Lösungen

2

```
Remove["Global`*"]
```

a

```
(LaplaceTransform[y''[t] + a y'[t] + b y[t], t, s] /.
   {y[0] → y0, y′[0] → yS0, LaplaceTransform[y[t], t, s] → Y[s]}) ==
 (LaplaceTransform[f[t]] /. {LaplaceTransform[f[t]] → F[s]})
```

$-s\, y0 - yS0 + b\, Y[s] + s^2\, Y[s] + a\, (-y0 + s\, Y[s]) == F[s]$

b

```
Solve[-s y0 - yS0 + b Y[s] + s^2 Y[s] + a (-y0 + s Y[s]) == F[s], { Y[s]}]
```

$\left\{\left\{Y[s] \to \frac{a\, y0 + s\, y0 + yS0 + F[s]}{b + a\, s + s^2}\right\}\right\}$

c

```
InverseLaplaceTransform[
 Evaluate[((a y0 + s y0 + yS0 + F[s])/(b + a s + s^2) /. {a → 1, b → 1, F[s] → 0, y0 → 1, yS0 → 0})], s, t]
```

$\frac{1}{3}\, e^{-t/2} \left(3\, \mathrm{Cos}\left[\frac{\sqrt{3}\, t}{2}\right] + \sqrt{3}\, \mathrm{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]\right)$

```
((DSolve[{y''[t] + 1 y'[t] + 1 y[t] == 0, y[0] == 1, y'[0] == 0}, y, t] // Flatten)) //
 Simplify
```

$\left\{y \to \mathrm{Function}\left[\{t\},\ \frac{1}{3}\, e^{-t/2} \left(3\, \mathrm{Cos}\left[\frac{\sqrt{3}\, t}{2}\right] + \sqrt{3}\, \mathrm{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]\right)\right]\right\}$

d

```
InverseLaplaceTransform[
 Evaluate[((a y0 + s y0 + yS0 + F[s])/(b + a s + s^2) /. {a → 1, b → 1, F[s] → 0, y0 → 0, yS0 → 1})], s, t]
```

$\frac{2\, e^{-t/2}\, \mathrm{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]}{\sqrt{3}}$

```
((DSolve[{y''[t] + 1 y'[t] + 1 y[t] == 0, y[0] == 0, y'[0] == 1}, y, t] // Flatten)) //
 Simplify
```

$$\left\{y \to \text{Function}\left[\{t\}, \frac{2\, e^{-t/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]}{\sqrt{3}}\right]\right\}$$

e

```
InverseLaplaceTransform[
 Evaluate[((a y0 + s y0 + yS0 + F[s])/(b + a s + s^2) /. {a → 1, b → 1, F[s] → 0, y0 → 1, yS0 → 1})], s, t]
```

$$e^{-t/2} \left(\operatorname{Cos}\left[\frac{\sqrt{3}\, t}{2}\right] + \sqrt{3} \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]\right)$$

```
((DSolve[{y''[t] + 1 y'[t] + 1 y[t] == 0, y[0] == 1, y'[0] == 1}, y, t] // Flatten)) //
 Simplify
```

$$\left\{y \to \text{Function}\left[\{t\}, e^{-t/2} \left(\operatorname{Cos}\left[\frac{\sqrt{3}\, t}{2}\right] + \sqrt{3} \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]\right)\right]\right\}$$

f

```
InverseLaplaceTransform[
 Evaluate[((a y0 + s y0 + yS0 + F[s])/(b + a s + s^2) /. {a → -1, b → 1, F[s] → 0, y0 → 0, yS0 → 1})], s, t]
```

$$\frac{2\, e^{t/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]}{\sqrt{3}}$$

```
((DSolve[{y''[t] - 1 y'[t] + 1 y[t] == 0, y[0] == 0, y'[0] == 1}, y, t] // Flatten)) //
 Simplify
```

$$\left\{y \to \text{Function}\left[\{t\}, \frac{2\, e^{t/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, t}{2}\right]}{\sqrt{3}}\right]\right\}$$

g

```
InverseLaplaceTransform[
 Evaluate[((a y0 + s y0 + yS0 + F[s])/(b + a s + s^2) /. {a → -2, b → 1, F[s] → 0, y0 → 0, yS0 → 1})], s, t]
```

$$e^{t}\, t$$

```
((DSolve[{y''[t] - 2 y'[t] + 1 y[t] == 0, y[0] == 0, y'[0] == 1}, y, t] // Flatten)) //
 Simplify
```

$$\{y \to \text{Function}[\{t\}, e^{t}\, t]\}$$

h

InverseLaplaceTransform[
$$\textbf{Evaluate}\left[\left(\frac{\mathbf{a\,y0 + s\,y0 + yS0 + F[s]}}{\mathbf{b + a\,s + s^2}}\ \mathbf{/.\ \{a \to -3,\ b \to 1,\ F[s] \to 0,\ y0 \to 0,\ yS0 \to 1\}}\right)\right],\ \mathbf{s},\ \mathbf{t}\Big]$$

$$\frac{e^{-\frac{1}{2}\left(-3+\sqrt{5}\right)t}\left(-1+e^{\sqrt{5}\,t}\right)}{\sqrt{5}}$$

$$\frac{e^{-\frac{1}{2}\left(-3+\sqrt{5}\right)t}\left(-1+e^{\sqrt{5}\,t}\right)}{\sqrt{5}}\ \mathbf{//\ Expand}$$

$$-\frac{e^{-\frac{1}{2}\left(-3+\sqrt{5}\right)t}}{\sqrt{5}}+\frac{e^{\sqrt{5}\,t-\frac{1}{2}\left(-3+\sqrt{5}\right)t}}{\sqrt{5}}$$

```
((DSolve[{y''[t] - 3 y'[t] + 1 y[t] == 0, y[0] == 0, y'[0] == 1}, y, t] // Flatten)) //
 Simplify
```

$$\left\{y \to \text{Function}\left[\{t\},\ -\frac{e^{\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)t}-e^{\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right)t}}{\sqrt{5}}\right]\right\}$$

i

```
Remove["Global`*"]
```

InverseLaplaceTransform[
$$\textbf{Evaluate}\left[\left(\frac{\mathbf{a\,y0 + s\,y0 + yS0 + F[s]}}{\mathbf{b + a\,s + s^2}}\ \mathbf{/.\ \{a \to 0,\ b \to -1,\ F[s] \to 1,\ y0 \to 0,\ yS0 \to 0\}}\right)\right],\ \mathbf{s},\ \mathbf{t}\Big]$$

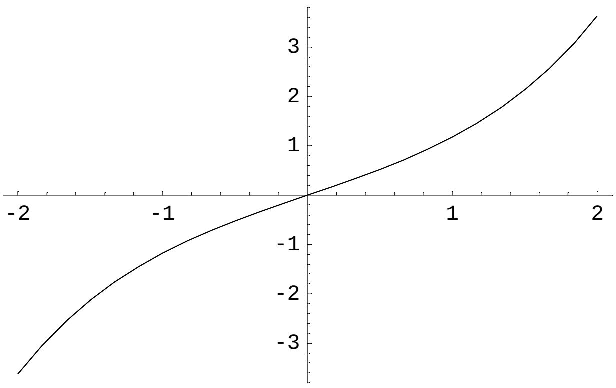
$$\frac{1}{2}\,e^{-t}\left(-1+e^{2t}\right)$$

$$\frac{1}{2}\,e^{-t}\left(-1+e^{2t}\right)\ \mathbf{//\ Expand}$$

$$-\frac{e^{-t}}{2}+\frac{e^{t}}{2}$$

$$\mathbf{res[t\_]\ :=\ \frac{e^{t}-e^{-t}}{2}}$$

```
Plot[{res[t], Sinh[t]}, {t, -2, 2}];
```



```
(res[5] // N) == (Sinh[5] // N)
```

```
True
```

```
((DSolve[{y''[t] + 0 y'[t] - 1 y[t] == DiracDelta[t], y[0] == 0, y'[0] == 0}, y, t] //
    Flatten)) // Simplify
```

$\{y \to \text{Function}[\{t\}, \frac{1}{2} e^{-t} (-1 + e^{2t}) (-1 + \text{UnitStep}[t])]\}$

```
((DSolve[{y''[t] + 0 y'[t] - 1 y[t] == DiracDelta[0], y[0] == 0, y'[0] == 0}, y, t] //
    Flatten)) // Simplify
```

$\{y \to \text{Function}[\{t\}, \frac{1}{2} e^{-t} (-1 + e^{t})^2 \text{DiracDelta}[0]]\}$

Problem in der Grenzsituation mit der Diracfunktion und den Anfangsbedingungen!

j

$$\text{InverseLaplaceTransform}\left[\text{Evaluate}\left[\left(\frac{a\,y0 + s\,y0 + yS0 + F[s]}{b + a\,s + s^2} \text{ /. } \{a \to 0, b \to 1, F[s] \to 1, y0 \to 0, yS0 \to 0\}\right)\right], s, t\right]$$

```
Sin[t]
```

```
((DSolve[{y''[t] + 0 y'[t] + y[t] == DiracDelta[0], y[0] == 0, y'[0] == 0}, y, t] //
    Flatten)) // Simplify
```

```
{y → Function[{t}, DiracDelta[0] - Cos[t] DiracDelta[0]]}
```

```
((DSolve[{y''[t] + 0 y'[t] + y[t] == DiracDelta[t], y[0] == 0, y'[0] == 0}, y, t] //
    Flatten)) // Simplify
```

```
{y → Function[{t}, -Sin[t] + Sin[t] UnitStep[t]]}
```

Problem in der Grenzsituation mit der Diracfunktion und den Anfangsbedingungen!

3

```
VK = 4 / 3 10^3 Pi
```

$\frac{4000\,\pi}{3}$

```
VK // N
```

```
4188.79
```

```
VZ = NIntegrate[1, {x, 1, 5}, {y, -Sqrt[2^2 - (x - 3)^2], Sqrt[2^2 - (x - 3)^2]},
   {z, -Sqrt[10^2 - x^2 - y^2], Sqrt[10^2 - x^2 - y^2]}]
```

```
236.96
```

```
V = VK - VZ
```

```
3951.83
```

```
Integrate[1, {x, 1, 5}, {y, -Sqrt[2^2 - (x - 3)^2], Sqrt[2^2 - (x - 3)^2]},
 {z, -Sqrt[10^2 - x^2 - y^2], Sqrt[10^2 - x^2 - y^2]}]
```

$$-\frac{1}{15\sqrt{11}}\left(2\,i\left(69267\,\text{EllipticE}\left[\frac{25}{33}\right]-4\left(18023\,\text{EllipticK}\left[\frac{25}{33}\right]-5000\left(\text{EllipticPi}\left[\frac{5}{11},\frac{25}{33}\right]-\text{EllipticPi}\left[\frac{5}{3},\frac{25}{33}\right]\right)\right)\right)\right)$$

4

a

$$\text{InverseLaplaceTransform}\left[\frac{1}{s^3-3s^2+3s-1},s,t\right]\ //\ \text{Expand}$$

$$\frac{e^t t^2}{2}$$

```
DSolve[{y'''[t] - 3 y''[t] + 3 y'[t] - y[t] == DiracDelta[0],
  y[0] == 0, y'[0] == 0, y''[0] == 0}, y, t]
```

$$\left\{\left\{y\to\text{Function}\left[\{t\},\frac{1}{2}\left(-2+2e^t-2e^t t+e^t t^2\right)\text{DiracDelta}[0]\right]\right\}\right\}$$

b

```
LaplaceTransform[E^(-t), t, s]
```

$$\frac{1}{1+s}$$

$$\text{InverseLaplaceTransform}\left[\frac{1}{1+s}\,\frac{1}{s^3-3s^2+3s-1},s,t\right]\ //\ \text{Expand}$$

$$-\frac{e^{-t}}{8}+\frac{e^t}{8}-\frac{e^t t}{4}+\frac{e^t t^2}{4}$$

```
DSolve[
 {y'''[t] - 3 y''[t] + 3 y'[t] - y[t] == E^(-t), y[0] == 0, y'[0] == 0, y''[0] == 0}, y, t]
```

$$\left\{\left\{y\to\text{Function}\left[\{t\},\frac{1}{8}e^{-t}\left(-1+e^{2t}-2e^{2t}t+2e^{2t}t^2\right)\right]\right\}\right\}$$

c

```
(LaplaceTransform[y'''[t] - 3 y''[t] + 3 y'[t] - y[t], t, s] /.
   LaplaceTransform[y[t], t, s] -> Y[s]) // Expand
```

$$-3y[0]+3s\,y[0]-s^2y[0]-Y[s]+3s\,Y[s]-3s^2Y[s]+s^3Y[s]+3y'[0]-s\,y'[0]-y''[0]$$

$$\text{InverseLaplaceTransform}\left[\left(\frac{1}{1+s}+s^{\wedge}2-3s+3\right)\left(\frac{1}{s^3-3s^2+3s-1}\right),s,t\right]\ //\ \text{Expand}$$

$$-\frac{e^{-t}}{8}+\frac{9e^t}{8}-\frac{5e^t t}{4}+\frac{3e^t t^2}{4}$$

```
DSolve[{y'''[t] - 3 y''[t] + 3 y'[t] - y[t] == E^(-t),
   y[0] == 1, y'[0] == 0, y''[0] == 0}, y, t] // ExpandAll
```

$$\left\{\left\{y \to \text{Function}\left[\{t\},\ \frac{1}{8}\, e^{-t}\, \left(-1 + 9\, e^{2\,t} - 10\, e^{2\,t}\, t + 6\, e^{2\,t}\, t^2\right)\right]\right\}\right\}$$

$$\frac{1}{8}\, e^{-t}\, \left(-1 + 9\, e^{2\,t} - 10\, e^{2\,t}\, t + 6\, e^{2\,t}\, t^2\right) \text{ // ExpandAll}$$

$$-\frac{e^{-t}}{8} + \frac{9\, e^{t}}{8} - \frac{5\, e^{t}\, t}{4} + \frac{3\, e^{t}\, t^2}{4}$$

5

```
<< Statistics`DescriptiveStatistics`
```

```
<< Statistics`StatisticsPlots`
```

```
<< Statistics`DataManipulation`
```

```
M2 = Reverse[{3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3, 2, 3, 8, 4, 6, 2, 6, 4,
   3, 3, 8, 3, 2, 7, 9, 5, 0, 2, 8, 8, 4, 1, 9, 7, 1, 6, 9, 3, 9, 9, 3, 7, 5, 1}]
```

```
{1, 5, 7, 3, 9, 9, 3, 9, 6, 1, 7, 9, 1, 4, 8, 8, 2, 0, 5, 9, 7, 2, 3, 8,
 3, 3, 4, 6, 2, 6, 4, 8, 3, 2, 3, 9, 7, 9, 8, 5, 3, 5, 6, 2, 9, 5, 1, 4, 1, 3}
```

```
M1 = Reverse[{0, 5, 8, 2, 0, 9, 7, 4, 9, 4, 4, 5, 9, 2, 3, 0, 7, 8, 1, 6, 4, 0, 6, 2,
   8, 6, 2, 0, 8, 9, 9, 8, 6, 2, 8, 0, 3, 4, 8, 2, 5, 3, 4, 2, 1, 1, 7, 0, 6, 8}]
```

```
{8, 6, 0, 7, 1, 1, 2, 4, 3, 5, 2, 8, 4, 3, 0, 8, 2, 6, 8, 9, 9, 8, 0, 2,
 6, 8, 2, 6, 0, 4, 6, 1, 8, 7, 0, 3, 2, 9, 5, 4, 4, 9, 4, 7, 9, 0, 2, 8, 5, 0}
```

a

```
Frequencies[M1]
```

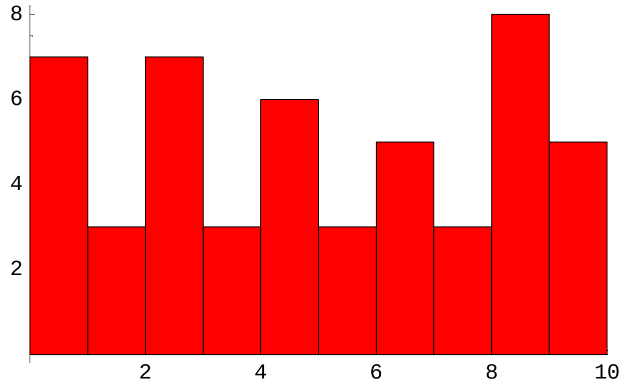
```
{{7, 0}, {3, 1}, {7, 2}, {3, 3}, {6, 4}, {3, 5}, {5, 6}, {3, 7}, {8, 8}, {5, 9}}
```

```
Frequencies[M2]
```

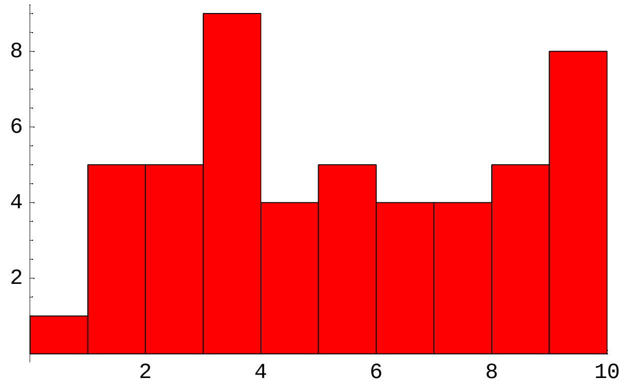
```
{{1, 0}, {5, 1}, {5, 2}, {9, 3}, {4, 4}, {5, 5}, {4, 6}, {4, 7}, {5, 8}, {8, 9}}
```

b

```
Histogram[M1];
```




```
Histogram[M2];
```



c

```
Length[M1]
```

50

```
LocationReport[M1] // N
```

{Mean → 4.5, HarmonicMean → Indeterminate, Median → 4.}

```
DispersionReport[M1] // N
```

{Variance → 9.39796, StandardDeviation → 3.06561, SampleRange → 9.,
MeanDeviation → 2.68, MedianDeviation → 3., QuartileDeviation → 3.}

```
Length[M2]
```

50

```
LocationReport[M2] // N
```

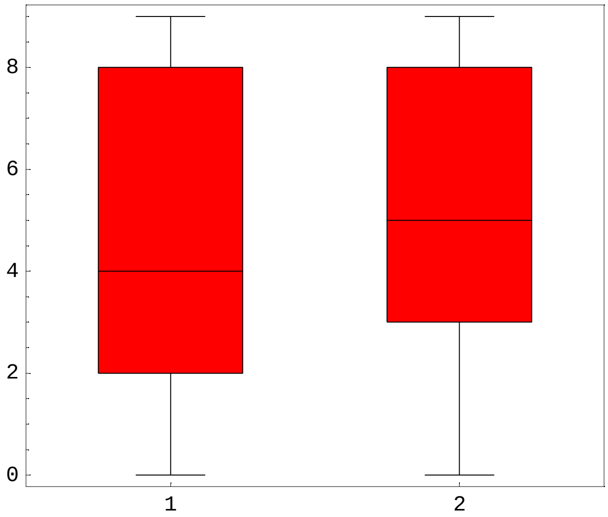
{Mean → 4.94, HarmonicMean → 0., Median → 5.}

```
DispersionReport[M2] // N
```

```
{Variance → 7.81265, StandardDeviation → 2.79511, SampleRange → 9.,
 MeanDeviation → 2.4224, MedianDeviation → 2., QuartileDeviation → 2.5}
```

d

```
BoxWhiskerPlot[Transpose[{M1, M2}]];
```



e

Es handelt sich um je 50 Stellen der Dezimalpruchentwicklung von .

6

a

```
14 !
```

```
87178291200
```

```
14 ! // N
```

8.71783×10^{10}

Bei Wiederholung:

```
14^14
```

```
11112006825558016
```

```
% // N
```

1.1112×10^{16}

b

```
50 / 7 // N
```

7.14286

```
7 Product[Binomial[50 - k 7, 7], {k, 0, 6}]
```

2577265483155016361393904911710617600000

```
% // N
```

2.57727×10^{39}