Lösungen

1

Abgabe

2

```
Remove["Global`*"]
```

a

```
(LaplaceTransform[y''[t] + a y'[t] + b y[t], t, s] /.
   {y[0] → y0, y′[0] → yS0, LaplaceTransform[y[t], t, s] → Y[s]}) ==
  (LaplaceTransform[f[t]] /. {LaplaceTransform[f[t]] → F[s]})
```

$-s\, y0 - yS0 + b\, Y[s] + s^2\, Y[s] + a\, (-y0 + s\, Y[s]) == F[s]$

b

```
Solve[-s y0 - yS0 + b Y[s] + s^2 Y[s] + a (-y0 + s Y[s]) == F[s], { Y[s]}]
```

$\left\{\left\{Y[s] \to \frac{a\, y0 + s\, y0 + yS0 + F[s]}{b + a\, s + s^2}\right\}\right\}$

```
((((DSolve[{y''[t] + a y'[t] + b y[t] == 0, y[0] == y0, y'[0] == yS0}, y, t] // Flatten) //.
     {Sqrt[a^2 - 4 b] → k}) //. 1/(2 Sqrt[a^2 - 4 b]) → 1 / (2 k)) // Simplify)
```

$\left\{y \to \text{Function}\left[\{t\}, \frac{1}{2\,k}\left(-a\, e^{\frac{1}{2}(-a-k)\,t}\, y0 + k\, e^{\frac{1}{2}(-a-k)\,t}\, y0 + a\, e^{\frac{1}{2}(-a+k)\,t}\, y0 + k\, e^{\frac{1}{2}(-a+k)\,t}\, y0 - 2\, e^{\frac{1}{2}(-a-k)\,t}\, yS0 + 2\, e^{\frac{1}{2}(-a+k)\,t}\, yS0\right)\right]\right\}$

c

```
InverseLaplaceTransform[
 Evaluate[((a y0 + s y0 + yS0 + F[s])/(b + a s + s^2) /. {a → 1, b → 1, F[s] → 0, y0 → 1, yS0 → 0})], s, t]
```

$\frac{1}{3}\, e^{-t/2} \left(3\, \text{Cos}\left[\frac{\sqrt{3}\, t}{2}\right] + \sqrt{3}\, \text{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]\right)$

```
((DSolve[{y''[t] + 1 y'[t] + 1 y[t] == 0, y[0] == 1, y'[0] == 0}, y, t] // Flatten)) //
 Simplify
```

$$\left\{y \to \text{Function}\left[\{t\}, \frac{1}{3} e^{-t/2} \left(3 \text{Cos}\left[\frac{\sqrt{3}\, t}{2}\right] + \sqrt{3}\, \text{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]\right)\right]\right\}$$

```
((DSolve[{y''[t] + a y'[t] + b y[t] == 0, y[0] == y0, y'[0] == yS0}, y, t] // Flatten) /.
   {a → 1, b → 1, F[s] → 0, y0 → 1, yS0 → 0}) // Simplify
```

$$\left\{y \to \text{Function}\left[\{t\}, \frac{1}{2\sqrt{1^2 - 4\,1}} \left(-e^{\frac{1}{2}\left(-1-\sqrt{1^2-4\,1}\right) t} + \sqrt{1^2 - 4\,1}\, e^{\frac{1}{2}\left(-1-\sqrt{1^2-4\,1}\right) t} + 1\, e^{\frac{1}{2}\left(-1+\sqrt{1^2-4\,1}\right) t}\, 1 + \sqrt{1^2 - 4\,1}\, e^{\frac{1}{2}\left(-1+\sqrt{1^2-4\,1}\right) t} - 2\, e^{\frac{1}{2}\left(-1-\sqrt{1^2-4\,1}\right) t}\, 0 + 2\, e^{\frac{1}{2}\left(-1+\sqrt{1^2-4\,1}\right) t}\, 0\right)\right]\right\}$$

```
ah = 1; bh = 1; y0h = 1; yS0h = 0;
((((DSolve[{y''[t] + a y'[t] + b y[t] == 0, y[0] == y0, y'[0] == yS0}, y, t] // Flatten) //.
      {Sqrt[a^2 - 4 b] → k}) //. 1/(2 Sqrt[a^2 - 4 b]) → 1/(2 k)) /.
    {a → ah, b → bh, F[s] → 0, y0 → y0h, yS0 → yS0h}) /. k → Sqrt[ah^2 - 4 bh] // Evaluate
```

$$\left\{y \to \text{Function}\left[\{t\}, \frac{1}{2\left(i\sqrt{3}\right)} \left(-e^{\frac{1}{2}\left(-1-i\sqrt{3}\right) t} + \left(i\sqrt{3}\right) e^{\frac{1}{2}\left(-1-i\sqrt{3}\right) t} + 1\, e^{\frac{1}{2}\left(-1+i\sqrt{3}\right) t}\, 1 + \left(i\sqrt{3}\right) e^{\frac{1}{2}\left(-1+i\sqrt{3}\right) t} - 2\, e^{\frac{1}{2}\left(-1-i\sqrt{3}\right) t}\, 0 + 2\, e^{\frac{1}{2}\left(-1+i\sqrt{3}\right) t}\, 0\right)\right]\right\}$$

d

```
InverseLaplaceTransform[
 Evaluate[((a y0 + s y0 + yS0 + F[s])/(b + a s + s^2) /. {a → 1, b → 1, F[s] → 0, y0 → 0, yS0 → 1})], s, t]
```

$$\frac{2\, e^{-t/2}\, \text{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]}{\sqrt{3}}$$

```
((DSolve[{y''[t] + 1 y'[t] + 1 y[t] == 0, y[0] == 0, y'[0] == 1}, y, t] // Flatten)) //
 Simplify
```

$$\left\{y \to \text{Function}\left[\{t\}, \frac{2\, e^{-t/2}\, \text{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]}{\sqrt{3}}\right]\right\}$$

```
((DSolve[{y''[t] + a y'[t] + b y[t] == 0, y[0] == y0, y'[0] == yS0}, y, t] // Flatten) /.
   {a → 1, b → 1, F[s] → 0, y0 → 0, yS0 → 1}) // Simplify
```

$$\left\{y \to \text{Function}\left[\{t\}, \frac{1}{2\sqrt{1^2 - 4\,1}} \left(-e^{\frac{1}{2}\left(-1-\sqrt{1^2-4\,1}\right) t}\, 0 + \sqrt{1^2 - 4\,1}\, e^{\frac{1}{2}\left(-1-\sqrt{1^2-4\,1}\right) t}\, 0 + e^{\frac{1}{2}\left(-1+\sqrt{1^2-4\,1}\right) t}\, 0 + \sqrt{1^2 - 4\,1}\, e^{\frac{1}{2}\left(-1+\sqrt{1^2-4\,1}\right) t}\, 0 - 2\, e^{\frac{1}{2}\left(-1-\sqrt{1^2-4\,1}\right) t} + 2\, e^{\frac{1}{2}\left(-1+\sqrt{1^2-4\,1}\right) t}\right)\right]\right\}$$

```
ah = 1; bh = 1; y0h = 0; yS0h = 1;
((((DSolve[{y''[t] + a y'[t] + b y[t] == 0, y[0] == y0, y'[0] == yS0}, y, t] // Flatten) //.
      {Sqrt[a^2 - 4 b] -> k}) //. 1/(2 Sqrt[a^2 - 4 b]) -> 1/(2 k)) /.
    {a -> ah, b -> bh, F[s] -> 0, y0 -> y0h, yS0 -> yS0h}) /. k -> Sqrt[ah^2 - 4 bh] // Evaluate
```

$$\left\{y \to \text{Function}\left[\{t\},\ \frac{1}{2\left(i\sqrt{3}\right)}\left(-e^{\frac{1}{2}\left(-1-i\sqrt{3}\right)t}\,0 + \left(i\sqrt{3}\right)e^{\frac{1}{2}\left(-1-i\sqrt{3}\right)t}\,0 + e^{\frac{1}{2}\left(-1+i\sqrt{3}\right)t}\,0 + \left(i\sqrt{3}\right)e^{\frac{1}{2}\left(-1+i\sqrt{3}\right)t}\,0 - 2\,e^{\frac{1}{2}\left(-1-i\sqrt{3}\right)t} + 2\,e^{\frac{1}{2}\left(-1+i\sqrt{3}\right)t}\right)\right]\right\}$$

e

```
InverseLaplaceTransform[
 Evaluate[((a y0 + s y0 + yS0 + F[s])/(b + a s + s^2) /. {a -> 1, b -> 1, F[s] -> 0, y0 -> 1, yS0 -> 1})], s, t]
```

$$e^{-t/2}\left(\text{Cos}\left[\frac{\sqrt{3}\,t}{2}\right] + \sqrt{3}\,\text{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]\right)$$

```
((DSolve[{y''[t] + 1 y'[t] + 1 y[t] == 0, y[0] == 1, y'[0] == 1}, y, t] // Flatten)) //
 Simplify
```

$$\left\{y \to \text{Function}\left[\{t\},\ e^{-t/2}\left(\text{Cos}\left[\frac{\sqrt{3}\,t}{2}\right] + \sqrt{3}\,\text{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]\right)\right]\right\}$$

```
((DSolve[{y''[t] + a y'[t] + b y[t] == 0, y[0] == y0, y'[0] == yS0}, y, t] // Flatten) /.
   {a -> 1, b -> 1, F[s] -> 0, y0 -> 1, yS0 -> 1}) // Simplify
```

$$\left\{y \to \text{Function}\left[\{t\},\ \frac{1}{2\sqrt{1^2-4\,1}}\left(-e^{\frac{1}{2}\left(-1-\sqrt{1^2-4\,1}\right)t} + \sqrt{1^2-4\,1}\;e^{\frac{1}{2}\left(-1-\sqrt{1^2-4\,1}\right)t} + 1\,e^{\frac{1}{2}\left(-1+\sqrt{1^2-4\,1}\right)t}\,1 + \sqrt{1^2-4\,1}\;e^{\frac{1}{2}\left(-1+\sqrt{1^2-4\,1}\right)t} - 2\,e^{\frac{1}{2}\left(-1-\sqrt{1^2-4\,1}\right)t} + 2\,e^{\frac{1}{2}\left(-1+\sqrt{1^2-4\,1}\right)t}\right)\right]\right\}$$

```
ah = 1; bh = 1; y0h = 1; yS0h = 1;
((((DSolve[{y''[t] + a y'[t] + b y[t] == 0, y[0] == y0, y'[0] == yS0}, y, t] // Flatten) //.
      {Sqrt[a^2 - 4 b] -> k}) //. 1/(2 Sqrt[a^2 - 4 b]) -> 1/(2 k)) /.
    {a -> ah, b -> bh, F[s] -> 0, y0 -> y0h, yS0 -> yS0h}) /. k -> Sqrt[ah^2 - 4 bh] // Evaluate
```

$$\left\{y \to \text{Function}\left[\{t\},\ \frac{1}{2\left(i\sqrt{3}\right)}\left(-e^{\frac{1}{2}\left(-1-i\sqrt{3}\right)t} + \left(i\sqrt{3}\right)e^{\frac{1}{2}\left(-1-i\sqrt{3}\right)t} + 1\,e^{\frac{1}{2}\left(-1+i\sqrt{3}\right)t}\,1 + \left(i\sqrt{3}\right)e^{\frac{1}{2}\left(-1+i\sqrt{3}\right)t} - 2\,e^{\frac{1}{2}\left(-1-i\sqrt{3}\right)t} + 2\,e^{\frac{1}{2}\left(-1+i\sqrt{3}\right)t}\right)\right]\right\}$$

f

```
InverseLaplaceTransform[
 Evaluate[((a y0 + s y0 + yS0 + F[s])/(b + a s + s^2) /. {a → -1, b → 1, F[s] → 0, y0 → 0, yS0 → 1})], s, t]
```

$$\frac{2\, e^{t/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]}{\sqrt{3}}$$

```
((DSolve[{y''[t] - 1 y'[t] + 1 y[t] == 0, y[0] == 0, y'[0] == 1}, y, t] // Flatten)) //
 Simplify
```

$$\left\{y \to \operatorname{Function}\left[\{t\}, \frac{2\, e^{t/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]}{\sqrt{3}}\right]\right\}$$

```
((DSolve[{y''[t] + a y'[t] + b y[t] == 0, y[0] == y0, y'[0] == yS0}, y, t] // Flatten) /.
   {a → -1, b → 1, F[s] → 0, y0 → 0, yS0 → 1}) // Simplify
```

$$\Big\{y \to \operatorname{Function}\Big[\{t\}, \frac{1}{2\sqrt{(-1)^2 - 4\,1}} \Big(-(-1)\, e^{\frac{1}{2}\left(-(-1)-\sqrt{(-1)^2-4\,1}\right) t}\, 0 + \sqrt{(-1)^2 - 4\,1}\, e^{\frac{1}{2}\left(-(-1)-\sqrt{(-1)^2-4\,1}\right) t}\, 0 - e^{\frac{1}{2}\left(-(-1)+\sqrt{(-1)^2-4\,1}\right) t}\, 0 + \sqrt{(-1)^2 - 4\,1}\, e^{\frac{1}{2}\left(-(-1)+\sqrt{(-1)^2-4\,1}\right) t}\, 0 - 2\, e^{\frac{1}{2}\left(-(-1)-\sqrt{(-1)^2-4\,1}\right) t} + 2\, e^{\frac{1}{2}\left(-(-1)+\sqrt{(-1)^2-4\,1}\right) t}\Big)\Big]\Big\}$$

```
ah = -1; bh = 1; y0h = 0; yS0h = 1;
((((DSolve[{y''[t] + a y'[t] + b y[t] == 0, y[0] == y0, y'[0] == yS0}, y, t] // Flatten) //.
     {Sqrt[a^2 - 4 b] → k}) //. 1/(2 Sqrt[a^2 - 4 b]) → 1/(2 k)) /.
   {a → ah, b → bh, F[s] → 0, y0 → y0h, yS0 → yS0h}) /. k → Sqrt[ah^2 - 4 bh] // Evaluate
```

$$\Big\{y \to \operatorname{Function}\Big[\{t\}, \frac{1}{2\left(i\sqrt{3}\right)} \Big(-(-1)\, e^{\frac{1}{2}\left(-(-1)-i\sqrt{3}\right) t}\, 0 + \left(i\sqrt{3}\right) e^{\frac{1}{2}\left(-(-1)-i\sqrt{3}\right) t}\, 0 - e^{\frac{1}{2}\left(-(-1)+i\sqrt{3}\right) t}\, 0 + \left(i\sqrt{3}\right) e^{\frac{1}{2}\left(-(-1)+i\sqrt{3}\right) t}\, 0 - 2\, e^{\frac{1}{2}\left(-(-1)-i\sqrt{3}\right) t} + 2\, e^{\frac{1}{2}\left(-(-1)+i\sqrt{3}\right) t}\Big)\Big]\Big\}$$

g

```
InverseLaplaceTransform[
 Evaluate[((a y0 + s y0 + yS0 + F[s])/(b + a s + s^2) /. {a → -2, b → 1, F[s] → 0, y0 → 0, yS0 → 1})], s, t]
```

$$e^t\, t$$

```
((DSolve[{y''[t] - 2 y'[t] + 1 y[t] == 0, y[0] == 0, y'[0] == 1}, y, t] // Flatten)) //
 Simplify
```

$$\{y \to \operatorname{Function}[\{t\}, e^t\, t]\}$$

```
((DSolve[{y''[t] + a y'[t] + b y[t] == 0, y[0] == y0, y'[0] == yS0}, y, t] // Flatten) /.
   {a → -2, b → 1, F[s] → 0, y0 → 0, yS0 → 1}) // Simplify
```

$$\Big\{y \to \text{Function}\Big[\{t\}, \frac{1}{2\sqrt{(-2)^2 - 4\,1}} \Big(-(-2)\, e^{\frac{1}{2}\left(-(-2)-\sqrt{(-2)^2-4\,1}\right) t}\, 0 + \sqrt{(-2)^2 - 4\,1}\, e^{\frac{1}{2}\left(-(-2)-\sqrt{(-2)^2-4\,1}\right) t}\, 0 - 2\, e^{\frac{1}{2}\left(-(-2)+\sqrt{(-2)^2-4\,1}\right) t}\, 0 + \sqrt{(-2)^2 - 4\,1}\, e^{\frac{1}{2}\left(-(-2)+\sqrt{(-2)^2-4\,1}\right) t}\, 0 - 2\, e^{\frac{1}{2}\left(-(-2)-\sqrt{(-2)^2-4\,1}\right) t} + 2\, e^{\frac{1}{2}\left(-(-2)+\sqrt{(-2)^2-4\,1}\right) t}\Big)\Big]\Big\}$$

```
ah = -2; bh = 1; y0h = 0; yS0h = 1;
((((DSolve[{y''[t] + a y'[t] + b y[t] == 0, y[0] == y0, y'[0] == yS0}, y, t] // Flatten) //.
      {√(a² - 4 b) → k}) //. 1/(2 √(a² - 4 b)) → 1 / (2 k)) /.
   {a → ah, b → bh, F[s] → 0, y0 → y0h, yS0 → yS0h}) /. k → Sqrt[ah^2 - 4 bh] // Evaluate
```

$$\Big\{y \to \text{Function}\Big[\{t\}, \frac{1}{2\,0}\Big(-(-2)\, e^{\frac{1}{2}(-(-2)-0) t}\, 0 + 0\, e^{\frac{1}{2}(-(-2)-0) t}\, 0 - 2\, e^{\frac{1}{2}(-(-2)+0) t}\, 0 + 0\, e^{\frac{1}{2}(-(-2)+0) t}\, 0 - 2\, e^{\frac{1}{2}(-(-2)-0) t} + 2\, e^{\frac{1}{2}(-(-2)+0) t}\Big)\Big]\Big\}$$

h

```
InverseLaplaceTransform[
 Evaluate[((a y0 + s y0 + yS0 + F[s]) / (b + a s + s²) /. {a → -3, b → 1, F[s] → 0, y0 → 0, yS0 → 1})], s, t]
```

$$\frac{e^{-\frac{1}{2}\left(-3+\sqrt{5}\right) t}\left(-1 + e^{\sqrt{5}\, t}\right)}{\sqrt{5}}$$

```
e^(-1/2 (-3+√5) t) (-1 + e^(√5 t)) / √5 // Expand
```

$$-\frac{e^{-\frac{1}{2}\left(-3+\sqrt{5}\right) t}}{\sqrt{5}} + \frac{e^{\sqrt{5}\, t-\frac{1}{2}\left(-3+\sqrt{5}\right) t}}{\sqrt{5}}$$

```
((DSolve[{y''[t] - 3 y'[t] + 1 y[t] == 0, y[0] == 0, y'[0] == 1}, y, t] // Flatten)) //
 Simplify
```

$$\Big\{y \to \text{Function}\Big[\{t\}, -\frac{e^{\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right) t} - e^{\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right) t}}{\sqrt{5}}\Big]\Big\}$$

```
((DSolve[{y''[t] + a y'[t] + b y[t] == 0, y[0] == y0, y'[0] == yS0}, y, t] // Flatten) /.
   {a → -3, b → 1, F[s] → 0, y0 → 0, yS0 → 1}) // Simplify
```

$$\Big\{y \to \text{Function}\Big[\{t\}, \frac{1}{2\sqrt{(-3)^2 - 4\,1}} \Big(-(-3)\, e^{\frac{1}{2}\left(-(-3)-\sqrt{(-3)^2-4\,1}\right) t}\, 0 + \sqrt{(-3)^2 - 4\,1}\, e^{\frac{1}{2}\left(-(-3)-\sqrt{(-3)^2-4\,1}\right) t}\, 0 - 3\, e^{\frac{1}{2}\left(-(-3)+\sqrt{(-3)^2-4\,1}\right) t}\, 0 + \sqrt{(-3)^2 - 4\,1}\, e^{\frac{1}{2}\left(-(-3)+\sqrt{(-3)^2-4\,1}\right) t}\, 0 - 2\, e^{\frac{1}{2}\left(-(-3)-\sqrt{(-3)^2-4\,1}\right) t} + 2\, e^{\frac{1}{2}\left(-(-3)+\sqrt{(-3)^2-4\,1}\right) t}\Big)\Big]\Big\}$$

```
ah = -3; bh = 1; y0h = 0; yS0h = 1;
((((DSolve[{y''[t] + a y'[t] + b y[t] == 0, y[0] == y0, y'[0] == yS0}, y, t] // Flatten) //.
      {Sqrt[a^2 - 4 b] -> k}) //. 1/(2 Sqrt[a^2 - 4 b]) -> 1/(2 k)) /.
    {a -> ah, b -> bh, F[s] -> 0, y0 -> y0h, yS0 -> yS0h}) /. k -> Sqrt[ah^2 - 4 bh] // Evaluate
```

$$\left\{y\to \text{Function}\left[\{t\},\ \frac{1}{2\sqrt{5}}\left(-(-3)\, e^{\frac{1}{2}\left(-(-3)-\sqrt{5}\right)t}\,0+\sqrt{5}\, e^{\frac{1}{2}\left(-(-3)-\sqrt{5}\right)t}\,0-3\, e^{\frac{1}{2}\left(-(-3)+\sqrt{5}\right)t}\,0+\sqrt{5}\, e^{\frac{1}{2}\left(-(-3)+\sqrt{5}\right)t}\,0-2\, e^{\frac{1}{2}\left(-(-3)-\sqrt{5}\right)t}+2\, e^{\frac{1}{2}\left(-(-3)+\sqrt{5}\right)t}\right)\right]\right\}$$

i

```
Remove["Global`*"]
```

```
InverseLaplaceTransform[
 Evaluate[((a y0 + s y0 + yS0 + F[s])/(b + a s + s^2) /. {a -> 0, b -> -1, F[s] -> 1, y0 -> 0, yS0 -> 0})], s, t]
```

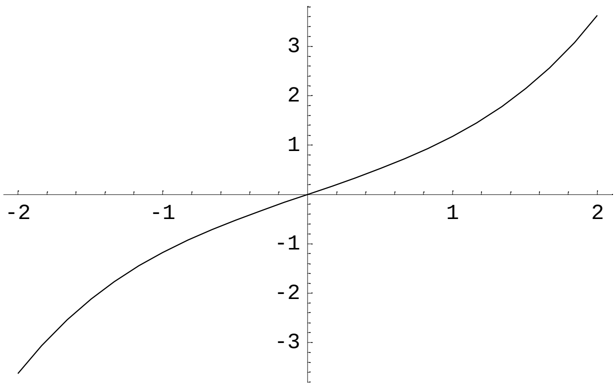
$$\frac{1}{2}e^{-t}\left(-1+e^{2t}\right)$$

```
1/2 E^-t (-1 + E^(2 t)) // Expand
```

$$-\frac{e^{-t}}{2}+\frac{e^{t}}{2}$$

```
res[t_] := (E^t - E^-t)/2
```

```
Plot[{res[t], Sinh[t]}, {t, -2, 2}];
```




```
(res[5] // N) == (Sinh[5] // N)
```

True

```
((DSolve[{y''[t] + 0 y'[t] - 1 y[t] == DiracDelta[t], y[0] == 0, y'[0] == 0}, y, t] //
    Flatten)) // Simplify
```

$$\left\{y\to \text{Function}\left[\{t\},\ \frac{1}{2}e^{-t}\left(-1+e^{2t}\right)\left(-1+\text{UnitStep}[t]\right)\right]\right\}$$

```
((DSolve[{y''[t] + 0 y'[t] - 1 y[t] == DiracDelta[0], y[0] == 0, y'[0] == 0}, y, t] //
    Flatten)) // Simplify
```

$$\left\{y \to \text{Function}\left[\{t\}, \frac{1}{2} e^{-t} (-1 + e^{t})^2 \,\text{DiracDelta}[0]\right]\right\}$$

Problem in der Grenzsituation mit der Diracfunktion und den Anfangsbedingungen!

```
((DSolve[{y''[t] + a y'[t] + b y[t] == f[t], y[0] == y0, y'[0] == yS0}, y, t] // Flatten) /.
   {a -> 0, b -> -1, f[t] -> DiracDelta[t], y0 -> 0, yS0 -> 0}) // Simplify
```

$$\Bigg\{y \to \text{Function}\Bigg[\{t\}, \frac{1}{2\sqrt{0^2-4(-1)}} \Bigg(-0\, e^{\frac{1}{2}\left(-0-\sqrt{0^2-4(-1)}\right)t}\, 0 + \sqrt{0^2-4(-1)}\, e^{\frac{1}{2}\left(-0-\sqrt{0^2-4(-1)}\right)t}\, 0 + 0\, e^{\frac{1}{2}\left(-0+\sqrt{0^2-4(-1)}\right)t}\, 0 +$$
$$\sqrt{0^2-4(-1)}\, e^{\frac{1}{2}\left(-0+\sqrt{0^2-4(-1)}\right)t}\, 0 - 2\, e^{\frac{1}{2}\left(-0-\sqrt{0^2-4(-1)}\right)t}\, 0 + 2\, e^{\frac{1}{2}\left(-0+\sqrt{0^2-4(-1)}\right)t}\, 0 -$$
$$2\sqrt{0^2-4(-1)}\, e^{\frac{1}{2}\left(-0-\sqrt{0^2-4(-1)}\right)t} \int_1^0 -\frac{e^{0\, K\$1187+\frac{1}{2}\left(-0+\sqrt{0^2-4(-1)}\right)K\$1187}\, f[K\$1187]}{\sqrt{0^2-4(-1)}}\, dK\$1187 +$$
$$2\sqrt{0^2-4(-1)}\, e^{\frac{1}{2}\left(-0-\sqrt{0^2-4(-1)}\right)t} \int_1^t -\frac{e^{0\, K\$1187+\frac{1}{2}\left(-0+\sqrt{0^2-4(-1)}\right)K\$1187}\, f[K\$1187]}{\sqrt{0^2-4(-1)}}\, dK\$1187 -$$
$$2\sqrt{0^2-4(-1)}\, e^{\frac{1}{2}\left(-0+\sqrt{0^2-4(-1)}\right)t} \int_1^0 \frac{e^{0\, K\$1208+\frac{1}{2}\left(-0-\sqrt{0^2-4(-1)}\right)K\$1208}\, f[K\$1208]}{\sqrt{0^2-4(-1)}}\, dK\$1208 +$$
$$2\sqrt{0^2-4(-1)}\, e^{\frac{1}{2}\left(-0+\sqrt{0^2-4(-1)}\right)t} \int_1^t \frac{e^{0\, K\$1208+\frac{1}{2}\left(-0-\sqrt{0^2-4(-1)}\right)K\$1208}\, f[K\$1208]}{\sqrt{0^2-4(-1)}}\, dK\$1208\Bigg)\Bigg]\Bigg\}$$

```
ah = 0; bh = -1; y0h = 0; yS0h = 0;
((((DSolve[{y''[t] + a y'[t] + b y[t] == f[t], y[0] == y0, y'[0] == yS0}, y, t] //
       Flatten) //. {Sqrt[a^2 - 4 b] -> k}) //. 1/(2 Sqrt[a^2 - 4 b]) -> 1/(2 k)) /.
    {a -> ah, b -> bh, f[t] -> DiracDelta[t], y0 -> y0h, yS0 -> yS0h}) /.
  k -> Sqrt[ah^2 - 4 bh] // Evaluate
```

$$\Bigg\{y \to \text{Function}\Bigg[\{t\},$$
$$\frac{1}{2\,2}\Bigg(-0\, e^{\frac{1}{2}(-0-2)t}\, 0 + 2\, e^{\frac{1}{2}(-0-2)t}\, 0 + 0\, e^{\frac{1}{2}(-0+2)t}\, 0 + 2\, e^{\frac{1}{2}(-0+2)t}\, 0 - 2\, e^{\frac{1}{2}(-0-2)t}\, 0 +$$
$$2\, e^{\frac{1}{2}(-0+2)t}\, 0 - 2\,2\, e^{\frac{1}{2}(-0-2)t} \int_1^0 -\frac{e^{0\, K\$1273+\frac{1}{2}(-0+2)K\$1273}\, f[K\$1273]}{\sqrt{0^2-4(-1)}}\, dK\$1273 +$$
$$2\,2\, e^{\frac{1}{2}(-0-2)t} \int_1^t -\frac{e^{0\, K\$1273+\frac{1}{2}(-0+2)K\$1273}\, f[K\$1273]}{\sqrt{0^2-4(-1)}}\, dK\$1273 -$$
$$2\,2\, e^{\frac{1}{2}(-0+2)t} \int_1^0 \frac{e^{0\, K\$1294+\frac{1}{2}(-0-2)K\$1294}\, f[K\$1294]}{\sqrt{0^2-4(-1)}}\, dK\$1294 +$$
$$2\,2\, e^{\frac{1}{2}(-0+2)t} \int_1^t \frac{e^{0\, K\$1294+\frac{1}{2}(-0-2)K\$1294}\, f[K\$1294]}{\sqrt{0^2-4(-1)}}\, dK\$1294\Bigg)\Bigg]\Bigg\}$$

j

```
InverseLaplaceTransform[
 Evaluate[((a y0 + s y0 + yS0 + F[s])/(b + a s + s^2) /. {a → 0, b → 1, F[s] → 1, y0 → 0, yS0 → 0})], s, t]
```

```
Sin[t]
```

```
((DSolve[{y''[t] + 0 y'[t] + y[t] == DiracDelta[0], y[0] == 0, y'[0] == 0}, y, t] //
    Flatten)) // Simplify
```

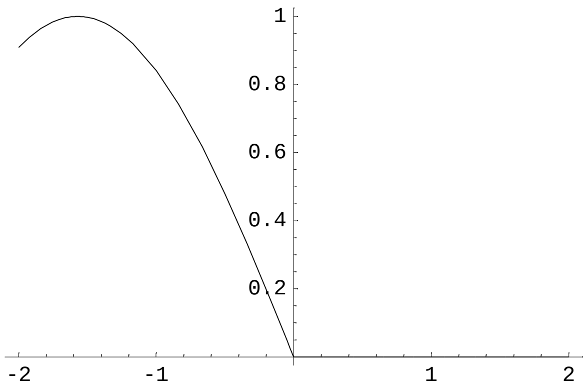
```
{y → Function[{t}, DiracDelta[0] - Cos[t] DiracDelta[0]]}
```

```
((DSolve[{y''[t] + 0 y'[t] + y[t] == DiracDelta[t], y[0] == 0, y'[0] == 0}, y, t] //
    Flatten)) // Simplify
```

```
{y → Function[{t}, -Sin[t] + Sin[t] UnitStep[t]]}
```

Problem in der Grenzsituation mit der Diracfunktion und den Anfangsbedingungen!

```
Plot[-Sin[t] + Sin[t] UnitStep[t], {t, -2, 2}];
```




```
((DSolve[{y''[t] + a y'[t] + b y[t] == f[t], y[0] == y0, y'[0] == yS0}, y, t] // Flatten) /.
   {a → 0, b → 1, f[t] → DiracDelta[t], y0 → 0, yS0 → 0}) // Simplify
```

$$\Big\{y \to \text{Function}\Big[\{t\},\ \frac{1}{2\sqrt{0^2-4\,1}}\Big(-0\, e^{\frac{1}{2}\left(-0-\sqrt{0^2-4\,1}\right)t}\,0+\sqrt{0^2-4\,1}\, e^{\frac{1}{2}\left(-0-\sqrt{0^2-4\,1}\right)t}\,0+0\, e^{\frac{1}{2}\left(-0+\sqrt{0^2-4\,1}\right)t}\,0+\sqrt{0^2-4\,1}\, e^{\frac{1}{2}\left(-0+\sqrt{0^2-4\,1}\right)t}\,0-2\, e^{\frac{1}{2}\left(-0-\sqrt{0^2-4\,1}\right)t}\,0+2\, e^{\frac{1}{2}\left(-0+\sqrt{0^2-4\,1}\right)t}\,0-$$

$$2\sqrt{0^2-4\,1}\, e^{\frac{1}{2}\left(-0-\sqrt{0^2-4\,1}\right)t}\int_1^0 -\frac{e^{0\,K\$1362+\frac{1}{2}\left(-0+\sqrt{0^2-4\,1}\right)K\$1362}\, f[K\$1362]}{\sqrt{0^2-4\,1}}\,dK\$1362+$$

$$2\sqrt{0^2-4\,1}\, e^{\frac{1}{2}\left(-0-\sqrt{0^2-4\,1}\right)t}\int_1^t -\frac{e^{0\,K\$1362+\frac{1}{2}\left(-0+\sqrt{0^2-4\,1}\right)K\$1362}\, f[K\$1362]}{\sqrt{0^2-4\,1}}\,dK\$1362-$$

$$2\sqrt{0^2-4\,1}\, e^{\frac{1}{2}\left(-0+\sqrt{0^2-4\,1}\right)t}\int_1^0 \frac{e^{0\,K\$1383+\frac{1}{2}\left(-0-\sqrt{0^2-4\,1}\right)K\$1383}\, f[K\$1383]}{\sqrt{0^2-4\,1}}\,dK\$1383+$$

$$2\sqrt{0^2-4\,1}\, e^{\frac{1}{2}\left(-0+\sqrt{0^2-4\,1}\right)t}\int_1^t \frac{e^{0\,K\$1383+\frac{1}{2}\left(-0-\sqrt{0^2-4\,1}\right)K\$1383}\, f[K\$1383]}{\sqrt{0^2-4\,1}}\,dK\$1383\Big)\Big]\Big\}$$

```
ah = 0; bh = 1; y0h = 0; yS0h = 0;
(((DSolve[{y''[t] + a y'[t] + b y[t] == f[t], y[0] == y0, y'[0] == yS0}, y, t] //
        Flatten) //. {Sqrt[a^2 - 4 b] -> k}) //. 1/(2 Sqrt[a^2 - 4 b]) -> 1/(2 k)) /.
    {a -> ah, b -> bh, f[t] -> DiracDelta[t], y0 -> y0h, yS0 -> yS0h}) /.
  k -> Sqrt[ah^2 - 4 bh] // Evaluate
```

$$\Big\{y\to\text{Function}\Big[\{t\},\ \frac{1}{2\,2\,i}\Big(-0\,e^{\frac{1}{2}(-0-2i)t}\,0+2\,i\,e^{\frac{1}{2}(-0-2i)t}\,0+0\,e^{\frac{1}{2}(-0+2i)t}\,0+2\,i\,e^{\frac{1}{2}(-0+2i)t}\,0-2\,e^{\frac{1}{2}(-0-2i)t}\,0+$$
$$2\,e^{\frac{1}{2}(-0+2i)t}\,0-2\,2\,i\,e^{\frac{1}{2}(-0-2i)t}\int_1^0-\frac{e^{0\,K\$1448+\frac{1}{2}(-0+2i)K\$1448}\,f[K\$1448]}{\sqrt{0^2-4\,1}}\,dK\$1448+$$
$$2\,2\,i\,e^{\frac{1}{2}(-0-2i)t}\int_1^t-\frac{e^{0\,K\$1448+\frac{1}{2}(-0+2i)K\$1448}\,f[K\$1448]}{\sqrt{0^2-4\,1}}\,dK\$1448-$$
$$2\,2\,i\,e^{\frac{1}{2}(-0+2i)t}\int_1^0\frac{e^{0\,K\$1469+\frac{1}{2}(-0-2i)K\$1469}\,f[K\$1469]}{\sqrt{0^2-4\,1}}\,dK\$1469+$$
$$2\,2\,i\,e^{\frac{1}{2}(-0+2i)t}\int_1^t\frac{e^{0\,K\$1469+\frac{1}{2}(-0-2i)K\$1469}\,f[K\$1469]}{\sqrt{0^2-4\,1}}\,dK\$1469\Big)\Big]\Big\}$$

3

```
VK = 4 / 3 10^3 Pi
```

$$\frac{4000\,\pi}{3}$$

```
VK // N
```

4188.79

```
VZ = NIntegrate[1, {x, 1, 5}, {y, -Sqrt[2^2 - (x - 3)^2], Sqrt[2^2 - (x - 3)^2]},
  {z, -Sqrt[10^2 - x^2 - y^2], Sqrt[10^2 - x^2 - y^2]}]
```

236.96

```
V = VK - VZ
```

3951.83

4

a

```
InverseLaplaceTransform[1/(s^3 - 3 s^2 + 3 s - 1), s, t] // Expand
```

$\frac{e^t t^2}{2}$

```
DSolve[{y'''[t] - 3 y''[t] + 3 y'[t] - y[t] == DiracDelta[0],
  y[0] == 0, y'[0] == 0, y''[0] == 0}, y, t]
```

$\{\{y \to \text{Function}[\{t\}, \frac{1}{2} (-2 + 2 e^t - 2 e^t t + e^t t^2) \text{DiracDelta}[0]]\}\}$

b

```
LaplaceTransform[E^(-t), t, s]
```

$\frac{1}{1+s}$

```
InverseLaplaceTransform[1/(1 + s) 1/(s^3 - 3 s^2 + 3 s - 1), s, t] // Expand
```

$-\frac{e^{-t}}{8} + \frac{e^t}{8} - \frac{e^t t}{4} + \frac{e^t t^2}{4}$

```
DSolve[
 {y'''[t] - 3 y''[t] + 3 y'[t] - y[t] == E^(-t), y[0] == 0, y'[0] == 0, y''[0] == 0}, y, t]
```

$\{\{y \to \text{Function}[\{t\}, \frac{1}{8} e^{-t} (-1 + e^{2t} - 2 e^{2t} t + 2 e^{2t} t^2)]\}\}$

c

```
(LaplaceTransform[y'''[t] - 3 y''[t] + 3 y'[t] - y[t], t, s] /.
   LaplaceTransform[y[t], t, s] → Y[s]) // Expand
```

$-3 y[0] + 3 s y[0] - s^2 y[0] - Y[s] + 3 s Y[s] - 3 s^2 Y[s] + s^3 Y[s] + 3 y'[0] - s y'[0] - y''[0]$

```
InverseLaplaceTransform[(1/(1 + s) + s^2 - 3 s + 3) (1/(s^3 - 3 s^2 + 3 s - 1)), s, t] // Expand
```

$-\frac{e^{-t}}{8} + \frac{9 e^t}{8} - \frac{5 e^t t}{4} + \frac{3 e^t t^2}{4}$

```
DSolve[{y'''[t] - 3 y''[t] + 3 y'[t] - y[t] == E^(-t),
   y[0] == 1, y'[0] == 0, y''[0] == 0}, y, t] // ExpandAll
```

$\{\{y \to \text{Function}[\{t\}, \frac{1}{8} e^{-t} (-1 + 9 e^{2t} - 10 e^{2t} t + 6 e^{2t} t^2)]\}\}$

$$\frac{1}{8} e^{-t} \left(-1 + 9 e^{2t} - 10 e^{2t} t + 6 e^{2t} t^2\right) \text{ // ExpandAll}$$

$$-\frac{e^{-t}}{8} + \frac{9 e^t}{8} - \frac{5 e^t t}{4} + \frac{3 e^t t^2}{4}$$

5

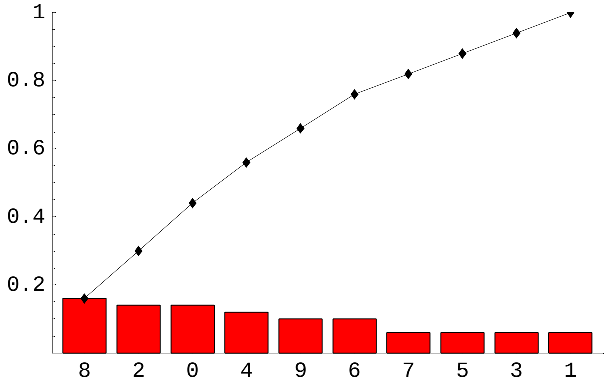
```
<< Statistics`DescriptiveStatistics`

<< Statistics`StatisticsPlots`

<< Statistics`DataManipulation`

M2 = Reverse[{3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3, 2, 3, 8, 4, 6, 2, 6, 4,
   3, 3, 8, 3, 2, 7, 9, 5, 0, 2, 8, 8, 4, 1, 9, 7, 1, 6, 9, 3, 9, 9, 3, 7, 5, 1}]

{1, 5, 7, 3, 9, 9, 3, 9, 6, 1, 7, 9, 1, 4, 8, 8, 2, 0, 5, 9, 7, 2, 3, 8,
 3, 3, 4, 6, 2, 6, 4, 8, 3, 2, 3, 9, 7, 9, 8, 5, 3, 5, 6, 2, 9, 5, 1, 4, 1, 3}

M1 = Reverse[{0, 5, 8, 2, 0, 9, 7, 4, 9, 4, 4, 5, 9, 2, 3, 0, 7, 8, 1, 6, 4, 0, 6, 2,
   8, 6, 2, 0, 8, 9, 9, 8, 6, 2, 8, 0, 3, 4, 8, 2, 5, 3, 4, 2, 1, 1, 7, 0, 6, 8}]

{8, 6, 0, 7, 1, 1, 2, 4, 3, 5, 2, 8, 4, 3, 0, 8, 2, 6, 8, 9, 9, 8, 0, 2,
 6, 8, 2, 6, 0, 4, 6, 1, 8, 7, 0, 3, 2, 9, 5, 4, 4, 9, 4, 7, 9, 0, 2, 8, 5, 0}

Mm1 = {3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3, 2, 3, 8, 4, 6, 2, 6, 4,
  3, 3, 8, 3, 2, 7, 9, 5, 0, 2, 8, 8, 4, 1, 9, 7, 1, 6, 9, 3, 9, 9, 3, 7, 5, 1}

{3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3, 2, 3, 8, 4, 6, 2, 6, 4,
 3, 3, 8, 3, 2, 7, 9, 5, 0, 2, 8, 8, 4, 1, 9, 7, 1, 6, 9, 3, 9, 9, 3, 7, 5, 1}

Mm2 = {0, 5, 8, 2, 0, 9, 7, 4, 9, 4, 4, 5, 9, 2, 3, 0, 7, 8, 1, 6, 4, 0, 6, 2,
  8, 6, 2, 0, 8, 9, 9, 8, 6, 2, 8, 0, 3, 4, 8, 2, 5, 3, 4, 2, 1, 1, 7, 0, 6, 8}

{0, 5, 8, 2, 0, 9, 7, 4, 9, 4, 4, 5, 9, 2, 3, 0, 7, 8, 1, 6, 4, 0, 6, 2,
 8, 6, 2, 0, 8, 9, 9, 8, 6, 2, 8, 0, 3, 4, 8, 2, 5, 3, 4, 2, 1, 1, 7, 0, 6, 8}
```

a

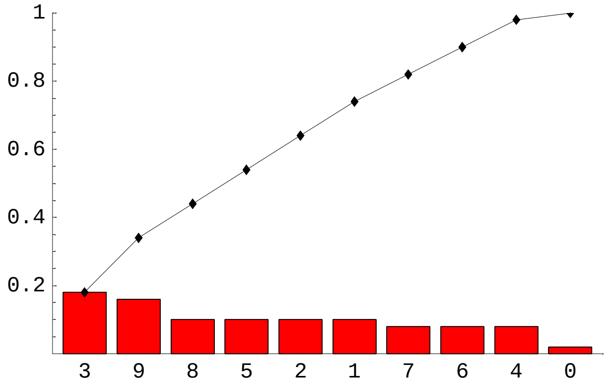
```
Frequencies[M1]

{{7, 0}, {3, 1}, {7, 2}, {3, 3}, {6, 4}, {3, 5}, {5, 6}, {3, 7}, {8, 8}, {5, 9}}

Frequencies[M2]

{{1, 0}, {5, 1}, {5, 2}, {9, 3}, {4, 4}, {5, 5}, {4, 6}, {4, 7}, {5, 8}, {8, 9}}
```

b

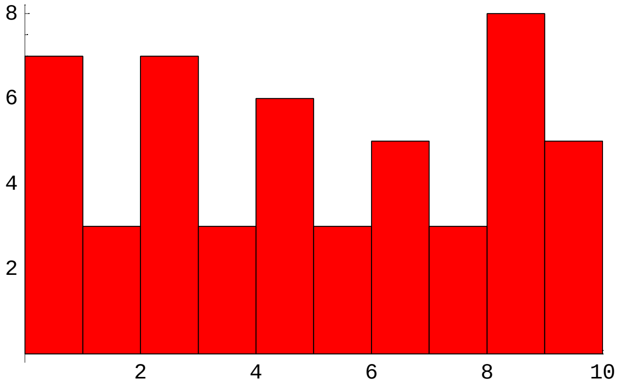
```
ParetoPlot[M1];
```



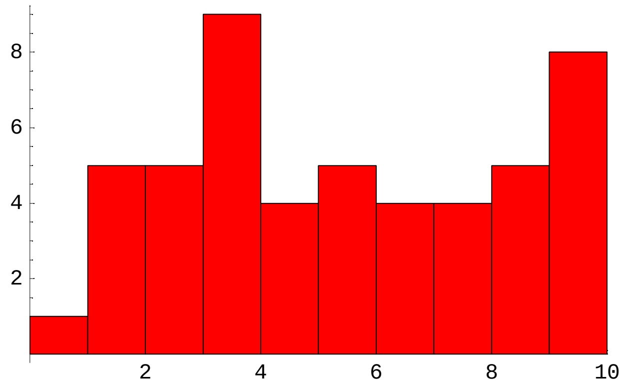

```
ParetoPlot[M2];
```




```
Histogram[M1];
```



Histogram[**M2**];



c

Length[**M1**]

50

LocationReport[**M1**] // **N**

Power::infy : Infinite expression $\frac{1}{0}$ encountered. Mehr...

Power::infy : Infinite expression $\frac{1}{0}$ encountered. Mehr...

Power::infy : Infinite expression $\frac{1}{0}$ encountered. Mehr...

General::stop : Further output of Power::infy will be suppressed during this calculation. Mehr...

∞::indet :
Indeterminate expression $\frac{7823}{630}$ + ComplexInfinity + ComplexInfinity + ComplexInfinity + ComplexInfinity + ComplexInfinity + ComplexInfinity + ComplexInfinity encountered. Mehr...

{Mean → 4.5, HarmonicMean → Indeterminate, Median → 4.}

DispersionReport[**M1**] // **N**

{Variance → 9.39796, StandardDeviation → 3.06561, SampleRange → 9.,
MeanDeviation → 2.68, MedianDeviation → 3., QuartileDeviation → 3.}

Length[**M2**]

50

LocationReport[**M2**] // **N**

Power::infy : Infinite expression $\frac{1}{0}$ encountered. Mehr...

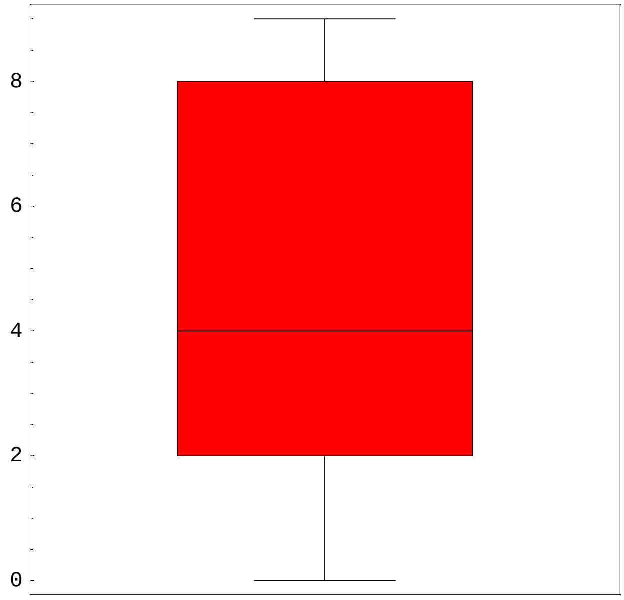
{Mean → 4.94, HarmonicMean → 0., Median → 5.}

DispersionReport[**M2**] // **N**

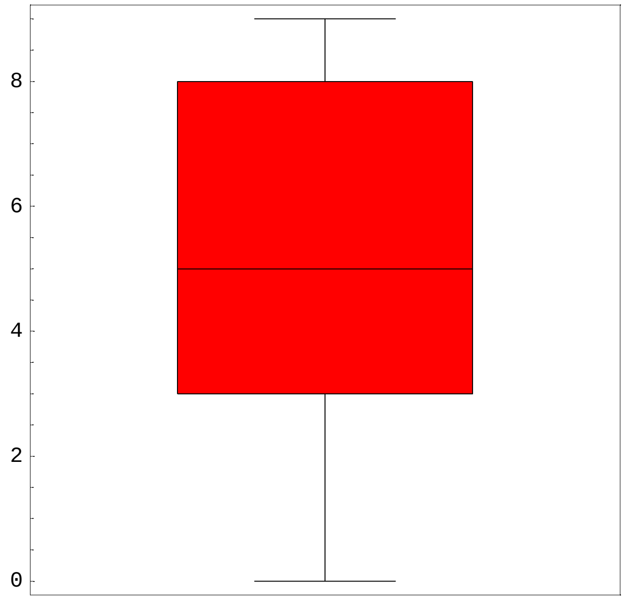
{Variance → 7.81265, StandardDeviation → 2.79511, SampleRange → 9.,
MeanDeviation → 2.4224, MedianDeviation → 2., QuartileDeviation → 2.5}

d

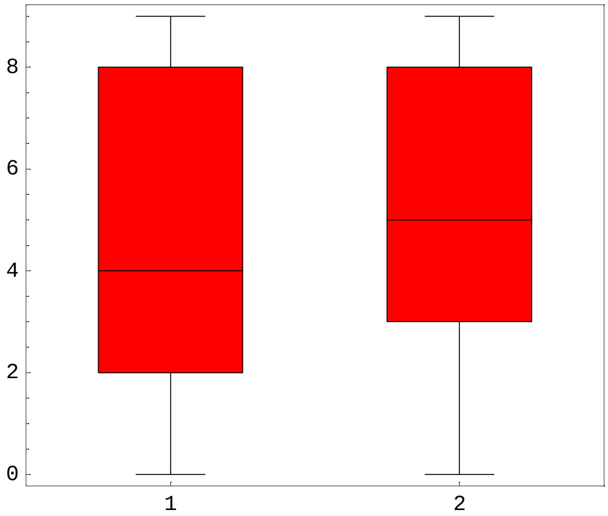
```
BoxWhiskerPlot[M1];
```




```
BoxWhiskerPlot[M2];
```



```
BoxWhiskerPlot[Transpose[{M1, M2}]];
```



e

Es handelt sich um je 50 Stellen der Dezimalpruchentwicklung von .

```
N[Pi, 150] - 314159 / 100000
```

2.65358979323846264338327950288419716939937510582097494459230781640628620899862803⋱
48253421170679821480865132823066470938446095505822317253594081 3$\times 10^{-6}$

```
N[Pi, 150]
```

3.14159265358979323846264338327950288419716939937510582097494459230781640628620899⋱
8628034825342117067982148086513282306647093844609550582231725359408 13

```
26535897932384626433832795028841971693993751058209749445923078164062862089986280 34⋱
825342
```

2653589793238462643383279502884197169399375105820974944592307816406286208998628034⋱
825342

```
3.14159265358979323846264338327950288419716939937510582097494459230781640628620899⋱
  8628034825342117067982148086513282306647093844609550582231725359408128481117450377
  78 10^30 // N
```

3.14159×10^{30}

```
N[Pi, 50]
```

3.1415926535897932384626433832795028841971693993751

```
N[Pi, 100]
```

3.14159265358979323846264338327950288419716939937510582097494459230781640628620899⋱
8628034825342117068

6

a

```
14!
```

87178291200

```
14! // N
```

8.71783×10^{10}

Bei Wiederholung:

```
14^14
```

11112006825558016

```
% // N
```

1.1112×10^{16}

b

```
50 / 7 // N
```

7.14286

```
7 Binomial[50, 7] Binomial[50 - 7, 7] Binomial[50 - 2 7, 7] Binomial[50 - 3 7, 7]
 Binomial[50 - 4 7, 7] Binomial[50 - 5 7, 7] Binomial[50 - 6 7, 7]
```

2577265483155016361393904911710617600000

```
% // N
```

2.57727×10^{39}

```
7 Product[Binomial[50 - k 7, 7], {k, 0, 6}]
```

2577265483155016361393904911710617600000

```
% // N
```

2.57727×10^{39}