Lösungen

1

a

Abgabe Papier

b

Abgabe elektronisch

2

```
Remove["Global`*"]
```

a

```
LaplaceTransform[Sin[3 t + Pi], t, s]
```

$$-\frac{3}{9+s^2}$$

```
LaplaceTransform[Cos[3 t], t, s]
```

$$\frac{s}{9+s^2}$$

```
LaplaceTransform[Sin[3 t + Pi] + Cos[3 t] + t E^-(3 t), t, s]
```

$$\frac{1}{(3+s)^2}-\frac{3}{9+s^2}+\frac{s}{9+s^2}$$

```
% // Together
```

$$\frac{-18-9\,s+4\,s^2+s^3}{(3+s)^2\,(9+s^2)}$$

b

```
LaplaceTransform[Sin[3 t + Pi] + Cos[3 t] + t E^-(3 t), t, s] /. s → 3
```

$$\frac{1}{36}$$

c

```
LaplaceTransform[Sin[3 t + Pi] + Cos[3 t] + t E^-(3 t), t, s] /. s → -3
```

```
ComplexInfinity
```

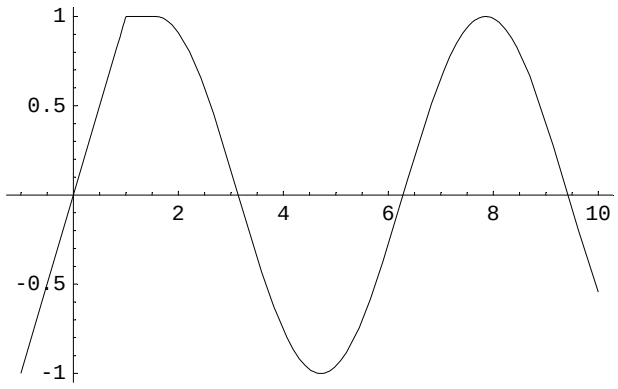
Kommentar: s < 0 > s ausserhalb des Definitionsbereiches von F(s)! Die Polstelle von F(s) ist bedeutungslos.

3

```
Remove["Global`*"]
```

```
f[t_] := Sin[t];
f[t_ /; t < 1] := t;
f[t_ /; 1 ≤ t && t < Pi / 2] := 1;
```

```
Plot[f[t], {t, -1, 10}];
```




```
Integrate[E^(-s t) t, {t, 0, 1}] // ExpandAll
```

$$\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$$

```
Integrate[E^(-s t) 1, {t, 1, Pi / 2}] // ExpandAll
```

$$\frac{e^{-s}}{s} - \frac{e^{-\frac{\pi s}{2}}}{s}$$

```
Integrate[E^(-s t) Sin[t], {t, Pi / 2, Infinity}] // ExpandAll
```

$$\text{If}\left[\text{Re}[s] > 0,\ \frac{e^{-\frac{\pi s}{2}}\, s}{1 + s^2},\ \text{Integrate}\left[e^{-s t}\, \text{Sin}[t],\ \left\{t,\ \frac{\pi}{2},\ \infty\right\},\ \text{Assumptions} \to \text{Re}[s] \le 0\right]\right]$$

```
Integrate[E^(-s t) t, {t, 0, 1}] + Integrate[E^(-s t) 1, {t, 1, Pi / 2}] +
  Integrate[E^(-s t) Sin[t], {t, Pi / 2, Infinity}] // ExpandAll
```

$$\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-\frac{\pi s}{2}}}{s} +$$
$$\text{If}\left[\text{Re}[s] > 0,\ \frac{e^{-\frac{\pi s}{2}}\, s}{1 + s^2},\ \text{Integrate}\left[e^{-s t}\, \text{Sin}[t],\ \left\{t,\ \frac{\pi}{2},\ \infty\right\},\ \text{Assumptions} \to \text{Re}[s] \le 0\right]\right]$$

```
(1/s^2 - e^-s/s^2 - e^-(π s/2)/s + e^-(π s/2) s/(1 + s^2)) /. {E^(-s) → ϱ[s], E^(-Pi s/2) → ϱ[s]^(Pi/2)}
```

$$\frac{1}{s^2} - \frac{\varrho[s]}{s^2} - \frac{\varrho[s]^{\pi/2}}{s} + \frac{s\,\varrho[s]^{\pi/2}}{1+s^2}$$

4

a

```
x[0] = 0; y[0] = 1;
```

```
(LaplaceTransform[x'[t] - 2 y[t], t, s] == LaplaceTransform[DiracDelta[t], t, s]) /.
 {LaplaceTransform[x[t], t, s] → Xs, LaplaceTransform[y[t], t, s] → Ys}
```

$$s\,Xs - 2\,Ys == 1$$

```
(LaplaceTransform[x[t] + y'[t], t, s] == LaplaceTransform[-Sin[t], t, s]) /.
 {LaplaceTransform[x[t], t, s] → Xs, LaplaceTransform[y[t], t, s] → Ys}
```

$$-1 + Xs + s\,Ys == -\frac{1}{1+s^2}$$

b

```
solv = Solve[{s Xs - 2 Ys == 1, -1 + Xs + s Ys == -1/(1 + s^2)}, {Xs, Ys}] // Flatten
```

$$\left\{Xs \to \frac{s\,(1+2s+s^2)}{2+3s^2+s^4},\ Ys \to -\frac{1+s^2-s^3}{2+3s^2+s^4}\right\}$$

```
X[s_] := Xs /. solv; X[s]
```

$$\frac{s\,(1+2s+s^2)}{2+3s^2+s^4}$$

```
Apart[X[s]]
```

$$-\frac{2}{1+s^2} + \frac{4+s}{2+s^2}$$

```
Y[s_] := Ys /. solv; Y[s]
```

$$-\frac{1+s^2-s^3}{2+3s^2+s^4}$$

```
Apart[Y[s]]
```

$$-\frac{s}{1+s^2} + \frac{-1+2s}{2+s^2}$$

c

```
InverseLaplaceTransform[X[s], s, t] // Expand
```

$$\text{Cos}\left[\sqrt{2}\ t\right] - 2\,\text{Sin}[t] + 2\sqrt{2}\,\text{Sin}\left[\sqrt{2}\ t\right]$$

```
InverseLaplaceTransform[Y[s], s, t] // Expand
```

$$-\text{Cos}[t] + 2\,\text{Cos}\left[\sqrt{2}\ t\right] - \frac{\text{Sin}\left[\sqrt{2}\ t\right]}{\sqrt{2}}$$

5

```
Remove["Global`*"]
```

a

```
y[0] = 0; y'[0] = 1;
```

```
u = (LaplaceTransform[y''[t] - y'[t] + 2 y[t], t, s] == LaplaceTransform[1, t, s]) /.
  {LaplaceTransform[y[t], t, s] → Ys}
```

$$-1 + 2\,\text{Ys} - s\,\text{Ys} + s^2\,\text{Ys} == \frac{1}{s}$$

```
solv = Solve[u, {Ys}] // Flatten
```

$$\left\{\text{Ys} \to \frac{1+s}{s\,(2-s+s^2)}\right\}$$

```
(Y[s_] := Ys /. solv) ; (Y[s]) // Apart
```

$$\frac{1}{2\,s} + \frac{3-s}{2\,(2-s+s^2)}$$

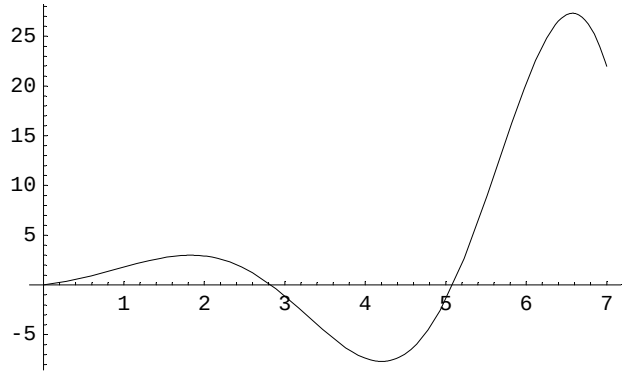
b

```
InverseLaplaceTransform[Y[s], s, t] // Expand
```

$$\frac{1}{2} - \frac{1}{2}\,e^{t/2}\,\text{Cos}\left[\frac{\sqrt{7}\ t}{2}\right] + \frac{5\,e^{t/2}\,\text{Sin}\left[\frac{\sqrt{7}\ t}{2}\right]}{2\sqrt{7}}$$

c

$$\text{Plot}\left[\frac{1}{14}\left(7+e^{t/2}\left(-7\,\text{Cos}\left[\frac{\sqrt{7}\,t}{2}\right]+5\sqrt{7}\,\text{Sin}\left[\frac{\sqrt{7}\,t}{2}\right]\right)\right),\{t,0,7\}\right];$$



d

$e^{t/2}$ wächst über alle Grenzen. Cosinus und Sinus sorgen für ein Hin – und Herpendeln. Der Limes für t gegen unendlich ist nicht definiert.

6

```
Remove["Global`*"]

<<Statistics`StatisticsPlots`

M1 = Round[100 * Table[6.2 + 0.4 Random[], {t, 1, 20}]] / 100 // N
```

```
{6.34, 6.5, 6.44, 6.51, 6.26, 6.34, 6.33, 6.33, 6.58,
 6.4, 6.6, 6.58, 6.57, 6.36, 6.5, 6.55, 6.48, 6.26, 6.42, 6.21}
```

```
(* M1={6.36`,6.56`,6.42`,6.27`,6.36`,6.47`,6.29`,6.54`,6.3`,
   6.55`,6.46`,6.49`,6.38`,6.4`,6.35`,6.41`,6.56`,6.25`,6.34`,6.52`} *)

M1 = {6.36`, 6.56`, 6.42`, 6.27`, 6.36`, 6.47`, 6.29`, 6.54`,
  6.3`, 6.55`, 6.46`, 6.49`, 6.38`, 6.4`, 6.35`, 6.41`, 6.56`, 6.25`, 6.34`}
```

```
{6.36, 6.56, 6.42, 6.27, 6.36, 6.47, 6.29, 6.54,
 6.3, 6.55, 6.46, 6.49, 6.38, 6.4, 6.35, 6.41, 6.56, 6.25, 6.34}
```

```
M2 = Round[100 * Table[6.3 + 0.3 Random[], {t, 1, 20}]] / 100 // N
```

```
{6.46, 6.49, 6.43, 6.44, 6.35, 6.57, 6.56, 6.5, 6.31,
 6.47, 6.46, 6.4, 6.32, 6.32, 6.46, 6.42, 6.34, 6.5, 6.53, 6.46}
```

```
(* M2={6.55`,6.48`,6.51`,6.51`,6.43`,6.51`,6.35`,6.46`,6.31`,
   6.31`,6.58`,6.51`,6.54`,6.35`,6.39`,6.59`,6.4`,6.5`,6.57`,6.44`} *)
```

```
M2 = {6.55`, 6.48`, 6.51`, 6.51`, 6.43`, 6.51`, 6.35`, 6.46`,
   6.31`, 6.31`, 6.58`, 6.51`, 6.54`, 6.35`, 6.39`, 6.59`, 6.4`, 6.5`, 6.57`}
```

```
{6.55, 6.48, 6.51, 6.51, 6.43, 6.51, 6.35, 6.46,
 6.31, 6.31, 6.58, 6.51, 6.54, 6.35, 6.39, 6.59, 6.4, 6.5, 6.57}
```

```
MM = {M1, M2} // Transpose
```

```
{{6.36, 6.55}, {6.56, 6.48}, {6.42, 6.51}, {6.27, 6.51},
 {6.36, 6.43}, {6.47, 6.51}, {6.29, 6.35}, {6.54, 6.46}, {6.3, 6.31},
 {6.55, 6.31}, {6.46, 6.58}, {6.49, 6.51}, {6.38, 6.54}, {6.4, 6.35},
 {6.35, 6.39}, {6.41, 6.59}, {6.56, 6.4}, {6.25, 6.5}, {6.34, 6.57}}
```

a

```
<< Statistics`DescriptiveStatistics`
```

```
minmax[t_] := {Max[t], Min[t], Max[t] - Mean[t],  Mean[t] - Min[t], Max[t] - Min[t]}
```

```
minmax[M1]
```

```
{6.56, 6.25, 0.151579, 0.158421, 0.31}
```

```
minmax[M2]
```

```
{6.59, 6.31, 0.124211, 0.155789, 0.28}
```

```
LocationReport[M1]
```

```
{Mean → 6.40842, HarmonicMean → 6.40694, Median → 6.4}
```

```
LocationReport[M2]
```

```
{Mean → 6.46579, HarmonicMean → 6.46458, Median → 6.5}
```

b

```
DispersionReport[M1]
```

```
{Variance → 0.010014, StandardDeviation → 0.10007, SampleRange → 0.31,
 MeanDeviation → 0.0825485, MedianDeviation → 0.07, QuartileDeviation → 0.07125}
```

```
DispersionReport[M2]
```

```
{Variance → 0.00821462, StandardDeviation → 0.0906345, SampleRange → 0.28,
 MeanDeviation → 0.0764543, MedianDeviation → 0.07, QuartileDeviation → 0.07}
```

```
q[M_] := Quartiles[M]
```

```
q[M1]
```

```
{6.3425, 6.4, 6.485}
```

```
q[M1][[3]] - q[M1][[1]] (* Quartilsdifferenz *)
```

```
0.1425
```

```
q[M2]
```

```
{6.3925, 6.5, 6.5325}
```

```
q[M2][[3]] - q[M2][[1]] (* Quartilsdifferenz *)
```

```
0.14
```

c

```
Abs[Mean[M1] - Min[M1]] > 2 StandardDeviation[M1]
```

```
False
```

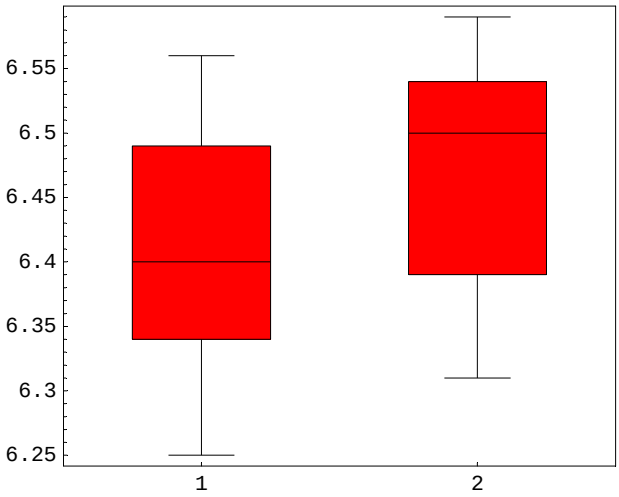
```
Abs[Mean[M2] - Min[M2]] > 2 StandardDeviation[M2]
```

```
False
```

Keine schwachen Ausreisser!

d

```
BoxWhiskerPlot[MM];
```



7

a

14 > 13 nummerierte Zwischenstellen. 2 daraus auswählen.

```
Binomial[13, 2]
```

```
78
```

b

5 auswählen, aus dem Rest nochmals 5 und dann die bleibenden 4

```
Binomial[14 – 5 2, 4]
```

```
1
```

```
Binomial[14, 5] Binomial[14 – 5, 5] Binomial[14 – 5 2, 4]
```

```
252252
```

c

```
Binomial[14, 5] 5 Binomial[14 – 5, 5] 5 Binomial[14 – 5 2, 4] 4
```

```
25225200
```

8

1000 Leute:

Nicht infisziert, Test richtig (richtigerweise nicht behandlet)

```
1000 0.9 0.7
```

```
630.
```

Nicht infisziert, Test falsch (fälschlicherweise behandlet)

```
1000 0.9 0.3
```

```
270.
```

Infisziert, Test richtig (richtigerweise behandlet)

```
1000 0.1 0.7
```

```
70.
```

Infisziert, Test falsch (fälschlicherweise nicht behandlet)

```
1000 0.1 0.3
```

```
30.
```

270 Menschen werden hier fälschlicherweise behandlet mit grossen Folgen und 30 fälschlicherweise nicht behandlet mit noch grösseren Folgen