Lösungen

1

```
Remove["Global`*"]

v1[t1_] := {-2, 1, -1} + t1 {-1, 2, 1};
v2[t2_] := {1, 4, 2} + t2 {-2, 4, 2};

Solve[k {-1, 2, 1} == {-2, 4, 2}, {k}]

{{k → 2}}
```

==> Geraden parallel ==> Abstand:

```
a = Norm[Cross[(v1[0] - v2[0]), {-1, 2, 1}]] / Norm[{-1, 2, 1}]
```

$\sqrt{21}$

```
N[%]

4.58258
```

2

```
Remove["Global`*"]
```

in den Ursprung verschoben, Verschiebungsvektor:

```
vVersch = -{-1, 1, 2};

OQ = {5, 15, 1};
OQversch = OQ + vVersch

{6, 14, -1}
```

in den Ursprung verschoben, dann Spiegelungsmatrix

```
aVec = {1, 2, 1}; bVec = {2, 1, -1}; cVec = Cross[aVec, bVec]

{-3, 3, -3}

Φ[λ_, μ_] := -vVersch + λ aVec + μ bVec;
```

```
mX = Transpose[{aVec, bVec, cVec}]; mX // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & -3 \\ 2 & 1 & 3 \\ 1 & -1 & -3 \end{pmatrix}$$

```
Dλ = {{1, 0, 0}, {0, 1, 0}, {0, 0, -1}}; Dλ // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

```
mS = mX.Dλ.Inverse[mX]; mS // MatrixForm
```

$$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

```
PStrichVersch = mS.OQversch
```

{12, 8, 5}

```
LVersch = (PStrichVersch + OQversch) / 2
```

{9, 11, 2}

```
PStrich = PStrichVersch - vVersch
```

{11, 9, 7}

a

```
Lvec = LVersch - vVersch
```

{8, 12, 4}

Kontrolle ob Lvec in liegt :

```
Solve[Φ[λ, μ] == Lvec, {λ, μ}]
```

$\left\{\left\{\lambda \to \frac{13}{3}, \mu \to \frac{7}{3}\right\}\right\}$

```
(OQ - Lvec).(aVec)
```

0

```
(OQ - Lvec).(bVec)
```

0

b

```
PStrichStrich = Lvec + 2 (PStrich - Lvec)
```

{14, 6, 10}

3

```
Remove["Global`*"]
```

Vektornormierung

```
nor[m_] := Table[m[[k]] / Norm[m[[k]]], {k, 1, Length[m]}]
```

Daten

```
M1 = {{-11, 8}, {8, 1}}; M1 // MatrixForm
```

$$\begin{pmatrix} -11 & 8 \\ 8 & 1 \end{pmatrix}$$

```
M2 = {{2, -6}, {-6, -7}}; M2 // MatrixForm
```

$$\begin{pmatrix} 2 & -6 \\ -6 & -7 \end{pmatrix}$$

a

```
ew1 = Eigenvalues[M1]
```

{-15, 5}

b

```
ev1 = Eigenvectors[M1]
```

{{-2, 1}, {1, 2}}

```
nor[ev1]
```

$$\left\{\left\{-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\}, \left\{\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\}\right\}$$

```
nor[ev1] // N
```

{{-0.894427, 0.447214}, {0.447214, 0.894427}}

c

```
ew2 = Eigenvalues[M2]
```

{-10, 5}

d

```
ev2 = Eigenvectors[M2]
```

{{1, 2}, {-2, 1}}

```
nor[ev2]
```

$\left\{\left\{\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\}, \left\{-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\}\right\}$

```
nor[ev2] // N
```

{{0.447214, 0.894427}, {-0.894427, 0.447214}}

e

Ein Eigenwert gleich
Eigenvektoren gleich

f

```
M1.M2 // MatrixForm
```

$\begin{pmatrix} -70 & 10 \\ 10 & -55 \end{pmatrix}$

```
M2.M1 // MatrixForm
```

$\begin{pmatrix} -70 & 10 \\ 10 & -55 \end{pmatrix}$

Identisch beim Vertauschen von Faktoren

g

```
ew3 = Eigenvalues[M1.M2]
```

{-75, -50}

{-10, 5}, {-15,5}, {-75,-50}
5 * (-15) = -75
5 * (-10) = -50

h

```
ev3 = Eigenvectors[M1.M2]
```

{{-2, 1}, {1, 2}}

```
nor[ev3]
```

$\{\{-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\}, \{\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\}\}$

```
nor[ev3] // N
```

```
{{-0.894427, 0.447214}, {0.447214, 0.894427}}
```

Eigenvektoren wie bei M1 und bei M2

i

```
ew4 = Eigenvalues[M1.M2.M1]
```

```
{1125, -250}
```

{-15, 5}, {-10, 5}, {-15, 5}, {1125, -250}
(-15) * 5 * (-15) = 1125
5 * (-10) * 5 = -250

```
ev4 = Eigenvectors[M1.M2.M1]
```

```
{{-2, 1}, {1, 2}}
```

```
nor[ev4] // N
```

```
{{-0.894427, 0.447214}, {0.447214, 0.894427}}
```

4

```
Remove["Global`*"]
```

Vektornormierung

```
nor[m_] := Table[m[[k]] / Norm[m[[k]]], {k, 1, Length[m]}]
```

Daten

```
a1 = {1, 1, 1}; a2 = {1, -1, 1}; a3 = {1, 1, -1};
b1 = 2 a1;
OQ = {0, -2, 2};
```

```
A = Transpose[{a1, a2, a3}]; A // MatrixForm
```

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

```
B = Transpose[{b1, a2, a3}]; B // MatrixForm
```

$$\begin{pmatrix} 2 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -1 \end{pmatrix}$$

a

```
ew1 = Eigenvalues[A]
```

```
{-2, 2, -1}
```

b

```
ew2 = Eigenvalues[B]
```

$\left\{1+\sqrt{5},\ -2,\ 1-\sqrt{5}\right\}$

```
% // N
```

```
{3.23607, -2., -1.23607}
```

c

Der Eigenwert -2 ist gemeinsam

d

```
ev1 = Eigenvectors[A]
```

```
{{0, -1, 1}, {2, 1, 1}, {-1, 1, 1}}
```

```
nor[ev1] // N
```

```
{{0., -0.707107, 0.707107},
 {0.816497, 0.408248, 0.408248}, {-0.57735, 0.57735, 0.57735}}
```

```
ev2 = Eigenvectors[B]
```

$\left\{\left\{\frac{1}{2}\left(1+\sqrt{5}\right),\ 1,\ 1\right\},\ \{0,\ -1,\ 1\},\ \left\{\frac{1}{2}\left(1-\sqrt{5}\right),\ 1,\ 1\right\}\right\}$

```
nor[ev2] // N
```

```
{{0.752938, 0.465341, 0.465341},
 {0., -0.707107, 0.707107}, {-0.400447, 0.647936, 0.647936}}
```

Der Eigenvektor {0,-1,1} zum Eigenwert -2 ist gemeinsam

e

```
ew3 = Eigenvalues[A.B]
```

$\left\{2\left(2+\sqrt{2}\right),\ 4,\ 2\left(2-\sqrt{2}\right)\right\}$

```
% // N
```

{6.82843, 4., 1.17157}

```
ew3 = Eigenvalues[B.A]
```

$\left\{2\left(2+\sqrt{2}\right),\ 4,\ 2\left(2-\sqrt{2}\right)\right\}$

```
% // N
```

{6.82843, 4., 1.17157}

Gleiche Eigenwerte

f

```
Apply[Plus, Eigenvalues[A]]
```

-1

```
Apply[Plus, Eigenvalues[B]]
```

0

```
Apply[Plus, Eigenvalues[A.B]] // Simplify
```

12

```
Apply[Plus, Eigenvalues[B.A]] // Simplify
```

12

Gleiche Eigenwerte von A.B und B.A ==> gleiche Summen

g

```
Apply[Times, Eigenvalues[A]]
```

4

```
Apply[Times, Eigenvalues[B]] // Simplify
```

8

```
Apply[Times, Eigenvalues[A.B]] // Simplify
```

32

```
Apply[Times, Eigenvalues[B.A]] // Simplify
```

```
32
```

Gleiche Eigenwerte von A mal Eigenwerte von B gleich Eigenwerte von B.A oder A.B

h

```
Det[A]
```

```
4
```

```
Det[B]
```

```
8
```

```
Det[A.B]
```

```
32
```

```
Det[B.A]
```

```
32
```

Die Determinante ist das Produkt der Eigenwerte: Resultate der vorangehenden Teilaufgabe

i

```
OP1 = A.OQ
```

```
{0, 4, -4}
```

```
OP2 = B.OQ
```

```
{0, 4, -4}
```

OQ ist Eigenvektor von A und von B !!!!

5

```
Remove["Global`*"]
```

Daten

```
v1 = -{-1, 3, 1};  v2 = -{1, -4, 0};
OP = {-1, 2, 3};
v3 = Cross[v1, v2]
```

```
{4, 1, 1}
```

```
OP = {-1, 2, 3};
```

a

```
mX = Transpose[{v1, v2, v3}]; mX // MatrixForm
```

$$\begin{pmatrix} 1 & -1 & 4 \\ -3 & 4 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

```
Dλ = {{1, 0, 0}, {0, 1, 0}, {0, 0, -1}}; Dλ // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

```
mS = mX.Dλ.Inverse[mX]; mS // MatrixForm
```

$$\begin{pmatrix} -\frac{7}{9} & -\frac{4}{9} & -\frac{4}{9} \\ -\frac{4}{9} & \frac{8}{9} & -\frac{1}{9} \\ -\frac{4}{9} & -\frac{1}{9} & \frac{8}{9} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} -0.777778 & -0.444444 & -0.444444 \\ -0.444444 & 0.888889 & -0.111111 \\ -0.444444 & -0.111111 & 0.888889 \end{pmatrix}$$

b

```
OP1 = mS.OP
```

$$\left\{-\frac{13}{9}, \frac{17}{9}, \frac{26}{9}\right\}$$

```
N[%]
```

{-1.44444, 1.88889, 2.88889}

c

```
Dreh[φ_] := {{Cos[φ], -Sin[φ], 0}, {Sin[φ], Cos[φ], 0}, {0, 0, 1}};
Dreh[Pi / 5] // MatrixForm
```

$$\begin{pmatrix} \frac{1}{4}\left(1+\sqrt{5}\right) & -\frac{1}{2}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)} & 0 \\ \frac{1}{2}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)} & \frac{1}{4}\left(1+\sqrt{5}\right) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} 0.809017 & -0.587785 & 0. \\ 0.587785 & 0.809017 & 0. \\ 0. & 0. & 1. \end{pmatrix}$$

d

```
OP2 = Dreh[Pi / 5].OP1
```

$$\left\{-\frac{17}{18}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}-\frac{13}{36}\left(1+\sqrt{5}\right),\ -\frac{13}{18}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}+\frac{17}{36}\left(1+\sqrt{5}\right),\ \frac{26}{9}\right\}$$

```
N[%]
```

{-2.27884, 0.67912, 2.88889}

e

```
OP3 = mS.OP2 // N
```

{0.18665, 1.29549, 3.50526}

f

```
Dλ1 = {{1, 0, 0}, {0, 1, 0}, {0, 0, 0}}; Dλ1 // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
mP = mX.Dλ1.Inverse[mX]; mP // MatrixForm
```

$$\begin{pmatrix} \frac{1}{9} & -\frac{2}{9} & -\frac{2}{9} \\ -\frac{2}{9} & \frac{17}{18} & -\frac{1}{18} \\ -\frac{2}{9} & -\frac{1}{18} & \frac{17}{18} \end{pmatrix}$$

```
N[mP] // MatrixForm
```

$$\begin{pmatrix} 0.111111 & -0.222222 & -0.222222 \\ -0.222222 & 0.944444 & -0.0555556 \\ -0.222222 & -0.0555556 & 0.944444 \end{pmatrix}$$

g

```
mP.OP3 // N
```

{-1.0461, 0.987306, 3.19708}

6

```
Remove["Global`*"]
```

```
transl = {{1, 0, 5}, {0, 1, 8}, {0, 0, 1}};
transl // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{pmatrix}$$

```
transl.{0, 0, 1}
```

{5, 8, 1}

```
transl.{-5, -8, 1}
```

{0, 0, 1}

7

```
Remove["Global`*"]
```

$A.(A+X).A+A+A^{-1} = A.A^{T}+E$

$\Longrightarrow\ A^{-1}.A.(A+X).A.A^{-1}+A^{-1}.A.A^{-1}+A^{-1}.A^{-1}.A^{-1} = A^{-1}.A.A^{T}.A^{-1}+A^{-1}.E.A^{-1}$

$\Longrightarrow\ A+X+A^{-1}+A^{-3} = A^{T}.A^{-1}+A^{-2}$

$\Longrightarrow\ X = A^{T}.A^{-1}+A^{-2}-A-A^{-1}-A^{-3} = A^{T}.A^{-1} - A - A^{-1} + A^{-2} - A^{-3}$