Lösungen

1

a

ans =
 -1.41421
 1.00000

b

ans = 0.00019000

c

Inf

d

ans = 0 -3 2

e

ans =

1.0e+01 *

3.40000
2.90000
6.00000
-4.76000
8.00000

f

ans = 8.9000

2

```
Remove["Global`*"]
```

a

```
u = (3 a x + 2 y + 3 z == 1);
v = (2 x + 2 a y + 4 z == 1);
w = (3 x + 4 y + 1 a z == 1);
Solve[{u, v, w}, {x, y, z}]
```

$$\left\{\left\{x \to -\frac{-2+4a-a^2}{24-35a+3a^3},\ y \to -\frac{-9+14a-3a^2}{2\,(24-35a+3a^3)},\ z \to -\frac{-5+9a-3a^2}{24-35a+3a^3}\right\}\right\}$$

```
Solve[24 - 35 a + 3 a^3 == 0, {a}] // N
```

{{a → 3.}, {a → −3.71736}, {a → 0.717356}}

b

```
Solve[{u, v, w}, {x, y, z}] /. a → 1
```

$$\left\{\left\{x \to \frac{1}{8},\ y \to \frac{1}{8},\ z \to \frac{1}{8}\right\}\right\}$$

c

```
v1 = {a 3, 2, 3}; v2 = {2, a 2, 4}; v3 = {3, 4, a 1};
v1 - v2 + 3 v3
```

{7 + 3 a, 14 − 2 a, −1 + 3 a}

```
Det[{v1, v2, v3}] /. a → 3
```

0

Keine Dim. da keine Lösung

```
Solve[{u, v, w}, {x, y, z}] /. a → 3
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. Mehr…

General::stop : Further output of Power::infy will be suppressed during this calculation. Mehr…

{{x → ComplexInfinity, y → ComplexInfinity, z → ComplexInfinity}}

d

```
Solve[{u, v}, {x, y, z}] /. a → 2
```

Solve::svars : Equations may not give solutions for all "solve" variables. Mehr…

$\left\{\left\{x \to \frac{1}{10} - \frac{z}{5},\ y \to \frac{1}{5} - \frac{9 z}{10}\right\}\right\}$

```
g[t_] := {1/10 - t/5, 1/5 - 9 t/10, t}; P0 = {1, 1, 1};
v = g[1] - g[0]; u = P0 - g[0];
d = Norm[Cross[v, u]] / Norm[v]
```

$\frac{3 \sqrt{\frac{201}{185}}}{2}$

```
N[%]
```

1.56352

3

a

```
v1 = {3, 2, 3}; v2 = {2, 2, 4}; v3 = {3, 4, 1}; w = {10, 12, 2};
Det[{v1, v2, v3}]
```

-16

b

```
Solve[w == λ1 v1 + λ2 v2 + λ3 v3, {λ1, λ2, λ3}]
```

{{λ1 → 1, λ2 → -1, λ3 → 3}}

4

```
Remove["Global`*"]
```

a

```
a = {10, 10, 10}; e1 = {1, 0, 0}; e2 = {0, 1, 0}; e3 = {0, 0, 1};
LaengeHalbdiagonale = Norm[a]
```

$10 \sqrt{3}$

```
N[%]
```

17.3205

Winkel Diagonale-Achse

```
WinkelDiagonaleAchse = ArcCos[a.e1 / (Norm[a] Norm[e1])]
```

$\text{ArcCos}\left[\frac{1}{\sqrt{3}}\right]$

```
N[%]
```

0.955317

```
α = N[%] / Degree
```

54.7356

Test mit Richtungscosinusen o.k.:

```
solv = Solve[3 Cos[x]^2 == 1, {x}] // Flatten
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. Mehr…

$\left\{x \to -\text{ArcCos}\left[-\frac{1}{\sqrt{3}}\right],\ x \to \text{ArcCos}\left[-\frac{1}{\sqrt{3}}\right],\ x \to -\text{ArcCos}\left[\frac{1}{\sqrt{3}}\right],\ x \to \text{ArcCos}\left[\frac{1}{\sqrt{3}}\right]\right\}$

```
solv4 = x /. solv[[4]]
```

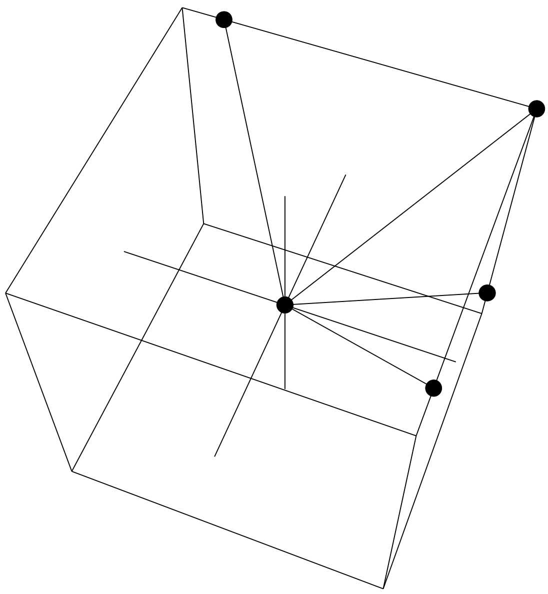
$\text{ArcCos}\left[\frac{1}{\sqrt{3}}\right]$

```
WinkelDiagonaleAchse == solv4
```

True

Skizze Würfel:

```
P1 = {1, 1, 1}; P2 = {-1, 1, 1}; P3 = {-1, -1, 1}; P4 = {1, -1, 1};
P5 = {1, 1, -1}; P6 = {-1, 1, -1}; P7 = {-1, -1, -1}; P8 = {1, -1, -1};
x1 = {1, 0, 0}; x2 = {-1, 0, 0}; y1 = {0, 1, 0};
y2 = {0, -1, 0}; z1 = {0, 0, 1}; z2 = {0, 0, -1};
linX = Line[{x1, x2}];
linY = Line[{y1, y2}];
linZ = Line[{z1, z2}];
linO1 = Line[{{0, 0, 0}, {-0.75, 1, 1}}];
lin02 = Line[{{0, 0, 0}, {1, 1, -0.75}}];
lin03 = Line[{{0, 0, 0}, {1, -0.75, 1}}];
lin04 = Line[{{0, 0, 0}, P1}];
linP = {PointSize[0.03], Point[P1], Point[{-0.75, 1, 1}],
   Point[{1, 1, -0.75}], Point[{1, -0.75, 1}], Point[{0, 0, 0}],
   Line[{P1, P2, P3, P4, P1, P5, P6, P2, P6, P7, P3, P7, P8, P4, P8, P5}],
   linX, linY, linZ, linO1, lin02, lin03, lin04};
Show [Graphics3D[linP ],
  (*ViewPoint->{-1.424, 4.258, 2.660}*)
  ViewPoint -> {0.614, -1.560, 2.660}, Boxed → False];
```



Winkel zwischen zwei Diagonalen:

```
WinkelDiagonaleDiagonale = ArcCos[P1.P2 / (Norm[P1] Norm[P2])]
```

$\text{ArcCos}\left[\frac{1}{3}\right]$

```
N[%]
```

1.23096

```
β = N[%] / Degree
```

70.5288

Winkel Kanten-Diagonalen:

```
ArcCos[P1.(P1 - P2) / (Norm[P1] Norm[P1 - P2])]
```

$\text{ArcCos}\left[\frac{1}{\sqrt{3}}\right]$

```
N[%]
```

0.955317

```
δ = N[%] / Degree
```

54.7356

Nochmals Winkel Kanten-Diagonalen:

```
γ = (180 - β) / 2
```

54.7356

```
WinkelKanteDiagonale = Pi - ArcCos[P1.(P2 - P1) / (Norm[P1] Norm[P2 - P1] )]
```

$\pi - \text{ArcCos}\left[-\frac{1}{\sqrt{3}}\right]$

```
N[%]
```

0.955317

```
δ = N[%] / Degree
```

54.7356

b

Winkel Kante Achse

Diagonalwinkel mit Achse : $\text{ArcCos}\left[\frac{1}{\sqrt{3}}\right]$,

Winkel Kanten − Diagonalen : $\pi - \text{ArcCos}\left[-\frac{1}{\sqrt{3}}\right]$ oder $\text{ArcCos}\left[\frac{1}{\sqrt{3}}\right]$

$\left\{\pi - \text{ArcCos}\left[-\frac{1}{\sqrt{3}}\right], \text{ArcCos}\left[\frac{1}{\sqrt{3}}\right]\right\}$ // N

{0.955317, 0.955317}

```
WinkelKanteAchse = Pi - WinkelKanteDiagonale - WinkelDiagonaleAchse
```

$\text{ArcCos}\left[-\frac{1}{\sqrt{3}}\right] - \text{ArcCos}\left[\frac{1}{\sqrt{3}}\right]$

```
N[%]
```

1.23096

```
WinkelKanteAchse = Pi - 2 WinkelDiagonaleAchse // Simplify
```

$\pi - 2\,\text{ArcCos}\left[\frac{1}{\sqrt{3}}\right]$

```
N[%]
```

1.23096

```
N[%] / Degree
```

70.5288

```
(Sin[WinkelKanteAchse]) // N
```

0.942809

```
LaengeKanteAchse =
 LaengeHalbdiagonale / Sin[WinkelKanteAchse] * Sin[WinkelDiagonaleAchse]
```

$10\sqrt{2}\,\text{Csc}\left[2\,\text{ArcCos}\left[\frac{1}{\sqrt{3}}\right]\right]$

```
N[%]
```

15.

```
VolumenEckstueck1 = LaengeKanteAchse^3 / 6
```

$\frac{1000}{3}\sqrt{2}\,\text{Csc}\left[2\,\text{ArcCos}\left[\frac{1}{\sqrt{3}}\right]\right]^3$

```
N[%]
```

562.5

```
VKoerper = 2 VolumenEckstueck1
```

$\frac{2000}{3}\sqrt{2}\,\text{Csc}\left[2\,\text{ArcCos}\left[\frac{1}{\sqrt{3}}\right]\right]^3$

```
N[%]
```

1125.

Andere Variante

```
v1[t_] := a - t e1; v2[t_] := a - t e2; Solve[v1[t].v2[t] == 0, {t}]
```

{{t → 15}}

```
v1[t].v2[t]
```

100 + 20 (10 - t)

```
v1[15]
```

{-5, 10, 10}

```
v2[15]
```

{10, -5, 10}

```
Norm[v1[15]]
```

15

```
{15 e1, 15 e2, 15 e3}
```

{{15, 0, 0}, {0, 15, 0}, {0, 0, 15}}

```
(15 e1 - a) . (15 e2 - a)
```

0

```
(15 e1 - a) . (15 e3 - a)
```

0

```
(15 e2 - a) . (15 e3 - a)
```

0

```
VolumenEckstueck2 = 15^3 / 6
```

$\frac{1125}{2}$

```
N[%]
```

562.5

```
VolumenKoerper = 2 VolumenEckstueck2
```

1125

Kontrollen:

```
WinkelKanteAchse = ArcCos[v1[15].e1 / Norm[v1[15]]]
```

ArcCos$\left[-\frac{1}{3}\right]$

```
N[%]
```

1.91063

```
N[%] / Degree
```

109.471

```
180 - %
```

70.5288

```
Sin[% Degree]
```

0.942809

```
Sin[%]
```

```
0.809212
```

```
e[λ_] := λ e1; e[λ].(a - e[λ]) == 0
```

```
(10 - λ) λ == 0
```

```
Solve[(10 - λ) λ == 0, {λ}]
```

```
{{λ → 0}, {λ → 10}}
```

c

```
kantenlaenge = 2 LaengeHalbdiagonale / Sqrt[3]
```

```
20
```

d

```
VolumenKoerper = 2 VolumenEckstueck2
```

```
1125
```

5

```
Remove["Global`*"]
```

```
p1 = {4, 1}; p2 = {2, 3}; φ = 68.44 Degree;
```

a

```
m = {{Cos[φ], -Sin[φ]}, {Sin[φ], Cos[φ]}}; m // MatrixForm
```

$$\begin{pmatrix} 0.367475 & -0.930033 \\ 0.930033 & 0.367475 \end{pmatrix}$$

```
p3 = m.p2
```

```
{-2.05515, 2.96249}
```

b

```
Det[{p2 - p1, p2 + (p3 - p1)}]
```

```
-1.81469
```

6

```
Remove["Global`*"]

r1 = {1, 0, 1}; r2 = {1, 2, -1}; a1 = {4, -1, 1}; a2 = {-1, -1, 6}; p0 = {10, 1, -2};
```

a

```
Vol = Det[{a1, a2, r2 - r1}]
```

-40

> Schneiden sich nicht

```
Solve[a1 == α a2, {α}]
```

{}

> windschief

b

```
d = Vol / (Norm[Cross[a1, a2]])
```

$-\frac{8}{3\sqrt{3}}$

```
N[%]
```

-1.5396

c

```
d = Det[{a1, a2, p0 - r1}] / (Norm[Cross[a1, a2]])
```

$-\frac{11}{3\sqrt{3}}$

```
N[%]
```

-2.11695

7

```
Remove["Global`*"]

p1 = {4, 1}; p2 = {2, 3}; p3 = {-6, -2};
```

a

```
k[x_, y_, r_] := ({x, y} - p1).({x, y} - p1) - r^2;
k[vec_, r_] := (vec - p1).(vec - p1) - r^2
```

```
k[p2, r]
```

$8 - r^2$

```
Solve[k[p2, r] == 0, {r}]
```

$\{\{r \to -2\sqrt{2}\}, \{r \to 2\sqrt{2}\}\}$

```
N[%]
```

{{r → -2.82843}, {r → 2.82843}}

```
k[vec_] := k[vec, 2 Sqrt[2]]
```

```
p4 = (p3 + p1) / 2
```

$\{-1, -\frac{1}{2}\}$

```
r1 = Norm[p1 - p4]
```

$\frac{\sqrt{109}}{2}$

```
N[%]
```

5.22015

```
k1[vec_] := (vec - p4).(vec - p4) - r1^2
```

```
{k[{x, y}] == 0, k1[{x, y}] == 0}
```

$\{-8 + (-4 + x)^2 + (-1 + y)^2 == 0, -\frac{109}{4} + (1 + x)^2 + \left(\frac{1}{2} + y\right)^2 == 0\}$

```
Solve[{k[{x, y}] == 0, k1[{x, y}] == 0}, {x, y}]
```

$$\{\{x \to \frac{2}{109}(178 - 3\sqrt{202}), y \to \frac{5}{109}(17 + 4\sqrt{202})\},$$
$$\{x \to \frac{2}{109}(178 + 3\sqrt{202}), y \to \frac{5}{109}(17 - 4\sqrt{202})\}\}$$

```
solv = N[%]
```

{{x → 2.48371, y → 3.38765}, {x → 4.0484, y → -1.82801}}

```
solv[[1]]
```

{x → 2.48371, y → 3.38765}

```
p5 = {x, y} /. solv[[1]]; T1 = p5
```

{2.48371, 3.38765}

```
p6 = {x, y} /. solv[[2]]; T2 = p6
```

```
{4.0484, -1.82801}
```

```
<< Graphics`ImplicitPlot`
```

```
pl = ImplicitPlot[{k[{x, y}] == 0, k1[{x, y}] == 0 }, {x, -7, 7}];
```

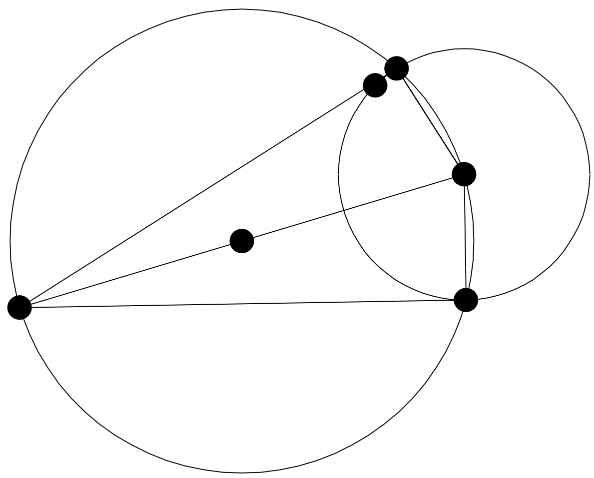
```
Prepend[Map[Point, {p1, p2, p3, p4, p5, p6}], PointSize[0.04]]
```

$\{\text{PointSize}[0.04], \text{Point}[\{4, 1\}], \text{Point}[\{2, 3\}], \text{Point}[\{-6, -2\}], \text{Point}\left[\left\{-1, -\frac{1}{2}\right\}\right], \text{Point}[\{2.48371, 3.38765\}], \text{Point}[\{4.0484, -1.82801\}]\}$

```
lin = Show[Graphics[Line[{p5, p3, p1, p5, p1, p6, p3}]]];
```

```
gra = Show[Graphics[Prepend[Map[Point, {p1, p2, p3, p4, p5, p6}], PointSize[0.04]]],
   pl, AspectRatio → Automatic];
```

```
Show[gra, lin];
```



b

```
len = Norm[p3 - p6]
```

```
10.0499
```

```
Norm[p3 - p5]
```

```
10.0499
```

c

```
len^2 == (s + 1) * s
```

```
101. == s (1 + s)
```

```
solv = Solve[len^2 == (s + 1) * s, {s}]
```

```
{{s → -10.5623}, {s → 9.56231}}
```

```
s = s /. solv[[2]]
```

```
9.56231
```

```
s * (s + 1) == len^2
```

```
True
```