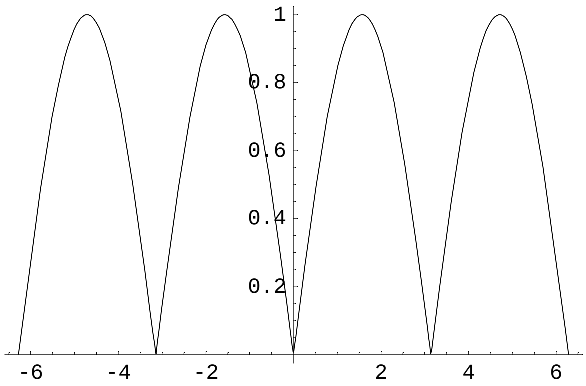
# Lösungen

## 1

```
Remove["Global`*"];
```

### a Gerade Funktion ==> Cosinusreihe

```
f1[t_] := Sin[t];  f1New[t_] := Sqrt[(f1[t])^2];
Plot[Evaluate[f1New[t]], {t, -2 Pi, 2 Pi}];
```




### b  Die ersten 6 Fourierkoeffizienten, Graphik

```
T = Pi;
cc = 0;
f1[t_] := Sin[t];  f1New[t_] := Sqrt[f1[t]^2]
ω = 2 Pi / T;
a[0] := 2 / T Integrate[f1[t], {t, cc, cc + T}];
a[k_] := 2 / T Integrate[f1[t] Cos[k ω t], {t, cc, cc + T}];
b[k_] := 2 / T Integrate[f1[t] Sin[k ω t], {t, cc, cc + T}];
c[k_] := 1 / T Integrate[f1[t] E^(-I k ω t), {t, cc, cc + T}];
ffInf[t_] := a[0] / 2 + Sum[a[n] Cos[n ω t] + b[n] Sin[n ω t], {n, 1, Infinity}];
fFh[t_, h_] := a[0] / 2 + Sum[a[n] Cos[n ω t] + b[n] Sin[n ω t], {n, 1, h}];
fFInf[t_] := Sum[c[n] E^(I n ω t), {n, -Infinity, Infinity}];
fFh[t_, h_] := Sum[c[n] E^(I n ω t), {n, -h, h}];
fFh[t, 6]
```

$$\frac{2}{\pi} - \frac{2\,e^{-2 i t}}{3\pi} - \frac{2\,e^{2 i t}}{3\pi} - \frac{2\,e^{-4 i t}}{15\pi} - \frac{2\,e^{4 i t}}{15\pi} - \frac{2\,e^{-6 i t}}{35\pi} - \frac{2\,e^{6 i t}}{35\pi} - \frac{2\,e^{-8 i t}}{63\pi} - \frac{2\,e^{8 i t}}{63\pi} - \frac{2\,e^{-10 i t}}{99\pi} - \frac{2\,e^{10 i t}}{99\pi} - \frac{2\,e^{-12 i t}}{143\pi} - \frac{2\,e^{12 i t}}{143\pi}$$

```
% // N
```

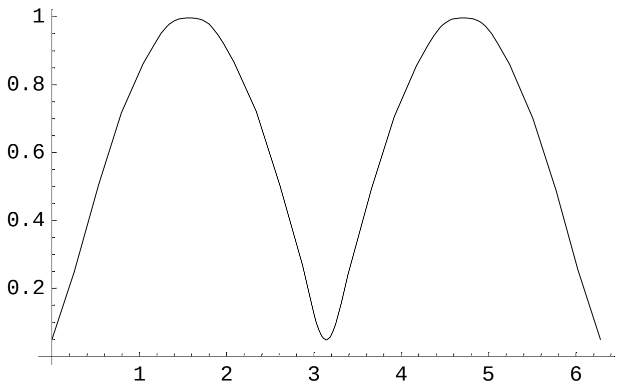
$$0.63662 - 0.212207\, 2.71828^{(0.-2.\, i)\, t} - 0.212207\, 2.71828^{(0.+2.\, i)\, t} - 0.0424413\, 2.71828^{(0.-4.\, i)\, t} - 0.0424413\, 2.71828^{(0.+4.\, i)\, t} - 0.0181891\, 2.71828^{(0.-6.\, i)\, t} - 0.0181891\, 2.71828^{(0.+6.\, i)\, t} - 0.0101051\, 2.71828^{(0.-8.\, i)\, t} - 0.0101051\, 2.71828^{(0.+8.\, i)\, t} - 0.0064305\, 2.71828^{(0.-10.\, i)\, t} - 0.0064305\, 2.71828^{(0.+10.\, i)\, t} - 0.00445189\, 2.71828^{(0.-12.\, i)\, t} - 0.00445189\, 2.71828^{(0.+12.\, i)\, t}$$

```
fFh[t_, 6] :=
```

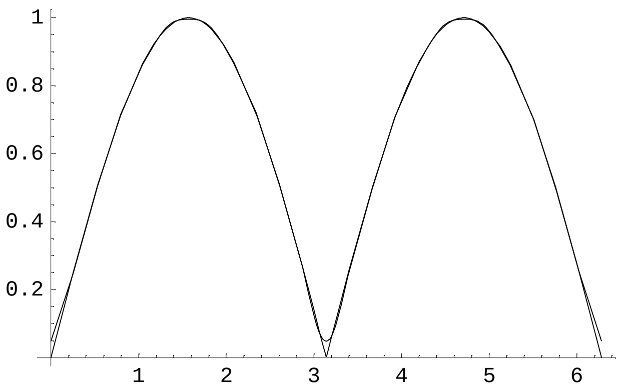
$$\frac{2}{\pi} - \frac{4\,\mathrm{Cos}[2\,t]}{3\,\pi} - \frac{4\,\mathrm{Cos}[4\,t]}{15\,\pi} - \frac{4\,\mathrm{Cos}[6\,t]}{35\,\pi} - \frac{4\,\mathrm{Cos}[8\,t]}{63\,\pi} - \frac{4\,\mathrm{Cos}[10\,t]}{99\,\pi} - \frac{4\,\mathrm{Cos}[12\,t]}{143\,\pi}$$

```
N[%]

Plot[fFh[t, 6], {t, 0, 2 Pi}];
```




```
Plot[{fFh[t, 6], f1New[t]}, {t, 0, 2 Pi}];
```




## c Die ersten 6 Fourierkoeffizienten von |Sin(t)| = $\sqrt{\mathrm{Sin}[t]^2}$ = f[t] ( ==> dieselbe Funktion!! )

```
a[0] / 2
```

$$\frac{2}{\pi}$$

```
a[k] /. {Cos[2 k π] → 1, Sin[2 k π] → 0}
```

$$\frac{4\,\mathrm{Cos}[k\,\pi]^2}{(1 - 4\,k^2)\,\pi}$$

```
b[k] /. { Cos[2 k π] → 1, Sin[2 k π] → 0}
```

0

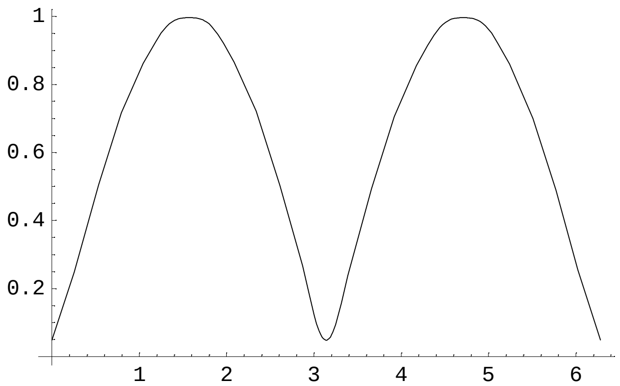
```
fF2h[t_, h_] := a[0] / 2 + Sum[a[n] Cos[n ω t] + b[n] Sin[n ω t], {n, 1, h}];
fF2h[t, 6]
```

$$\frac{2}{\pi} - \frac{4\,\mathrm{Cos}[2\,t]}{3\,\pi} - \frac{4\,\mathrm{Cos}[4\,t]}{15\,\pi} - \frac{4\,\mathrm{Cos}[6\,t]}{35\,\pi} - \frac{4\,\mathrm{Cos}[8\,t]}{63\,\pi} - \frac{4\,\mathrm{Cos}[10\,t]}{99\,\pi} - \frac{4\,\mathrm{Cos}[12\,t]}{143\,\pi}$$

```
N[%]
```

0.63662 − 0.424413 Cos[2. t] − 0.0848826 Cos[4. t] − 0.0363783 Cos[6. t] −
 0.0202102 Cos[8. t] − 0.012861 Cos[10. t] − 0.00890377 Cos[12. t]

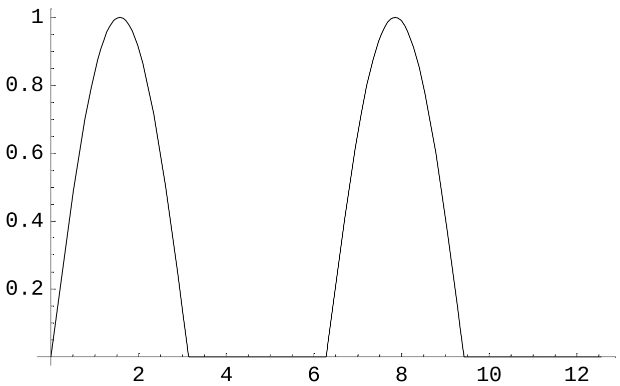
```
Plot[fF2h[t, 6], {t, 0, 2 Pi}];
```




## d Gleiche Funktion

```
f3[t_] := (Abs[Sin[t]] + Sin[t]) / 2
```

```
Plot[f3[t], {t, 0, 4 Pi}];
```



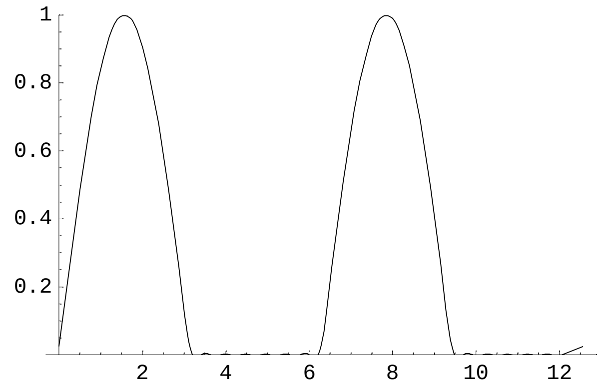

```
fF3h[t_, h_] := (fFh[t, h] + Sin[t]) / 2;
fF3h[t, 6]
```

$$\frac{1}{2}\left(\frac{2}{\pi} - \frac{4\,\mathrm{Cos}[2\,t]}{3\,\pi} - \frac{4\,\mathrm{Cos}[4\,t]}{15\,\pi} - \frac{4\,\mathrm{Cos}[6\,t]}{35\,\pi} - \frac{4\,\mathrm{Cos}[8\,t]}{63\,\pi} - \frac{4\,\mathrm{Cos}[10\,t]}{99\,\pi} - \frac{4\,\mathrm{Cos}[12\,t]}{143\,\pi} + \mathrm{Sin}[t]\right)$$

```
N[%]
```

```
0.5 (0.63662 - 0.424413 Cos[2. t] - 0.0848826 Cos[4. t] - 0.0363783 Cos[6. t] -
   0.0202102 Cos[8. t] - 0.012861 Cos[10. t] - 0.00890377 Cos[12. t] + Sin[t])
```

```
Plot[Evaluate[fF3h[t, 6]], {t, 0, 4 Pi}, PlotRange → {0, 1}];
```




## e

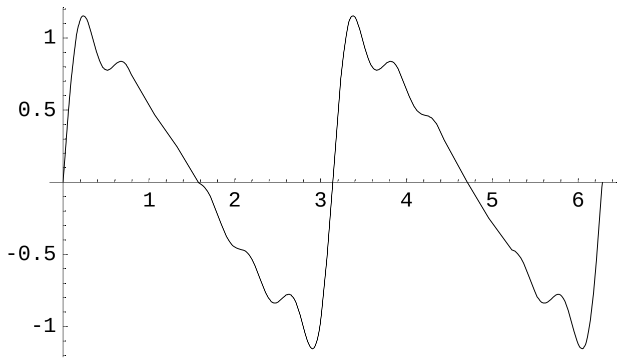
```
f4Dh[t_, h_] := D[a[0] / 2 + Sum[a[n] Cos[n ω t] + b[n] Sin[n ω t], {n, 1, h}], t];
f4Dh[t, 6]
```

$$\frac{8\,\mathrm{Sin}[2\,t]}{3\,\pi} + \frac{16\,\mathrm{Sin}[4\,t]}{15\,\pi} + \frac{24\,\mathrm{Sin}[6\,t]}{35\,\pi} + \frac{32\,\mathrm{Sin}[8\,t]}{63\,\pi} + \frac{40\,\mathrm{Sin}[10\,t]}{99\,\pi} + \frac{48\,\mathrm{Sin}[12\,t]}{143\,\pi}$$

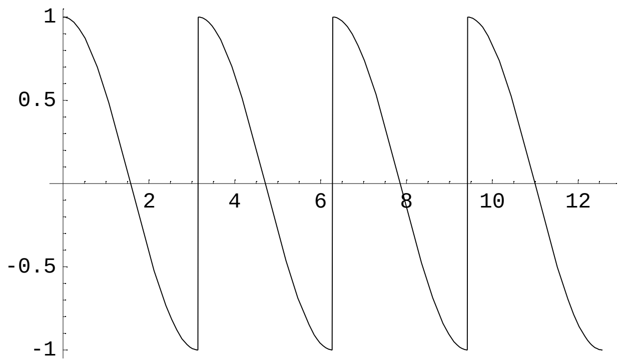
```
N[%]
```

```
0.848826 Sin[2. t] + 0.339531 Sin[4. t] + 0.21827 Sin[6. t] +
 0.161681 Sin[8. t] + 0.12861 Sin[10. t] + 0.106845 Sin[12. t]
```

```
Plot[Evaluate[f4Dh[t, 6]], {t, 0, 2 Pi}];
```



```
f4D[t_] := D[Sqrt[Sin[t]^2], t];
Plot[Evaluate [f4D[t]], {t, 0, 4 Pi}];
```




# 2

```
Remove["Global`*"];
```

## a

```
T = 2;
cc = 0;
g1[t_] := t;
ω = 2 Pi / T;
a[0] := 2 / T Integrate[g1[t], {t, cc, cc + T}];
a[k_] := 2 / T Integrate[g1[t] Cos[k ω t], {t, cc, cc + T}];
b[k_] := 2 / T Integrate[g1[t] Sin[k ω t], {t, cc, cc + T}];
gG1h[t_, h_] := a[0] / 2 + Sum[a[n] Cos[n ω t] + b[n] Sin[n ω t], {n, 1, h}];
gG1h[t, 10]
```

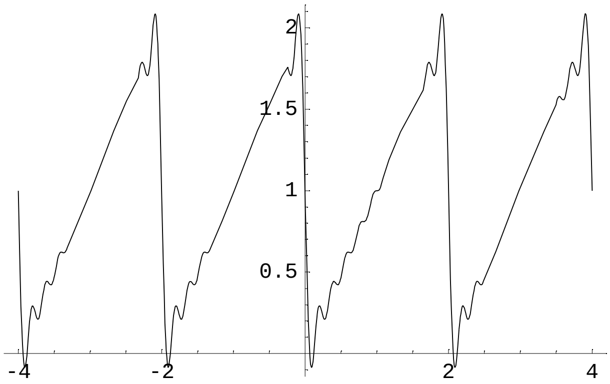
$$1-\frac{2\,\mathrm{Sin}[\pi\,t]}{\pi}-\frac{\mathrm{Sin}[2\,\pi\,t]}{\pi}-\frac{2\,\mathrm{Sin}[3\,\pi\,t]}{3\,\pi}-\frac{\mathrm{Sin}[4\,\pi\,t]}{2\,\pi}-\frac{2\,\mathrm{Sin}[5\,\pi\,t]}{5\,\pi}-\frac{\mathrm{Sin}[6\,\pi\,t]}{3\,\pi}-\frac{2\,\mathrm{Sin}[7\,\pi\,t]}{7\,\pi}-\frac{\mathrm{Sin}[8\,\pi\,t]}{4\,\pi}-\frac{2\,\mathrm{Sin}[9\,\pi\,t]}{9\,\pi}-\frac{\mathrm{Sin}[10\,\pi\,t]}{5\,\pi}$$

```
N[%]
```

```
1. - 0.63662 Sin[3.14159 t] - 0.31831 Sin[6.28319 t] -
 0.212207 Sin[9.42478 t] - 0.159155 Sin[12.5664 t] - 0.127324 Sin[15.708 t] -
 0.106103 Sin[18.8496 t] - 0.0909457 Sin[21.9911 t] -
 0.0795775 Sin[25.1327 t] - 0.0707355 Sin[28.2743 t] - 0.063662 Sin[31.4159 t]
```

$$\mathrm{gG1hZ}[t\_,\,10] := 1-\frac{2\,\mathrm{Sin}[\pi\,t]}{\pi}-\frac{\mathrm{Sin}[2\,\pi\,t]}{\pi}-\frac{2\,\mathrm{Sin}[3\,\pi\,t]}{3\,\pi}-\frac{\mathrm{Sin}[4\,\pi\,t]}{2\,\pi}-\frac{2\,\mathrm{Sin}[5\,\pi\,t]}{5\,\pi}-\frac{\mathrm{Sin}[6\,\pi\,t]}{3\,\pi}-\frac{2\,\mathrm{Sin}[7\,\pi\,t]}{7\,\pi}-\frac{\mathrm{Sin}[8\,\pi\,t]}{4\,\pi}-\frac{2\,\mathrm{Sin}[9\,\pi\,t]}{9\,\pi}-\frac{\mathrm{Sin}[10\,\pi\,t]}{5\,\pi}$$

```
Plot[gG1hZ[t, 10], {t, -4, 4}];
```




## b1 g2[t] = t^2

```
T = 2;
cc = 0;
g3[t_] := t^2;
ω = 2 Pi / T;
a[0] := 2 / T Integrate[g3[t], {t, cc, cc + T}];
a[k_] := 2 / T Integrate[g3[t] Cos[k ω t], {t, cc, cc + T}];
b[k_] := 2 / T Integrate[g3[t] Sin[k ω t], {t, cc, cc + T}];
gG3h[t_, h_] := a[0] / 2 + Sum[a[n] Cos[n ω t] + b[n] Sin[n ω t], {n, 1, h}];
gG3h[t, 10]
```

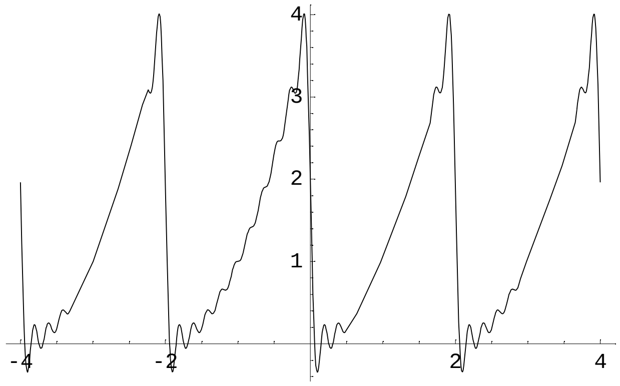
$$\frac{4}{3}+\frac{4\,\mathrm{Cos}[\pi t]}{\pi^2}+\frac{\mathrm{Cos}[2\pi t]}{\pi^2}+\frac{4\,\mathrm{Cos}[3\pi t]}{9\pi^2}+\frac{\mathrm{Cos}[4\pi t]}{4\pi^2}+\frac{4\,\mathrm{Cos}[5\pi t]}{25\pi^2}+\frac{\mathrm{Cos}[6\pi t]}{9\pi^2}+\frac{4\,\mathrm{Cos}[7\pi t]}{49\pi^2}+\frac{\mathrm{Cos}[8\pi t]}{16\pi^2}+\frac{4\,\mathrm{Cos}[9\pi t]}{81\pi^2}+\frac{\mathrm{Cos}[10\pi t]}{25\pi^2}-\frac{4\,\mathrm{Sin}[\pi t]}{\pi}-\frac{2\,\mathrm{Sin}[2\pi t]}{\pi}-\frac{4\,\mathrm{Sin}[3\pi t]}{3\pi}-\frac{\mathrm{Sin}[4\pi t]}{\pi}-\frac{4\,\mathrm{Sin}[5\pi t]}{5\pi}-\frac{2\,\mathrm{Sin}[6\pi t]}{3\pi}-\frac{4\,\mathrm{Sin}[7\pi t]}{7\pi}-\frac{\mathrm{Sin}[8\pi t]}{2\pi}-\frac{4\,\mathrm{Sin}[9\pi t]}{9\pi}-\frac{2\,\mathrm{Sin}[10\pi t]}{5\pi}$$

```
N[%]
```

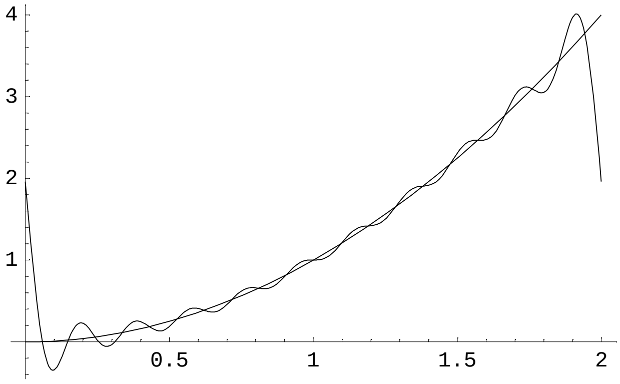
```
1.33333 + 0.405285 Cos[3.14159 t] + 0.101321 Cos[6.28319 t] +
 0.0450316 Cos[9.42478 t] + 0.0253303 Cos[12.5664 t] + 0.0162114 Cos[15.708 t] +
 0.0112579 Cos[18.8496 t] + 0.00827112 Cos[21.9911 t] + 0.00633257 Cos[25.1327 t] +
 0.00500352 Cos[28.2743 t] + 0.00405285 Cos[31.4159 t] - 1.27324 Sin[3.14159 t] -
 0.63662 Sin[6.28319 t] - 0.424413 Sin[9.42478 t] - 0.31831 Sin[12.5664 t] -
 0.254648 Sin[15.708 t] - 0.212207 Sin[18.8496 t] - 0.181891 Sin[21.9911 t] -
 0.159155 Sin[25.1327 t] - 0.141471 Sin[28.2743 t] - 0.127324 Sin[31.4159 t]
```

$$\mathbf{gG3hZ[t\_,\,10]} := \frac{4}{3}+\frac{4\,\mathrm{Cos}[\pi t]}{\pi^2}+\frac{\mathrm{Cos}[2\pi t]}{\pi^2}+\frac{4\,\mathrm{Cos}[3\pi t]}{9\pi^2}+\frac{\mathrm{Cos}[4\pi t]}{4\pi^2}+\frac{4\,\mathrm{Cos}[5\pi t]}{25\pi^2}+\frac{\mathrm{Cos}[6\pi t]}{9\pi^2}+\frac{4\,\mathrm{Cos}[7\pi t]}{49\pi^2}+\frac{\mathrm{Cos}[8\pi t]}{16\pi^2}+\frac{4\,\mathrm{Cos}[9\pi t]}{81\pi^2}+\frac{\mathrm{Cos}[10\pi t]}{25\pi^2}-\frac{4\,\mathrm{Sin}[\pi t]}{\pi}-\frac{2\,\mathrm{Sin}[2\pi t]}{\pi}-\frac{4\,\mathrm{Sin}[3\pi t]}{3\pi}-\frac{\mathrm{Sin}[4\pi t]}{\pi}-\frac{4\,\mathrm{Sin}[5\pi t]}{5\pi}-\frac{2\,\mathrm{Sin}[6\pi t]}{3\pi}-\frac{4\,\mathrm{Sin}[7\pi t]}{7\pi}-\frac{\mathrm{Sin}[8\pi t]}{2\pi}-\frac{4\,\mathrm{Sin}[9\pi t]}{9\pi}-\frac{2\,\mathrm{Sin}[10\pi t]}{5\pi}$$

```
Plot[gG3hZ[t, 10], {t, -4, 4}];
```




```
Plot[{gG3hZ[t, 10], t^2}, {t, 0, 2}];
```




## b2 g2[t] = t^2 --- Achtung: Die folgende Mehtode ist trickreich:

```
gG2hZa[t_, 10] := 2 Integrate[gG1hZ[t, 10], t];
gG2hZa[t, 10] // Expand
```

$$2\,t + \frac{4\,\mathrm{Cos}[\pi\,t]}{\pi^2} + \frac{\mathrm{Cos}[2\,\pi\,t]}{\pi^2} + \frac{4\,\mathrm{Cos}[3\,\pi\,t]}{9\,\pi^2} + \frac{\mathrm{Cos}[4\,\pi\,t]}{4\,\pi^2} + \frac{4\,\mathrm{Cos}[5\,\pi\,t]}{25\,\pi^2} + \frac{\mathrm{Cos}[6\,\pi\,t]}{9\,\pi^2} + \frac{4\,\mathrm{Cos}[7\,\pi\,t]}{49\,\pi^2} + \frac{\mathrm{Cos}[8\,\pi\,t]}{16\,\pi^2} + \frac{4\,\mathrm{Cos}[9\,\pi\,t]}{81\,\pi^2} + \frac{\mathrm{Cos}[10\,\pi\,t]}{25\,\pi^2}$$

Hier stört der Term t, der nicht zu einer Fourierreihe gehört. Man kann ihn aber durch die ursprünlgiche Fourierreihe ersetzen:

```
gG2hZb[t_, 10] := (gG2hZa[t, 10] // Expand ) - 2 t ;
gG2hZb[t, 10]
```

$$\frac{4\,\mathrm{Cos}[\pi\,t]}{\pi^2} + \frac{\mathrm{Cos}[2\,\pi\,t]}{\pi^2} + \frac{4\,\mathrm{Cos}[3\,\pi\,t]}{9\,\pi^2} + \frac{\mathrm{Cos}[4\,\pi\,t]}{4\,\pi^2} + \frac{4\,\mathrm{Cos}[5\,\pi\,t]}{25\,\pi^2} + \frac{\mathrm{Cos}[6\,\pi\,t]}{9\,\pi^2} + \frac{4\,\mathrm{Cos}[7\,\pi\,t]}{49\,\pi^2} + \frac{\mathrm{Cos}[8\,\pi\,t]}{16\,\pi^2} + \frac{4\,\mathrm{Cos}[9\,\pi\,t]}{81\,\pi^2} + \frac{\mathrm{Cos}[10\,\pi\,t]}{25\,\pi^2}$$

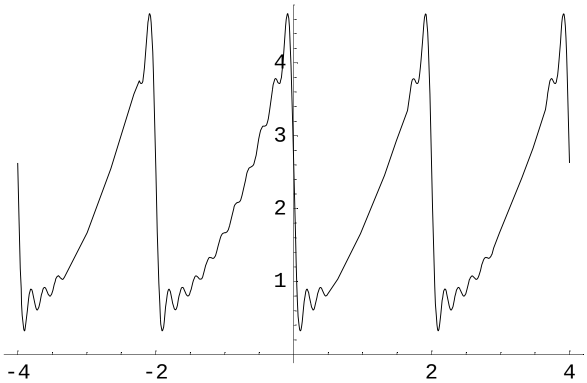
```
gG2hZ[t_, 10] := gG2hZb[t, 10] + 2 gG1hZ[t, 10];
gG2hZ[u, 10]
```

$$\frac{4\,\mathrm{Cos}[\pi\,u]}{\pi^2}+\frac{\mathrm{Cos}[2\,\pi\,u]}{\pi^2}+\frac{4\,\mathrm{Cos}[3\,\pi\,u]}{9\,\pi^2}+\frac{\mathrm{Cos}[4\,\pi\,u]}{4\,\pi^2}+\frac{4\,\mathrm{Cos}[5\,\pi\,u]}{25\,\pi^2}+\frac{\mathrm{Cos}[6\,\pi\,u]}{9\,\pi^2}+\frac{4\,\mathrm{Cos}[7\,\pi\,u]}{49\,\pi^2}+\frac{\mathrm{Cos}[8\,\pi\,u]}{16\,\pi^2}+\frac{4\,\mathrm{Cos}[9\,\pi\,u]}{81\,\pi^2}+\frac{\mathrm{Cos}[10\,\pi\,u]}{25\,\pi^2}+2\left(1-\frac{2\,\mathrm{Sin}[\pi\,u]}{\pi}-\frac{\mathrm{Sin}[2\,\pi\,u]}{\pi}-\frac{2\,\mathrm{Sin}[3\,\pi\,u]}{3\,\pi}-\frac{\mathrm{Sin}[4\,\pi\,u]}{2\,\pi}-\frac{2\,\mathrm{Sin}[5\,\pi\,u]}{5\,\pi}-\frac{\mathrm{Sin}[6\,\pi\,u]}{3\,\pi}-\frac{2\,\mathrm{Sin}[7\,\pi\,u]}{7\,\pi}-\frac{\mathrm{Sin}[8\,\pi\,u]}{4\,\pi}-\frac{2\,\mathrm{Sin}[9\,\pi\,u]}{9\,\pi}-\frac{\mathrm{Sin}[10\,\pi\,u]}{5\,\pi}\right)$$

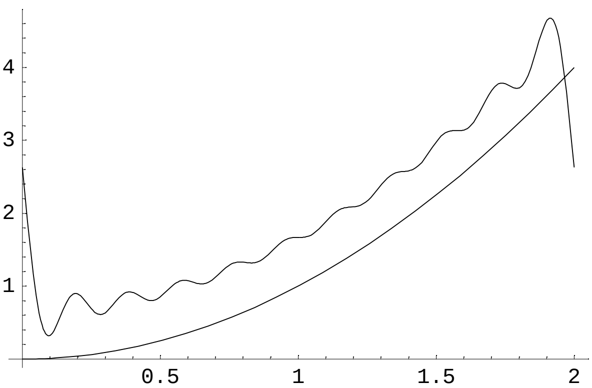
```
N[%]
```

```
0.405285 Cos[3.14159 u] + 0.101321 Cos[6.28319 u] +
 0.0450316 Cos[9.42478 u] + 0.0253303 Cos[12.5664 u] + 0.0162114 Cos[15.708 u] +
 0.0112579 Cos[18.8496 u] + 0.00827112 Cos[21.9911 u] + 0.00633257 Cos[25.1327 u] +
 0.00500352 Cos[28.2743 u] + 0.00405285 Cos[31.4159 u] +
 2. (1. - 0.63662 Sin[3.14159 u] - 0.31831 Sin[6.28319 u] - 0.212207 Sin[9.42478 u] -
    0.159155 Sin[12.5664 u] - 0.127324 Sin[15.708 u] - 0.106103 Sin[18.8496 u] -
    0.0909457 Sin[21.9911 u] - 0.0795775 Sin[25.1327 u] -
    0.0707355 Sin[28.2743 u] - 0.063662 Sin[31.4159 u])
```

```
Plot[Evaluate[gG2hZ[u, 10]], {u, -4, 4}];
```




```
Plot[Evaluate[{gG2hZ[u, 10], u^2}], {u, 0, 2}];
```




Man hat hier erst die Funktion bis auf die Integrationskonstante gefunden. Die Integrationskonstante muss noch bestimmt werden etwa durch Normierung der Funktion bei 1: gG2hZ[1,10] + c = 1

```
solv = Solve[(gG2hZ[u, 10] + c == 1) /. u → 1, {c}] // Flatten
```

$$\left\{c \to \frac{5194387-1587600\,\pi^2}{1587600\,\pi^2}\right\}$$

```
(c = c /. solv) // N
```

−0.668492

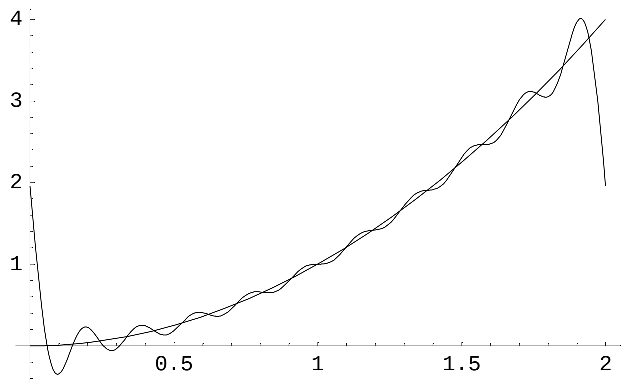
```
gG4hZ[u_, 10] := gG2hZ[u, 10] + c;
gG4hZ[u, 10]
```

$$\frac{5194387-1587600\pi^2}{1587600\pi^2}+\frac{4\,\mathrm{Cos}[\pi u]}{\pi^2}+\frac{\mathrm{Cos}[2\pi u]}{\pi^2}+\frac{4\,\mathrm{Cos}[3\pi u]}{9\pi^2}+\frac{\mathrm{Cos}[4\pi u]}{4\pi^2}+\frac{4\,\mathrm{Cos}[5\pi u]}{25\pi^2}+\frac{\mathrm{Cos}[6\pi u]}{9\pi^2}+\frac{4\,\mathrm{Cos}[7\pi u]}{49\pi^2}+\frac{\mathrm{Cos}[8\pi u]}{16\pi^2}+\frac{4\,\mathrm{Cos}[9\pi u]}{81\pi^2}+\frac{\mathrm{Cos}[10\pi u]}{25\pi^2}+2\left(1-\frac{2\,\mathrm{Sin}[\pi u]}{\pi}-\frac{\mathrm{Sin}[2\pi u]}{\pi}-\frac{2\,\mathrm{Sin}[3\pi u]}{3\pi}-\frac{\mathrm{Sin}[4\pi u]}{2\pi}-\frac{2\,\mathrm{Sin}[5\pi u]}{5\pi}-\frac{\mathrm{Sin}[6\pi u]}{3\pi}-\frac{2\,\mathrm{Sin}[7\pi u]}{7\pi}-\frac{\mathrm{Sin}[8\pi u]}{4\pi}-\frac{2\,\mathrm{Sin}[9\pi u]}{9\pi}-\frac{\mathrm{Sin}[10\pi u]}{5\pi}\right)$$

```
gG4hZ[u, 10] // N
```

```
-0.668492 + 0.405285 Cos[3.14159 u] + 0.101321 Cos[6.28319 u] +
 0.0450316 Cos[9.42478 u] + 0.0253303 Cos[12.5664 u] + 0.0162114 Cos[15.708 u] +
 0.0112579 Cos[18.8496 u] + 0.00827112 Cos[21.9911 u] + 0.00633257 Cos[25.1327 u] +
 0.00500352 Cos[28.2743 u] + 0.00405285 Cos[31.4159 u] +
 2. (1. - 0.63662 Sin[3.14159 u] - 0.31831 Sin[6.28319 u] - 0.212207 Sin[9.42478 u] -
    0.159155 Sin[12.5664 u] - 0.127324 Sin[15.708 u] - 0.106103 Sin[18.8496 u] -
    0.0909457 Sin[21.9911 u] - 0.0795775 Sin[25.1327 u] -
    0.0707355 Sin[28.2743 u] - 0.063662 Sin[31.4159 u])
```

```
Plot[Evaluate[{gG4hZ[u, 10], u^2}], {u, 0, 2}];
```



---

# 3

## a

Aus Aufgabe 1 : $|\mathrm{Sin}(t)| =$

$$fF[t] = \frac{2}{\pi} - \frac{4\,\mathrm{Cos}[2\,t]}{3\,\pi} - \frac{4\,\mathrm{Cos}[4\,t]}{15\,\pi} - \frac{4\,\mathrm{Cos}[6\,t]}{35\,\pi} - \frac{4\,\mathrm{Cos}[8\,t]}{63\,\pi} - \frac{4\,\mathrm{Cos}[10\,t]}{99\,\pi} - \frac{4\,\mathrm{Cos}[12\,t]}{143\,\pi} - ....$$

$$===> \; 0 = \mathrm{Sin}[0] = \frac{2}{\pi} - \frac{4\,\mathrm{Cos}[0]}{3\,\pi} - \frac{4\,\mathrm{Cos}[0]}{15\,\pi} - \frac{4\,\mathrm{Cos}[0]}{35\,\pi} - \frac{4\,\mathrm{Cos}[0]}{63\,\pi} - \frac{4\,\mathrm{Cos}[0]}{99\,\pi} - \frac{4\,\mathrm{Cos}[0]}{143\,\pi} - ....$$

$$===> \; \frac{2}{\pi} = \frac{4}{\pi}\left(\frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \frac{1}{63} + \frac{1}{99} + \frac{1}{143} - ....\right) \; ===> \; \frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \frac{1}{63} + \frac{1}{99} + \frac{1}{143} - .... = \frac{1}{2}$$

$$===> \; \frac{1}{15} + \frac{1}{35} + \frac{1}{63} + \frac{1}{99} + \frac{1}{143} - .... = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

```
Sum[1 / ((2 k + 1) (2 k + 3)), {k, 1, Infinity}]
```

$\frac{1}{6}$

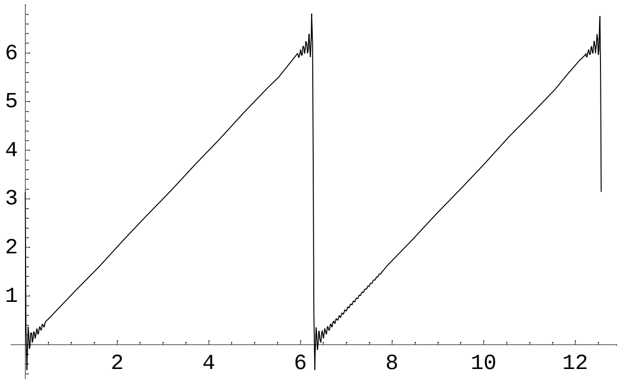
## b Sehr einfaches Beispiel mit nicht so rascher Konvergenz: Gerade y = t mit den Grundbereich für die Periode von 0 bis $2\,\pi$

```
Remove["Global`*"];
```

```
T = 2 Pi;
cc = 0;
Gerade1[t_] := t
ω = 2 Pi / T;
a[0] := 2 / T Integrate[Gerade1[t], {t, cc, cc + T}];
a[k_] := 2 / T Integrate[Gerade1[t] Cos[k ω t], {t, cc, cc + T}];
b[k_] := 2 / T Integrate[Gerade1[t] Sin[k ω t], {t, cc, cc + T}];
c[k_] := 1 / T Integrate[Gerade1[t] E^(-I k ω t), {t, cc, cc + T}];
Gerade1Gh[t_, h_] := a[0] / 2 + Sum[a[n] Cos[n ω t] + b[n] Sin[n ω t], {n, 1, h}];
Gerade1Gh[t, 100]
```

$$\pi - 2\,\mathrm{Sin}[t] - \mathrm{Sin}[2\,t] - \frac{2}{3}\mathrm{Sin}[3\,t] - \frac{1}{2}\mathrm{Sin}[4\,t] - \frac{2}{5}\mathrm{Sin}[5\,t] - \frac{1}{3}\mathrm{Sin}[6\,t] - \frac{2}{7}\mathrm{Sin}[7\,t] -$$
$$\frac{1}{4}\mathrm{Sin}[8\,t] - \frac{2}{9}\mathrm{Sin}[9\,t] - \frac{1}{5}\mathrm{Sin}[10\,t] - \frac{2}{11}\mathrm{Sin}[11\,t] - \frac{1}{6}\mathrm{Sin}[12\,t] - \frac{2}{13}\mathrm{Sin}[13\,t] -$$
$$\frac{1}{7}\mathrm{Sin}[14\,t] - \frac{2}{15}\mathrm{Sin}[15\,t] - \frac{1}{8}\mathrm{Sin}[16\,t] - \frac{2}{17}\mathrm{Sin}[17\,t] - \frac{1}{9}\mathrm{Sin}[18\,t] - \frac{2}{19}\mathrm{Sin}[19\,t] -$$
$$\frac{1}{10}\mathrm{Sin}[20\,t] - \frac{2}{21}\mathrm{Sin}[21\,t] - \frac{1}{11}\mathrm{Sin}[22\,t] - \frac{2}{23}\mathrm{Sin}[23\,t] - \frac{1}{12}\mathrm{Sin}[24\,t] - \frac{2}{25}\mathrm{Sin}[25\,t] -$$
$$\frac{1}{13}\mathrm{Sin}[26\,t] - \frac{2}{27}\mathrm{Sin}[27\,t] - \frac{1}{14}\mathrm{Sin}[28\,t] - \frac{2}{29}\mathrm{Sin}[29\,t] - \frac{1}{15}\mathrm{Sin}[30\,t] -$$
$$\frac{2}{31}\mathrm{Sin}[31\,t] - \frac{1}{16}\mathrm{Sin}[32\,t] - \frac{2}{33}\mathrm{Sin}[33\,t] - \frac{1}{17}\mathrm{Sin}[34\,t] - \frac{2}{35}\mathrm{Sin}[35\,t] -$$
$$\frac{1}{18}\mathrm{Sin}[36\,t] - \frac{2}{37}\mathrm{Sin}[37\,t] - \frac{1}{19}\mathrm{Sin}[38\,t] - \frac{2}{39}\mathrm{Sin}[39\,t] - \frac{1}{20}\mathrm{Sin}[40\,t] -$$
$$\frac{2}{41}\mathrm{Sin}[41\,t] - \frac{1}{21}\mathrm{Sin}[42\,t] - \frac{2}{43}\mathrm{Sin}[43\,t] - \frac{1}{22}\mathrm{Sin}[44\,t] - \frac{2}{45}\mathrm{Sin}[45\,t] -$$
$$\frac{1}{23}\mathrm{Sin}[46\,t] - \frac{2}{47}\mathrm{Sin}[47\,t] - \frac{1}{24}\mathrm{Sin}[48\,t] - \frac{2}{49}\mathrm{Sin}[49\,t] - \frac{1}{25}\mathrm{Sin}[50\,t] -$$
$$\frac{2}{51}\mathrm{Sin}[51\,t] - \frac{1}{26}\mathrm{Sin}[52\,t] - \frac{2}{53}\mathrm{Sin}[53\,t] - \frac{1}{27}\mathrm{Sin}[54\,t] - \frac{2}{55}\mathrm{Sin}[55\,t] -$$
$$\frac{1}{28}\mathrm{Sin}[56\,t] - \frac{2}{57}\mathrm{Sin}[57\,t] - \frac{1}{29}\mathrm{Sin}[58\,t] - \frac{2}{59}\mathrm{Sin}[59\,t] - \frac{1}{30}\mathrm{Sin}[60\,t] -$$
$$\frac{2}{61}\mathrm{Sin}[61\,t] - \frac{1}{31}\mathrm{Sin}[62\,t] - \frac{2}{63}\mathrm{Sin}[63\,t] - \frac{1}{32}\mathrm{Sin}[64\,t] - \frac{2}{65}\mathrm{Sin}[65\,t] -$$
$$\frac{1}{33}\mathrm{Sin}[66\,t] - \frac{2}{67}\mathrm{Sin}[67\,t] - \frac{1}{34}\mathrm{Sin}[68\,t] - \frac{2}{69}\mathrm{Sin}[69\,t] - \frac{1}{35}\mathrm{Sin}[70\,t] -$$
$$\frac{2}{71}\mathrm{Sin}[71\,t] - \frac{1}{36}\mathrm{Sin}[72\,t] - \frac{2}{73}\mathrm{Sin}[73\,t] - \frac{1}{37}\mathrm{Sin}[74\,t] - \frac{2}{75}\mathrm{Sin}[75\,t] -$$
$$\frac{1}{38}\mathrm{Sin}[76\,t] - \frac{2}{77}\mathrm{Sin}[77\,t] - \frac{1}{39}\mathrm{Sin}[78\,t] - \frac{2}{79}\mathrm{Sin}[79\,t] - \frac{1}{40}\mathrm{Sin}[80\,t] -$$
$$\frac{2}{81}\mathrm{Sin}[81\,t] - \frac{1}{41}\mathrm{Sin}[82\,t] - \frac{2}{83}\mathrm{Sin}[83\,t] - \frac{1}{42}\mathrm{Sin}[84\,t] - \frac{2}{85}\mathrm{Sin}[85\,t] -$$
$$\frac{1}{43}\mathrm{Sin}[86\,t] - \frac{2}{87}\mathrm{Sin}[87\,t] - \frac{1}{44}\mathrm{Sin}[88\,t] - \frac{2}{89}\mathrm{Sin}[89\,t] - \frac{1}{45}\mathrm{Sin}[90\,t] -$$
$$\frac{2}{91}\mathrm{Sin}[91\,t] - \frac{1}{46}\mathrm{Sin}[92\,t] - \frac{2}{93}\mathrm{Sin}[93\,t] - \frac{1}{47}\mathrm{Sin}[94\,t] - \frac{2}{95}\mathrm{Sin}[95\,t] -$$
$$\frac{1}{48}\mathrm{Sin}[96\,t] - \frac{2}{97}\mathrm{Sin}[97\,t] - \frac{1}{49}\mathrm{Sin}[98\,t] - \frac{2}{99}\mathrm{Sin}[99\,t] - \frac{1}{50}\mathrm{Sin}[100\,t]$$

```
Plot[Evaluate[Gerade1Gh[t, 100] // N], {t, 0, 4 Pi}];
```




```
Gerade1Gh[Pi / 2, 100]
```

$$-\frac{17003027382321032229774367383405751761 64}{10893808629642574556958407646142547430 75}+\pi$$

Es gilt: Gerade1Gh[ /2,100] ist ungefähr /2. Die in der Reihe vorkommenden Sinus- und Cosinus-Werte lassen sich an der Stelle k/2 alle exakt berechnen.

Es gilt dann: /2 ist ungefähr Gerade1Gh[Pi/2,100]. Diese Approsimationsgleichung lässt sich algebraisch nach Pi auflösen:

```
piUngefaehr = -2 (Gerade1Gh[Pi / 2, 100] - Pi)
```

$$\frac{34006054764642064459548734766811503523 28}{10893808629642574556958407646142547430 75}$$

```
N[%]
```

3.12159

Nicht sehr genau mit 101 Koeffizienten!!

Setzt man /2 in die Sinusreihe ein, so bekommt man /2. Die Sinuswerte werden dann 1 oder -1 oder 0. Damit lässt sich approximieren.

```
4 (Sum[(-1) ^ (k + 1) / (2 k - 1), {k, 1, Infinity}])
```

$\pi$

---

# 4

## a

```
Remove["Global`*"];
```

```
u[t_, x_, c_, a_, b_] := c E^(-a t) Sin[b x]
```

```
D[u[t, x, c, a, b], {t, 1}]
```

$-a\,c\,e^{-a\,t}\,\mathrm{Sin}[b\,x]$

```
D[u[t, x, c, a, b], {x, 2}]
```

$-b^2\, c\, e^{-a\, t}\, \mathrm{Sin}[b\, x]$

```
Solve[Evaluate[D[u[t, x, c, a, b], {t, 1}] == D[u[t, x, c, a, b], {x, 2}]], {a}]
```

$\{\{a \to b^2\}\}$

```
u[t, 0, c, a, b]
```

0

```
u[Pi, 0, c, a, b]
```

0

Die Funktion löst die Gleichung falls $a = b^2$ gilt .

## b

Eine Linearkombination von Lösungen ist wieder Lösung, da

1. der Differentaloperator ein linearer Operator ist und da
2. die Randbedingungen wegen dem Wert 0 bei Linearkombinationen erhalten bleiben.

## c

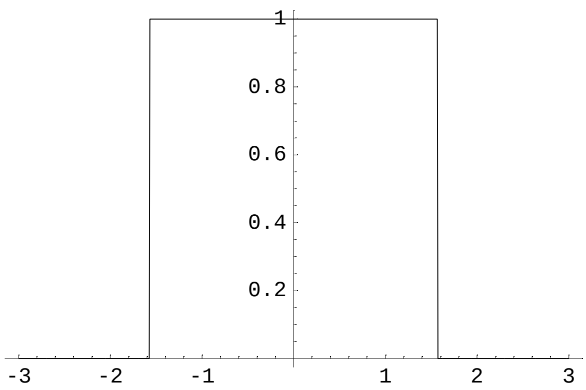
Bei fixem t gilt   u[t,x,c,a,b] = c E^(-a t_fix)  Sin[b x].
Falls  die b natürliche Zahlen sind, so hat man eine Fourierreihe mit den Koeffizienten c E^(-a t_fix).
Die a wären damit Quadratzahlen von natürlichen Zahlen.

---

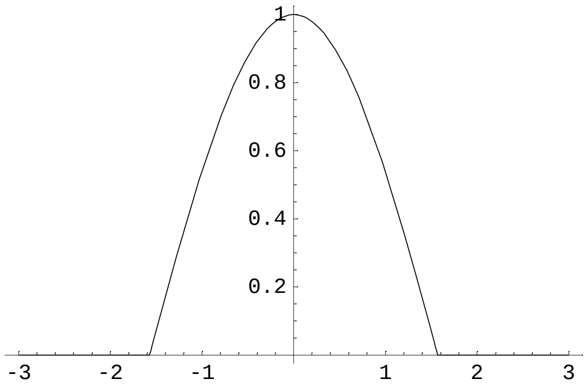
# 5

```
Remove["Global`*"];
```

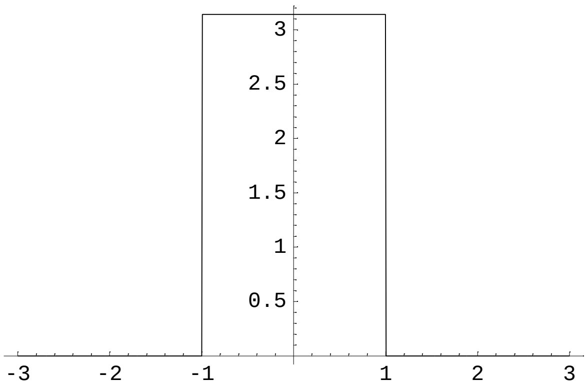
```
h1[t_] := UnitStep[t + Pi / 2] - UnitStep[t - Pi / 2];
Plot[h1[t], {t, -3, 3}];
```



```
v1[t_] := Cos[t] h1[t];
Plot[v1[x], {x, -3, 3}];
```




```
v2[t_] := Pi (UnitStep[t + 1] - UnitStep[t - 1]);
Plot[v2[x], {x, -3, 3}];
```




## a Achtung Faktor $\frac{1}{\sqrt{\pi}}$ bei anderer Definition der Fouriertransformation!!!

```
1 / (2 Pi) Integrate[Cos[t] E^(-I Ω t), {t, -Pi / 2, Pi / 2}]
```

$$-\frac{\mathrm{Cos}[\frac{\pi\,\Omega}{2}]}{\pi\,(-1+\Omega^2)}$$

Mit andererem üblichen Normierungsfaktor

```
FourierTransform[v1[t], t, ω] // Simplify
```

$$-\frac{\sqrt{\frac{2}{\pi}}\,\mathrm{Cos}[\frac{\pi\,\omega}{2}]}{-1+\omega^2}$$

## b Siehe Skript Mathematik II, z.B. deutsche Version Seite 121

Dort findet man eine ähnliche D'Gl., in der sich v2[t] nur um den Faktor unterscheidet: v2[t] = f[t]. Multipliziert man die dortige Lösung mit , so erhält man die Lösung zum vorliegenden Problem.

## Hier mit der symmetrischen Faktornormierung:

```
FourierTransform[v2[t], t, ω] // Simplify
```

$$\frac{\sqrt{2\pi}\,\mathrm{Sin}[\omega]}{\omega}$$

```
(FourierTransform[y'[t], t, ω] + FourierTransform[y[t], t, ω]) /.
 FourierTransform[y[t], t, ω] -> Y[ω]
```

$$Y[\omega] - i\,\omega\,Y[\omega]$$

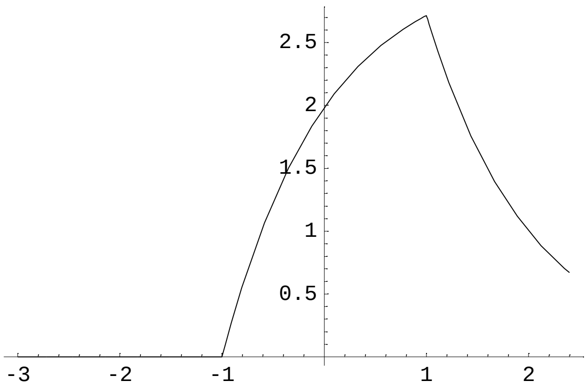
```
Solve[Y[ω] - ⅈ ω Y[ω] == (√(2 π) Sin[ω])/ω, {Y[ω]}] // Flatten
```

$$\left\{Y[\omega] \to \frac{\sqrt{2\pi}\,\omega\,\mathrm{Sin}[\omega] + i\,\sqrt{2\pi}\,\omega^2\,\mathrm{Sin}[\omega]}{\omega^2+\omega^4}\right\}$$

```
InverseFourierTransform[(√(2 π) ω Sin[ω] + ⅈ √(2 π) ω^2 Sin[ω])/(ω^2 + ω^4), {ω}, {t}] // Simplify
```

$$\frac{1}{2\,e}\left(\pi\left(e\,\mathrm{Sign}[1-t] + e\,\mathrm{Sign}[1+t] + 2\left(\mathrm{Cosh}[t]-\mathrm{Sinh}[t]\right)\left(e^2\,\mathrm{UnitStep}[-1+t] - \mathrm{UnitStep}[1+t]\right)\right)\right)$$

```
pl1 = Plot[1/(2 ⅇ) (π (ⅇ Sign[1 - t] + ⅇ Sign[1 + t] +
        2 (Cosh[t] - Sinh[t]) (ⅇ^2 UnitStep[-1 + t] - UnitStep[1 + t]))), {t, -3, 2.4}];
```




## Lösung aus dem Skript auf elementare Weise:

```
(* Remove["Global`*"] *)
```

```
s[x_] := 0 /; x < -1;
s[x_] := (1 - E^(-(x + 1))) Pi /; ((x ≥ -1) && (x < 1));
s[x_] := (E^(1 - x) - E^(-(x + 1))) Pi /; (x ≥ 1);
?s
```

Global`s

$s[x\_] := 0 \;/;\; x < -1$

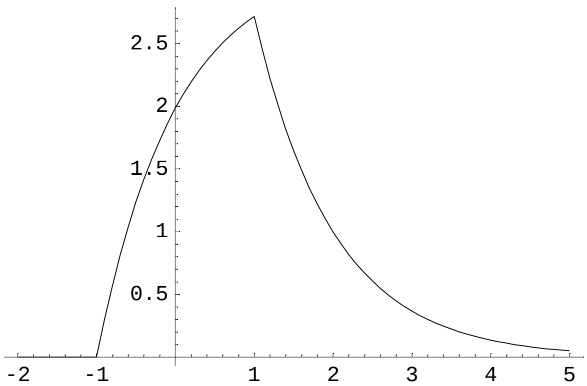
$s[x\_] := \left(1 - e^{-(x+1)}\right)\pi \;/;\; x \geq -1 \;\&\&\; x < 1$

$s[x\_] := \left(e^{1-x} - e^{-(x+1)}\right)\pi \;/;\; x \geq 1$

```
ta = Table[{x, s[x] // N}, {x, -2, 5, 0.1}] // Chop
```

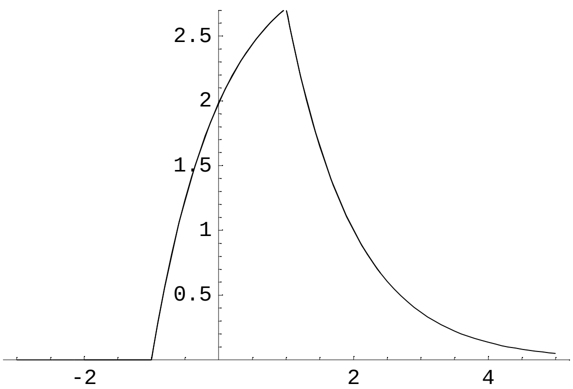
```
{{-2, 0}, {-1.9, 0}, {-1.8, 0}, {-1.7, 0}, {-1.6, 0}, {-1.5, 0}, {-1.4, 0},
 {-1.3, 0}, {-1.2, 0}, {-1.1, 0}, {-1., 0}, {-0.9, 0.298962}, {-0.8, 0.569474},
 {-0.7, 0.814244}, {-0.6, 1.03572}, {-0.5, 1.23612}, {-0.4, 1.41745},
 {-0.3, 1.58152}, {-0.2, 1.72998}, {-0.1, 1.86432}, {0, 1.98587}, {0.1, 2.09585},
 {0.2, 2.19536}, {0.3, 2.28541}, {0.4, 2.36689}, {0.5, 2.44061}, {0.6, 2.50732},
 {0.7, 2.56768}, {0.8, 2.62229}, {0.9, 2.67171}, {1., 2.71642}, {1.1, 2.45792},
 {1.2, 2.22402}, {1.3, 2.01238}, {1.4, 1.82087}, {1.5, 1.64759}, {1.6, 1.49081},
 {1.7, 1.34894}, {1.8, 1.22057}, {1.9, 1.10442}, {2., 0.999317}, {2.1, 0.904219},
 {2.2, 0.818171}, {2.3, 0.740312}, {2.4, 0.669862}, {2.5, 0.606116}, {2.6, 0.548437},
 {2.7, 0.496246}, {2.8, 0.449022}, {2.9, 0.406292}, {3., 0.367628}, {3.1, 0.332644},
 {3.2, 0.300988}, {3.3, 0.272346}, {3.4, 0.246428}, {3.5, 0.222978}, {3.6, 0.201759},
 {3.7, 0.182559}, {3.8, 0.165186}, {3.9, 0.149466}, {4., 0.135243}, {4.1, 0.122373},
 {4.2, 0.110727}, {4.3, 0.10019}, {4.4, 0.090656}, {4.5, 0.0820289}, {4.6, 0.0742228},
 {4.7, 0.0671596}, {4.8, 0.0607685}, {4.9, 0.0549856}, {5., 0.049753}}
```

```
pl2 = ListPlot[ta, PlotJoined → True];
```




### Vergleich beider Plots: die Plots decken sich

```
Show[pl1, pl2, PlotRange → {0, 2.7}];
```




---

## 6

Die Antworten auf die gestellten Fragen finden sich dargestellt im Skript Mathematik II,z.B.deutsche Version Seite 111.