[x_, y_] := x + 10 + (y - 6)^2 - 25;
grad[g]
```

$\{1,\ 2(-6 + y)\}$

```
λ grad[g][[1]] == grad[f][[1]]
```

$\lambda == 2\ (-2 + x) + 2\ x$

```
λ grad[g][[2]] == grad[f][[2]]
```

$2\ (-6 + y)\ \lambda == 2\ y + 2\ (1 + y)$

```
{λ grad[g][[1]] == grad[f][[1]], λ grad[g][[2]] == grad[f][[2]], g[x, y] == 0}
```

$\{\lambda == 2\ (-2 + x) + 2\ x,\ 2\ (-6 + y)\ \lambda == 2\ y + 2\ (1 + y),\ -15 + x + (-6 + y)^2 == 0\}$

```
Solve[Evaluate[{λ grad[g][[1]] == grad[f][[1]],
    λ grad[g][[2]] == grad[f][[2]], g[x, y] == 0}], {x, y, λ}] // N // Chop
```

```
{{x → 2.41617, λ → 5.66469, y → 9.54737},
 {x → 14.9415, λ → 55.7662, y → 6.24179}, {x → 0.642289, λ → -1.43084, y → 2.21084}}
```

```
x1 = 2.416171911191869`; y1 = 9.547369178533316`;
x2 = 14.94153866199081`; y2 = 6.241787795409923`;
x3 = 0.6422894268173227` ; y3 = 2.2108430260567613` ;
```

```
(* Kontrolle *)
FindRoot[Evaluate[
   {λ grad[g][[1]] == grad[f][[1]], λ grad[g][[2]] == grad[f][[2]], g[x, y] == 0}],
  {x, 1.86}, {y, 9.6}, {λ, 3.4}] // N // Chop
```

```
{x → 2.41617, y → 9.54737, λ → 5.66469}
```

```
f[x1, y1]
```

208.41

```
f[x2, y2]
```

482.136

```
f[x3, y3]
```

17.4533

8

```
DSolve[y'[x] == (x^3) / y[x], y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, -\frac{\sqrt{x^4 + 4\,C[1]}}{\sqrt{2}}\right]\right\}, \left\{y \to \text{Function}\left[\{x\}, \frac{\sqrt{x^4 + 4\,C[1]}}{\sqrt{2}}\right]\right\}\right\}$$

$$\text{Solve}\left[-\frac{\sqrt{1^4 + 4\,C[1]}}{\sqrt{2}} == 1, \{C[1]\}\right]$$

{}

```
Solve[Sqrt[1^4 + 4 C[1]]/Sqrt[2] == 1, {C[1]}]
```

$$\left\{\left\{C[1] \to \frac{1}{4}\right\}\right\}$$

$$\frac{\sqrt{x^4 + 4\,\frac{1}{4}}}{\sqrt{2}}$$

$$\frac{\sqrt{1 + x^4}}{\sqrt{2}}$$

```
DSolve[{y'[x] == (x^3) / y[x], y[1] == 1}, y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, \frac{\sqrt{1 + x^4}}{\sqrt{2}}\right]\right\}\right\}$$

$$\frac{\sqrt{1 + x^4}}{\sqrt{2}}$$

$$\frac{\sqrt{1 + x^4}}{\sqrt{2}}$$

9

```
Series[Cos[x], {x, 0, 4}]
```

$$1 - \frac{x^2}{2} + \frac{x^4}{24} + O[x]^5$$

```
Series[Cos[x], {x, 0, 4}] // Normal
```

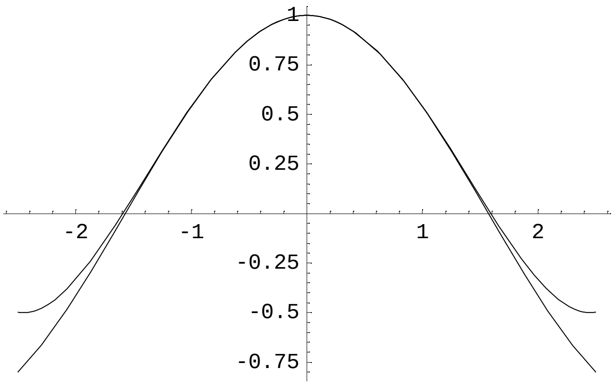
$$1 - \frac{x^2}{2} + \frac{x^4}{24}$$

```
a0 = 1; a1 = 0; a2 = 2 (-1 / 2) // Simplify; a3 = 0; a4 = 4 ! 1 / 24 // Simplify;
{a0, a1, a2, a3, a4}
```

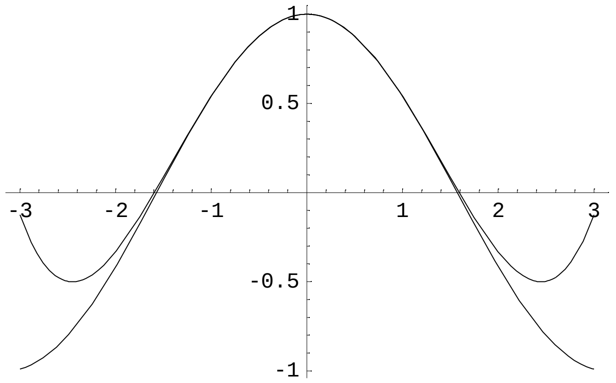
{1, 0, -1, 0, 1}

```
a0 = 1; a1 = 0; a2 = 2 (-1 / 2); a3 = 0; a4 = 4 ! 1 / 24;
p[x_] := a0 + a1 x + a2 / 2 x^2 + a3 / 3 ! x^3 + a4 / 4 ! x^4 // Simplify;
Print[p[x] // Expand];
m = 2.5;
Plot[{Cos[x], p[x]}, {x, -m, m}];
```

$1-\frac{x^2}{2}+\frac{x^4}{24}$




```
m = 3;
Plot[{Cos[x], p[x]}, {x, -m, m}];
```




```
u = 2.73;
(* Bei x = u ist die Distanz zwischen den Kurven = 0.5, im Diagramm rechts. *)
Plot[{Cos[x], p[x], Cos[u], Cos[u] + 0.5}, {x, 1.5, u}];
```