Lösungen

```
Remove["Global`*"]
```

1

```
x1 = {3, -2}; x2 = {1, 6}; λ1 = 20; λ2 = 30;
X = Transpose[{x1, x2}]; X // MatrixForm
```

$$\begin{pmatrix} 3 & 1 \\ -2 & 6 \end{pmatrix}$$

```
mD = {{λ1, 0}, {0, λ2}}
```

{{20, 0}, {0, 30}}

a

```
Eigensystem[X]
```

{{5, 4}, {{1, 2}, {1, 1}}}

```
Eigenvalues[X]
```

{5, 4}

Ganzzahlig.

b

Zwei Fixgeraden durch den Ursprung, Richtung der Eigenvektoren. Keine Fixpunktgerade, da ungleich 1.

c

```
A = X.mD.Inverse[X]; A // MatrixForm
```

$$\begin{pmatrix} 21 & \frac{3}{2} \\ 6 & 29 \end{pmatrix}$$

```
B = {{21, 6}, {3 / 2, 29}}; B // MatrixForm
```

$$\begin{pmatrix} 21 & 6 \\ \frac{3}{2} & 29 \end{pmatrix}$$

A und B sind zueinander transponiert.

d

```
Eigensystem[A]
```

$\left\{\{30, 20\}, \left\{\left\{\frac{1}{6}, 1\right\}, \left\{-\frac{3}{2}, 1\right\}\right\}\right\}$

Die Eigenwerte sind   1 und   2, X ist (x1,x2) von oben.

```
Eigensystem[mD]
```

{{30, 20}, {{0, 1}, {1, 0}}}

Die Eigenwerte von D sind ebenfalls sind   1 und   2,die Eigenvektoren von D sind die Einheitsvektoren.

e

```
Det[A - λ IdentityMatrix[2]]
```

$600 - 50\lambda + \lambda^2$

```
Det[mD - λ IdentityMatrix[2]]
```

$600 - 50\lambda + \lambda^2$

Die charakteristischen Polynome stimmen überein.

f

```
Tr[A]
```

50

```
Tr[mD]
```

50

```
Det[A]
```

600

```
Det[mD]
```

600

```
OP1 = {5, -3};
OQ = A.OP1
```

$\left\{\frac{201}{2}, -57\right\}$

```
N[%]
```

{100.5, -57.}

f

```
Eigenvalues[B]
```

```
{30, 20}
```

```
Eigenvalues[B] == Eigenvalues[A]
```

```
True
```

2

```
Remove["Global`*"]
```

b

```
Dm[φ_] := {{Cos[φ], -Sin[φ]}, {Sin[φ], Cos[φ]}};
Dm[φ] // MatrixForm
```

$$\begin{pmatrix} \text{Cos}[\varphi] & -\text{Sin}[\varphi] \\ \text{Sin}[\varphi] & \text{Cos}[\varphi] \end{pmatrix}$$

```
Dm[62 Degree] // N // MatrixForm
```

$$\begin{pmatrix} 0.469472 & -0.882948 \\ 0.882948 & 0.469472 \end{pmatrix}$$

```
OP[1] = {0, 0}; OP[2] = {2, 0}; OP[3] = {3, 2};
```

```
OQ[k_] := Dm[62 Degree].OP[k];
OQ[1] // N
```

```
{0., 0.}
```

```
OQ[2] // N
```

```
{0.938943, 1.7659}
```

```
OQ[3] // N
```

```
{-0.35748, 3.58779}
```

3

```
Remove["Global`*"]
```

a

```
x1 = {2, 3}; x2 = {-3, 2}; λ1 = 1; λ2 = -1;
X = Transpose[{x1, x2}]; X // MatrixForm
```

$$\begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix}$$

```
mD = {{λ1, 0}, {0, λ2}}
```

{{1, 0}, {0, -1}}

```
Sg = X.mD.Inverse[X]; Sg // MatrixForm
```

$$\begin{pmatrix} -\frac{5}{13} & \frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{pmatrix}$$

```
Sg // N // MatrixForm
```

$$\begin{pmatrix} -0.384615 & 0.923077 \\ 0.923077 & 0.384615 \end{pmatrix}$$

a 2. Anderes Vorgehen:

```
Sg1 = {{Cos[2 α], Sin[2 α]}, {Sin[2 α], -Cos[2 α]}}; Sg1 // MatrixForm
```

$$\begin{pmatrix} \mathrm{Cos}[2\,\alpha] & \mathrm{Sin}[2\,\alpha] \\ \mathrm{Sin}[2\,\alpha] & -\mathrm{Cos}[2\,\alpha] \end{pmatrix}$$

```
α = ArcTan[3 / 2]
```

$\mathrm{ArcTan}\left[\frac{3}{2}\right]$

```
αDegree = ArcTan[3 / 2] / Degree // N
```

56.3099

```
Sg1 // MatrixForm
```

$$\begin{pmatrix} \mathrm{Cos}\left[2\,\mathrm{ArcTan}\left[\frac{3}{2}\right]\right] & \mathrm{Sin}\left[2\,\mathrm{ArcTan}\left[\frac{3}{2}\right]\right] \\ \mathrm{Sin}\left[2\,\mathrm{ArcTan}\left[\frac{3}{2}\right]\right] & -\mathrm{Cos}\left[2\,\mathrm{ArcTan}\left[\frac{3}{2}\right]\right] \end{pmatrix}$$

```
Sg1 // N // MatrixForm
```

$$\begin{pmatrix} -0.384615 & 0.923077 \\ 0.923077 & 0.384615 \end{pmatrix}$$

b

```
OP = {7, -2};
OQ = Sg.OP
```

$\left\{-\frac{59}{13}, \frac{74}{13}\right\}$

```
N[%]
```

{−4.53846, 5.69231}

4

```
Remove["Global`*"]
```

```
a = {2, -1, 1}; b = {1, 2, -3}; u = {1, 1, -1};
```

a

```
Det[{a, b, u}]
```

3

> l.u.

b

Ovec={0,0,0};
P.Transpose[{a,b,u}]==Transpose[{a,b,Ovec}]   > P   ...

```
o = {0, 0, 0};
P.Transpose[{a, b, u}] == Transpose[{a, b, o}]
```

P.{{2, 1, 1}, {−1, 2, 1}, {1, −3, −1}} == {{2, 1, 0}, {−1, 2, 0}, {1, −3, 0}}

```
P = Transpose[{a, b, o}].Inverse[Transpose[{a, b, u}]]; P // MatrixForm
```

$$\begin{pmatrix} \frac{2}{3} & -\frac{7}{3} & -\frac{5}{3} \\ -\frac{1}{3} & -\frac{4}{3} & -\frac{5}{3} \\ \frac{1}{3} & \frac{7}{3} & \frac{8}{3} \end{pmatrix}$$

```
P // N // MatrixForm
```

$$\begin{pmatrix} 0.666667 & -2.33333 & -1.66667 \\ -0.333333 & -1.33333 & -1.66667 \\ 0.333333 & 2.33333 & 2.66667 \end{pmatrix}$$

c

```
P.{100, 100, 100}
```

$$\left\{-\frac{1000}{3}, -\frac{1000}{3}, \frac{1600}{3}\right\}$$

```
N[%]
```

{−333.333, −333.333, 533.333}

5

```
Remove["Global`*"]
```

a

```
a = {1, 1, -2};
b1 = a / Norm[a]
```

$$\left\{\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\sqrt{\frac{2}{3}}\right\}$$

```
{1, y1, 0}.a == 0
```

$1 + y1 == 0$

```
Solve[{1, y1, 0}.a == 0, {y1}]
```

{{y1 → -1}}

```
b2work = {1, -1, 0}
```

{1, -1, 0}

```
b2 = b2work / Norm[b2work]
```

$$\left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right\}$$

```
b2 // N
```

{0.707107, -0.707107, 0.}

```
b3 = Cross[b1, b2]
```

$$\left\{-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right\}$$

```
b3 // N
```

{-0.57735, -0.57735, -0.57735}

```
U = Transpose[{b1, b2, b3}]; U // MatrixForm
```

$$\begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} \\ -\sqrt{\frac{2}{3}} & 0 & -\frac{1}{\sqrt{3}} \end{pmatrix}$$

```
Dreh0[φ_] := {{1, 0, 0}, {0, Cos[φ], -Sin[φ]}, {0, Sin[φ], Cos[φ]}};
Dreh0[φ] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}[\varphi] & -\text{Sin}[\varphi] \\ 0 & \text{Sin}[\varphi] & \text{Cos}[\varphi] \end{pmatrix}$$

```
DrehAchse[φ_] := U.Dreh0[φ].Inverse[U]

Dreh = DrehAchse[36 Degree] // N; Dreh // MatrixForm
```

$$\begin{pmatrix} 0.840847 & 0.511755 & 0.176301 \\ -0.448094 & 0.840847 & -0.303623 \\ -0.303623 & 0.176301 & 0.936339 \end{pmatrix}$$

b

```
T1 = {5, 4, 6};

R1 = Dreh.T1

{7.30907, -0.698821, 4.80512}
```

c Bei einer Drehung um den negativen Winkel:

```
Dreh = DrehAchse[-36 Degree] // N; Dreh // MatrixForm
```

$$\begin{pmatrix} 0.840847 & -0.448094 & -0.303623 \\ 0.511755 & 0.840847 & 0.176301 \\ 0.176301 & -0.303623 & 0.936339 \end{pmatrix}$$

```
R1 = Dreh.T1

{0.590121, 6.97997, 5.28505}
```

6

```
Remove["Global`*"]

OP = {10, 10};
x[0] = OP;
mD = {{Cos[Pi / 7], -Sin[Pi / 7]}, {Sin[Pi / 7], Cos[Pi / 7]}}; mD // MatrixForm
```

$$\begin{pmatrix} \text{Cos}[\frac{\pi}{7}] & -\text{Sin}[\frac{\pi}{7}] \\ \text{Sin}[\frac{\pi}{7}] & \text{Cos}[\frac{\pi}{7}] \end{pmatrix}$$

```
mD // N // MatrixForm
```

$$\begin{pmatrix} 0.900969 & -0.433884 \\ 0.433884 & 0.900969 \end{pmatrix}$$

```
y[n_] := mD.x[n];
S = {{3 / 4, 0}, {0, 3 / 4}}; S // MatrixForm
```

$$\begin{pmatrix} \frac{3}{4} & 0 \\ 0 & \frac{3}{4} \end{pmatrix}$$

```
S // N // MatrixForm
```

$$\begin{pmatrix} 0.75 & 0. \\ 0. & 0.75 \end{pmatrix}$$

```
x[n_] := S.y[n - 1] // N;
Table[x[n], {n, 0, 5}]
```

```
{{10, 10}, {3.50314, 10.0114}, {-0.890672, 7.90493},
 {-3.17422, 5.05174}, {-3.7888, 2.38066}, {-3.33489, 0.375752}}
```

```
tab1 = Table[x[n], {n, 0, 20}]
```

```
{{10, 10}, {3.50314, 10.0114}, {-0.890672, 7.90493},
 {-3.17422, 5.05174}, {-3.7888, 2.38066}, {-3.33489, 0.375752},
 {-2.37575, -0.831311}, {-1.33484, -1.33484}, {-0.467613, -1.33636},
 {0.11889, -1.05518}, {0.423707, -0.674326}, {0.505744, -0.31778},
 {0.445155, -0.0501569}, {0.317124, 0.110967}, {0.178179, 0.178179},
 {0.0624187, 0.178383}, {-0.0158699, 0.14085}, {-0.056558, 0.0900116},
 {-0.0675087, 0.0424185}, {-0.059421, 0.00669513}, {-0.042331, -0.0148123}}
```

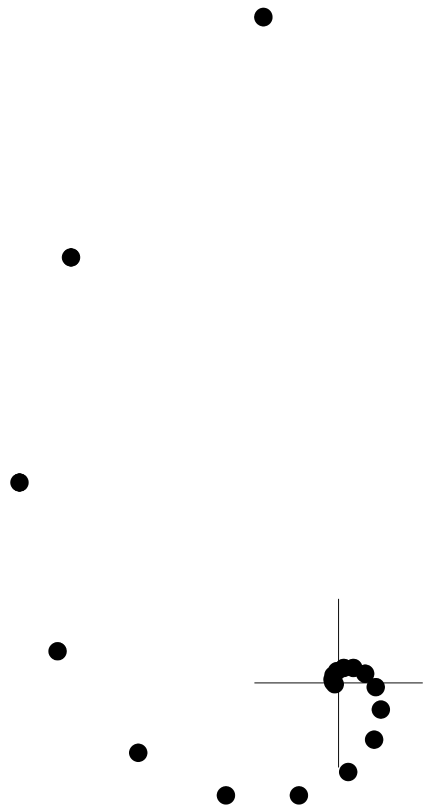
```
pretab = Prepend[Map[Point, tab1], PointSize[0.03]]
```

```
{PointSize[0.03], Point[{10, 10}], Point[{3.50314, 10.0114}],
 Point[{-0.890672, 7.90493}], Point[{-3.17422, 5.05174}], Point[{-3.7888, 2.38066}],
 Point[{-3.33489, 0.375752}], Point[{-2.37575, -0.831311}],
 Point[{-1.33484, -1.33484}], Point[{-0.467613, -1.33636}],
 Point[{0.11889, -1.05518}], Point[{0.423707, -0.674326}],
 Point[{0.505744, -0.31778}], Point[{0.445155, -0.0501569}],
 Point[{0.317124, 0.110967}], Point[{0.178179, 0.178179}],
 Point[{0.0624187, 0.178383}], Point[{-0.0158699, 0.14085}],
 Point[{-0.056558, 0.0900116}], Point[{-0.0675087, 0.0424185}],
 Point[{-0.059421, 0.00669513}], Point[{-0.042331, -0.0148123}]}
```

```
lin = {Line[{{-1, 0}, {1, 0}}], Line[{{0, -1}, {0, 1}}]}
```

```
{Line[{{-1, 0}, {1, 0}}], Line[{{0, -1}, {0, 1}}]}
```

```
Show[Graphics[Join[pretab, lin]], AspectRatio → Automatic];
```



Es ergibt sich eine Punktspierale, die gegen den Ursprung konvergiert.