Lösungen

1

```
Remove["Global`*"]

EVNorm[Matrix_] :=
 Transpose[Table[N[Transpose[Matrix][[k]] / Norm[Transpose[Matrix][[k]]]],
   {k, 1, Length[Transpose[Matrix]]}]]

X = {{0, 1, -2, 1}, {1, 2, 0, 1}, {1, 1, 3, 0}, {-2, 0, 4, 1}};
Dl = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}};
M = X.Dl.Inverse[X];
MatrixForm[M]
```

$$\begin{pmatrix} -27 & 24 & -20 & 2 \\ -32 & 29 & -24 & 2 \\ -6 & 6 & -5 & 0 \\ -40 & 36 & -32 & 3 \end{pmatrix}$$

```
(* MatrixForm[M]//TeXForm *)

h1 = Transpose[X][[1]]; h2 = Transpose[X][[2]];
h3 = Transpose[X][[3]]; h4 = Transpose[X][[4]];

{h1, h2, h3, h4}
```

```
{{0, 1, 1, -2}, {1, 2, 1, 0}, {-2, 0, 3, 4}, {1, 1, 0, 1}}
```

```
X // MatrixForm
```

$$\begin{pmatrix} 0 & 1 & -2 & 1 \\ 1 & 2 & 0 & 1 \\ 1 & 1 & 3 & 0 \\ -2 & 0 & 4 & 1 \end{pmatrix}$$

```
Det[X]
```

```
1
```

```
v1 = {-4, 0, -3, 5}; v2 = {-3, 4, 0, 2};  v3 = {1, 2, -4, 0};
v4 = {-1, 2, 3, 1}; Vmatr = {v1, v2, v3, v4};

Det[Vmatr]
```

```
148
```

■ a

```
Eigenvalues[M]
```

```
{-1, -1, 1, 1}
```

■ b Eigenvektoren zu doppelten Eigenwerten nicht eindeutig

```
Eigenvectors[M] // Transpose // MatrixForm
```

$$\begin{pmatrix} 1 & -6 & 1 & 1 \\ 1 & -4 & 1 & 2 \\ 0 & 3 & 0 & 1 \\ 1 & 0 & 2 & 0 \end{pmatrix}$$

```
EVNorm[Eigenvectors[M] // Transpose] // MatrixForm
```

$$\begin{pmatrix} 0.57735 & -0.768221 & 0.408248 & 0.408248 \\ 0.57735 & -0.512148 & 0.408248 & 0.816497 \\ 0. & 0.384111 & 0. & 0.408248 \\ 0.57735 & 0. & 0.816497 & 0. \end{pmatrix}$$

```
EV = Eigenvectors[M]; ev1 = EV[[1]]; ev2 = EV[[2]]; ev3 = EV[[3]]; ev4 = EV[[4]];
{ev1, ev2, ev3, ev4}
```

```
{{1, 1, 0, 1}, {-6, -4, 3, 0}, {1, 1, 0, 2}, {1, 2, 1, 0}}
```

```
{h1, h2, h3, h4}
```

```
{{0, 1, 1, -2}, {1, 2, 1, 0}, {-2, 0, 3, 4}, {1, 1, 0, 1}}
```

```
Solve[x1 ev3 + x2 ev4 == h1, {x1, x2}]
```

```
{{x1 → -1, x2 → 1}}
```

```
Solve[x1 ev3 + x2 ev4 == h2, {x1, x2}]
```

```
{{x1 → 0, x2 → 1}}
```

```
Solve[x1 ev1 + x2 ev2 == h3, {x1, x2}]
```

```
{{x1 → 4, x2 → 1}}
```

```
Solve[x1 ev1 + x2 ev2 == h4, {x1, x2}]
```

```
{{x1 → 1, x2 → 0}}
```

- **c**

```
Inverse[M] // MatrixForm
```

$$\begin{pmatrix} -27 & 24 & -20 & 2 \\ -32 & 29 & -24 & 2 \\ -6 & 6 & -5 & 0 \\ -40 & 36 & -32 & 3 \end{pmatrix}$$

```
M == Inverse[M]
```

```
True
```

```
Eigenvalues[Inverse[M]]
```

```
{-1, -1, 1, 1}
```

- **d Eigenvektoren zu doppelten Eigenwerten nicht eindeutig**

```
Eigenvectors[Inverse[M]] // Transpose // MatrixForm
```

$$\begin{pmatrix} 1 & -6 & 1 & 1 \\ 1 & -4 & 1 & 2 \\ 0 & 3 & 0 & 1 \\ 1 & 0 & 2 & 0 \end{pmatrix}$$

```
EVNorm[Eigenvectors[Inverse[M]] // Transpose] // MatrixForm
```

$$\begin{pmatrix} 0.57735 & -0.768221 & 0.408248 & 0.408248 \\ 0.57735 & -0.512148 & 0.408248 & 0.816497 \\ 0. & 0.384111 & 0. & 0.408248 \\ 0.57735 & 0. & 0.816497 & 0. \end{pmatrix}$$

- **e**

```
Eigenvalues[Transpose[M]]
```

```
{-1, -1, 1, 1}
```

f

```
Eigenvectors[Transpose[M]] // Transpose // MatrixForm
```

$$\begin{pmatrix} -4 & 1 & 2 & 3 \\ 2 & -1 & -3 & -3 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

```
EVNorm[Eigenvectors[Transpose[M]] // Transpose] // MatrixForm
```

$$\begin{pmatrix} -0.872872 & 0.57735 & 0.534522 & 0.639602 \\ 0.436436 & -0.57735 & -0.801784 & -0.639602 \\ 0. & 0.57735 & 0. & 0.426401 \\ 0.218218 & 0. & 0.267261 & 0. \end{pmatrix}$$

g

```
W = Transpose[Vmatr]; w // MatrixForm
```

w

```
VolW = Det[W]
```

148

h

```
Det[M]
```

1

```
VolMW = Det[M.W]
```

148

```
VolMW / VolW
```

1

2

a

```
X = {v1, v2, v3, v4} // Transpose; X // MatrixForm
```

$$\begin{pmatrix} -4 & -3 & 1 & -1 \\ 0 & 4 & 2 & 2 \\ -3 & 0 & -4 & 3 \\ 5 & 2 & 0 & 1 \end{pmatrix}$$

```
Dλ = {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -2, 0}, {0, 0, 0, 2}}; Dλ // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

```
A = X.Dλ.Inverse[X]; A // MatrixForm
```

$$\begin{pmatrix} -\frac{145}{37} & -\frac{5}{74} & -\frac{19}{37} & -\frac{157}{37} \\ \frac{108}{37} & -\frac{16}{37} & \frac{56}{37} & \frac{120}{37} \\ \frac{249}{37} & \frac{177}{74} & \frac{65}{74} & \frac{393}{74} \\ \frac{99}{37} & -\frac{17}{74} & \frac{41}{74} & \frac{257}{74} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} -3.91892 & -0.0675676 & -0.513514 & -4.24324 \\ 2.91892 & -0.432432 & 1.51351 & 3.24324 \\ 6.72973 & 2.39189 & 0.878378 & 5.31081 \\ 2.67568 & -0.22973 & 0.554054 & 3.47297 \end{pmatrix}$$

```
74 A // MatrixForm
```

$$\begin{pmatrix} -290 & -5 & -38 & -314 \\ 216 & -32 & 112 & 240 \\ 498 & 177 & 65 & 393 \\ 198 & -17 & 41 & 257 \end{pmatrix}$$

- **b**

```
OQStrich = A.(v1 + v2 + v3 + v4)
```

{-5, -4, 11, 5}

- **c**

```
OQStrichStrich = (A + Transpose[A]).OQStrich
```

$$\left\{\frac{3268}{37}, \frac{3495}{74}, \frac{72}{37}, \frac{3516}{37}\right\}$$

```
N[%]
```

{88.3243, 47.2297, 1.94595, 95.027}

3

```
Remove["Global`*"]
```

- **a**

```
a = {1, -1, -2}; b = {2, 1, -4}; c = {1, 1, 1};
```

```
X = Transpose[{a, b, c}]; X // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ -2 & -4 & 1 \end{pmatrix}$$

```
Dλ = {{1, 0, 0}, {0, 1, 0}, {0, 0, -1}}; Dλ // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

```
B = X.Dλ.Inverse[X]; B // MatrixForm
```

$$\begin{pmatrix} -\frac{1}{3} & 0 & -\frac{2}{3} \\ -\frac{4}{3} & 1 & -\frac{2}{3} \\ -\frac{4}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} -0.333333 & 0. & -0.666667 \\ -1.33333 & 1. & -0.666667 \\ -1.33333 & 0. & 0.333333 \end{pmatrix}$$

```
3 B // MatrixForm
```

$$\begin{pmatrix} -1 & 0 & -2 \\ -4 & 3 & -2 \\ -4 & 0 & 1 \end{pmatrix}$$

b

```
B.{2, 4, 5}
```

```
{-4, -2, -1}
```

c

```
Dλ^100
```

```
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

```
Bhoch100 = X.(Dλ^100).Inverse[X]; Bhoch100 // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
Bhoch100 == IdentityMatrix[3]
```

```
True
```

4

$$(U\,W^{-1})\,(U\,W^{-1})^{-1}\,(-X)\;U^T + E = \left(\left(\left(W^{-1}\right)^T U^T\right)^{-1}\right)^T \quad == > \quad (U\,W^{-1})\,(U\,W^{-1})^{-1} = E \text{ oder}$$

$$== > \quad U\,W^{-1}\,W\,U^{-1}\,(-X)\;U^T + E =$$

$$\left(\left(\left(U\,W^{-1}\right)^T\right)^{-1}\right)^T = \left(\left(\left(U\,W^{-1}\right)^T\right)^T\right)^{-1} = \left(U\,W^{-1}\right)^{-1} = W\,U^{-1}$$

$$== > \qquad (-X)\;U^T = W\,U^{-1} - E$$

$$== > \qquad (-X) = W\,U^{-1}\left(U^T\right)^{-1} - \left(U^T\right)^{-1} = W\,\left(U^T\,U\right)^{-1} - \left(U^T\right)^{-1}$$

$$== > \qquad X = -W\,\left(U^T\,U\right)^{-1} + \left(U^T\right)^{-1} = \left(-W\,U^{-1} + E\right)\left(U^T\right)^{-1}$$

```
U = {{-37, 32}, {20, -19}}; W = {{57, -5}, {32, -28}};
{U // MatrixForm, W // MatrixForm}
```

$$\left\{\begin{pmatrix} -37 & 32 \\ 20 & -19 \end{pmatrix}, \begin{pmatrix} 57 & -5 \\ 32 & -28 \end{pmatrix}\right\}$$

```
X = (-W.Inverse[U] + IdentityMatrix[2]).Inverse[Transpose[U]]
```

$$\left\{\left\{-\frac{72\,322}{3969}, -\frac{81\,563}{3969}\right\}, \left\{-\frac{848}{1323}, -\frac{949}{1323}\right\}\right\}$$

```
X // MatrixForm
```

$$\begin{pmatrix} -\frac{72\,322}{3969} & -\frac{81\,563}{3969} \\ -\frac{848}{1323} & -\frac{949}{1323} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} -18.2217 & -20.55 \\ -0.640967 & -0.717309 \end{pmatrix}$$

5

```
Remove["Global`*"]
```

a

```
OP1 = {0, 1, 1}; OP2 = {1, 0, -1}; OP3 = {2, 1, 0};
OP4 = {2, 6, 1}; OP5 = {-1, 5, 8}; OP6 = {-2, 12, 0};
```

```
G1 = {OP1, OP2, OP3} // Transpose;
G2 = {OP4, OP5, OP6} // Transpose;
Det[G1]
```

-1

```
Det[G2]
```

-290

```
G.G1 == G2
```

G.{{0, 1, 2}, {1, 0, 1}, {1, -1, 0}} == {{2, -1, -2}, {6, 5, 12}, {1, 8, 0}}

```
G = G2.Inverse[G1];  G // MatrixForm
```

$$\begin{pmatrix} -3 & 4 & -2 \\ 1 & 10 & -4 \\ -9 & 18 & -17 \end{pmatrix}$$

- **b**

```
Dreh[ϕ_] := {{Cos[ϕ], -Sin[ϕ], 0}, {Sin[ϕ], Cos[ϕ], 0}, {0, 0, 1}}; Dreh[ϕ] // MatrixForm
```

$$\begin{pmatrix} \mathrm{Cos}[\phi] & -\mathrm{Sin}[\phi] & 0 \\ \mathrm{Sin}[\phi] & \mathrm{Cos}[\phi] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
Dreh[30 Degree] // MatrixForm
```

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
Dreh[30 Degree] // N // MatrixForm
```

$$\begin{pmatrix} 0.866025 & -0.5 & 0. \\ 0.5 & 0.866025 & 0. \\ 0. & 0. & 1. \end{pmatrix}$$

- **c**

```
OP7 = Dreh[30 Degree].OP1
```

$$\left\{-\frac{1}{2}, \frac{\sqrt{3}}{2}, 1\right\}$$

```
Dreh[30 Degree].OP1 // N
```

{-0.5, 0.866025, 1.}

- **d**

```
G.OP7 // MatrixForm
```

$$\begin{pmatrix} -\frac{1}{2} + 2\sqrt{3} \\ -\frac{9}{2} + 5\sqrt{3} \\ -\frac{25}{2} + 9\sqrt{3} \end{pmatrix}$$

```
G.OP7 // N // MatrixForm
```

$$\begin{pmatrix} 2.9641 \\ 4.16025 \\ 3.08846 \end{pmatrix}$$

6

```
Remove["Global`*"]
```

a

```
Hmatrix = {{0, -2, 1}, {1, 0, -1}, {2, 2, 0}}; Det[Hmatrix]
```

6

```
Smatrix = Hmatrix + Transpose[Hmatrix]; Smatrix // MatrixForm
```

$$\begin{pmatrix} 0 & -1 & 3 \\ -1 & 0 & 1 \\ 3 & 1 & 0 \end{pmatrix}$$

Symmetrische Matrix

b

```
Det[Smatrix]
```

-6

```
Eigenvalues[Hmatrix]
```

{Root[-6 + 2 #1 + #1^3 &, 3], Root[-6 + 2 #1 + #1^3 &, 2], Root[-6 + 2 #1 + #1^3 &, 1]}

```
Eigenvalues[Hmatrix] // N
```

{-0.728082 + 1.89481 ⅈ, -0.728082 - 1.89481 ⅈ, 1.45616}

```
Inverse[Hmatrix] // MatrixForm
```

$$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{1}{6} \\ \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} 0.333333 & 0.333333 & 0.333333 \\ -0.333333 & -0.333333 & 0.166667 \\ 0.333333 & -0.666667 & 0.333333 \end{pmatrix}$$

```
Eigenvalues[Smatrix] // Simplify
```

$$\left\{\frac{1}{2}\left(-3-\sqrt{17}\right),\ 3,\ \frac{1}{2}\left(-3+\sqrt{17}\right)\right\}$$

```
Eigenvalues[Smatrix] // N
```

{-3.56155, 3., 0.561553}

```
Det[Smatrix - x IdentityMatrix[3]]
```

$-6 + 11\,x - x^3$

```
Solve[Det[Smatrix - x IdentityMatrix[3]] == 0, {x}] // Simplify
```

$$\left\{\{x \to 3\},\ \left\{x \to \frac{1}{2}\left(-3-\sqrt{17}\right)\right\},\ \left\{x \to \frac{1}{2}\left(-3+\sqrt{17}\right)\right\}\right\}$$

```
N[%]
```

{{x → 3.}, {x → -3.56155}, {x → 0.561553}}

c

```
a1 = {1, 1, 1};
Hmatrix.a1
```

{-1, 0, 4}

```
Transpose[Hmatrix].a1
```

{3, 0, 0}

```
Smatrix.a1
```

{2, 0, 4}

```
(Hmatrix + Transpose[Hmatrix]).a1
```

{2, 0, 4}

```
N[%]
```

{2., 0., 4.}

- **d**

```
A = Hmatrix + Inverse[Hmatrix]; A // MatrixForm
```

$$\begin{pmatrix} \frac{1}{3} & -\frac{5}{3} & \frac{4}{3} \\ \frac{2}{3} & -\frac{1}{3} & -\frac{5}{6} \\ \frac{7}{3} & \frac{4}{3} & \frac{1}{3} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} 0.333333 & -1.66667 & 1.33333 \\ 0.666667 & -0.333333 & -0.833333 \\ 2.33333 & 1.33333 & 0.333333 \end{pmatrix}$$

```
A.a1
```

$\left\{0, -\frac{1}{2}, 4\right\}$

7

```
Remove["Global`*"]
```

```
a = {-2, 2, 1};  OQ = {-1, 0, 4};
```

■ Matrixkonstruktion : Lokale Basis

```
(* Normiert einen Vektor *)
NVec[a_] := a / Norm[a];
(* Quadriert Komponenten eines Vektors *)
QVec[a_] := Table[a[[k]]^2, {k, 1, Length[a]}];
(* Numeriert Komponenten eines Vektors *)
QVecNr[a_] := Table[{k, a[[k]]^2}, {k, 1, Length[a]}];
(* Sucht die Nummer einer absolut maximal grossen Komponente *)
NrMaxQVec[a_] := Max[Table[If[a[[k]]^2 == Max[QVec[a]], k, 0], {k, 1, Length[a]}]];
(* Sucht die Nummer einer absolut minimal grossen Komponente *)
NrMinQVec[a_] :=
  Min[Table[If[a[[k]]^2 == Min[QVec[a]], k, Length[a] + 1], {k, 1, Length[a]}]];
b[a_, x_] := Table[ If[k == NrMaxQVec[a], 1, If[k == NrMinQVec[a], 0, x ] ]
     , {k, 1, Length[a]}];
solv = Solve[b[a, x].a == 0, {x}] // Flatten;
b[a_] := b[a, x] /. solv
e1 = {1, 0, 0}; e2 = {0, 1, 0}; e3 = {0, 0, 1};
 If[Element[NVec[a], Union[{e1, e2, e3}, -{e1, e2, e3}]],
  b[a_] := Cross[e1 + e2 + e3, NVec[a]], b[a] = b[a]];
basis[a_] := {NVec[a], NVec[b[a]], Cross[NVec[a], NVec[b[a]]]};
TrBasis[a_] := basis[a] // Transpose;
aVec1 = NVec[a]; aVec2 = NVec[b[a]];
aVec3 = Cross[NVec[a], NVec[b[a]]];
```

■ Kontrolle

```
basis[a]
```

$$\left\{\left\{-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right\}, \left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\}, \left\{-\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, -\frac{2\sqrt{2}}{3}\right\}\right\}$$

```
Cross[a, basis[a][[1]]]
```

{0, 0, 0}

```
basis[a][[1]].basis[a][[2]]
```

0

```
basis[a][[1]].basis[a][[3]]
```

0

```
basis[a][[2]].basis[a][[3]]
```

0

```
basis[a][[1]] // Norm
```

1

```
basis[a][[2]] // Norm
```

1

```
basis[a][[3]] // Norm
```

1

```
TrBasis[a].e1 == aVec1
```

True

```
TrBasis[a].e2 == aVec2
```

True

```
TrBasis[a].e3 == aVec3
```

True

■ Matrixzusammensetzung

```
Print[Inverse[TrBasis[a]] // MatrixForm];
mDrehung[ϕ_] := {{1, 0, 0}, {0, Cos[ϕ], -Sin[ϕ]}, {0, Sin[ϕ], Cos[ϕ]}};
mDrehung[Pi / 8];
Print[mDrehung[Pi / 8] // MatrixForm];
matrix[ϕ_] := TrBasis[a].mDrehung[ϕ].Inverse[TrBasis[a]];
Print[matrix[Pi / 8] // MatrixForm];
Print[matrix[Pi / 8] // N // MatrixForm];
```

$$\begin{pmatrix} -\frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{3\sqrt{2}} & \frac{1}{3\sqrt{2}} & -\frac{2\sqrt{2}}{3} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}\left[\frac{\pi}{8}\right] & -\text{Sin}\left[\frac{\pi}{8}\right] \\ 0 & \text{Sin}\left[\frac{\pi}{8}\right] & \text{Cos}\left[\frac{\pi}{8}\right] \end{pmatrix}$$

$$\begin{pmatrix} \frac{4}{9} - \frac{-\frac{\text{Cos}\left[\frac{\pi}{8}\right]}{3\sqrt{2}} - \frac{\text{Sin}\left[\frac{\pi}{8}\right]}{\sqrt{2}}}{3\sqrt{2}} + \frac{\frac{\text{Cos}\left[\frac{\pi}{8}\right]}{\sqrt{2}} - \frac{\text{Sin}\left[\frac{\pi}{8}\right]}{3\sqrt{2}}}{\sqrt{2}} & -\frac{4}{9} + \frac{-\frac{\text{Cos}\left[\frac{\pi}{8}\right]}{3\sqrt{2}} - \frac{\text{Sin}\left[\frac{\pi}{8}\right]}{\sqrt{2}}}{3\sqrt{2}} + \frac{\frac{\text{Cos}\left[\frac{\pi}{8}\right]}{\sqrt{2}} - \frac{\text{Sin}\left[\frac{\pi}{8}\right]}{3\sqrt{2}}}{\sqrt{2}} & -\frac{2}{9} - \frac{2}{3}\sqrt{2}\left(-\frac{\text{Cos}\left[\frac{\pi}{8}\right]}{3\sqrt{2}} - \frac{\text{Sin}\left[\frac{\pi}{8}\right]}{\sqrt{2}}\right) \\ -\frac{4}{9} - \frac{\frac{\text{Cos}\left[\frac{\pi}{8}\right]}{3\sqrt{2}} - \frac{\text{Sin}\left[\frac{\pi}{8}\right]}{\sqrt{2}}}{3\sqrt{2}} + \frac{\frac{\text{Cos}\left[\frac{\pi}{8}\right]}{\sqrt{2}} + \frac{\text{Sin}\left[\frac{\pi}{8}\right]}{3\sqrt{2}}}{\sqrt{2}} & \frac{4}{9} + \frac{\frac{\text{Cos}\left[\frac{\pi}{8}\right]}{3\sqrt{2}} - \frac{\text{Sin}\left[\frac{\pi}{8}\right]}{\sqrt{2}}}{3\sqrt{2}} + \frac{\frac{\text{Cos}\left[\frac{\pi}{8}\right]}{\sqrt{2}} + \frac{\text{Sin}\left[\frac{\pi}{8}\right]}{3\sqrt{2}}}{\sqrt{2}} & \frac{2}{9} - \frac{2}{3}\sqrt{2}\left(\frac{\text{Cos}\left[\frac{\pi}{8}\right]}{3\sqrt{2}} - \frac{\text{Sin}\left[\frac{\pi}{8}\right]}{\sqrt{2}}\right) \\ -\frac{2}{9} + \frac{2}{9}\text{Cos}\left[\frac{\pi}{8}\right] - \frac{2}{3}\text{Sin}\left[\frac{\pi}{8}\right] & \frac{2}{9} - \frac{2}{9}\text{Cos}\left[\frac{\pi}{8}\right] - \frac{2}{3}\text{Sin}\left[\frac{\pi}{8}\right] & \frac{1}{9} + \frac{8}{9}\text{Cos}\left[\frac{\pi}{8}\right] \end{pmatrix}$$

$$\begin{pmatrix} 0.957711 & -0.161392 & 0.238207 \\ 0.0937298 & 0.957711 & 0.272038 \\ -0.272038 & -0.238207 & 0.932337 \end{pmatrix}$$

■ Drehung

```
OQStrich = matrix[Pi / 8].OQ
```

$$\left\{-\frac{4}{9} + \frac{-\frac{\text{Cos}\left[\frac{\pi}{8}\right]}{3\sqrt{2}} - \frac{\text{Sin}\left[\frac{\pi}{8}\right]}{\sqrt{2}}}{3\sqrt{2}} - \frac{\frac{\text{Cos}\left[\frac{\pi}{8}\right]}{\sqrt{2}} - \frac{\text{Sin}\left[\frac{\pi}{8}\right]}{3\sqrt{2}}}{\sqrt{2}} + 4\left(-\frac{2}{9} - \frac{2}{3}\sqrt{2}\left(-\frac{\text{Cos}\left[\frac{\pi}{8}\right]}{3\sqrt{2}} - \frac{\text{Sin}\left[\frac{\pi}{8}\right]}{\sqrt{2}}\right)\right),\right.$$

$$\frac{4}{9} + \frac{\frac{\text{Cos}\left[\frac{\pi}{8}\right]}{3\sqrt{2}} - \frac{\text{Sin}\left[\frac{\pi}{8}\right]}{\sqrt{2}}}{3\sqrt{2}} - \frac{\frac{\text{Cos}\left[\frac{\pi}{8}\right]}{\sqrt{2}} + \frac{\text{Sin}\left[\frac{\pi}{8}\right]}{3\sqrt{2}}}{\sqrt{2}} + 4\left(\frac{2}{9} - \frac{2}{3}\sqrt{2}\left(\frac{\text{Cos}\left[\frac{\pi}{8}\right]}{3\sqrt{2}} - \frac{\text{Sin}\left[\frac{\pi}{8}\right]}{\sqrt{2}}\right)\right),$$

$$\left.\frac{2}{9} + 4\left(\frac{1}{9} + \frac{8}{9}\text{Cos}\left[\frac{\pi}{8}\right]\right) - \frac{2}{9}\text{Cos}\left[\frac{\pi}{8}\right] + \frac{2}{3}\text{Sin}\left[\frac{\pi}{8}\right]\right\}$$

```
OQStrich // N
```

{-0.00488434, 0.994422, 4.00139}

```
p1 = ParametricPlot3D[matrix[ϕ].OQ, {ϕ, 0, 2 Pi}];
```

```
p2 = ParametricPlot3D[ϕ aVec1, {ϕ, 0, 2 Pi}];
```

```
p3 = Graphics3D[{Sphere[OQ, 0.2], Sphere[OQStrich, 0.2]}];
Show[p1, p2, p3]
```