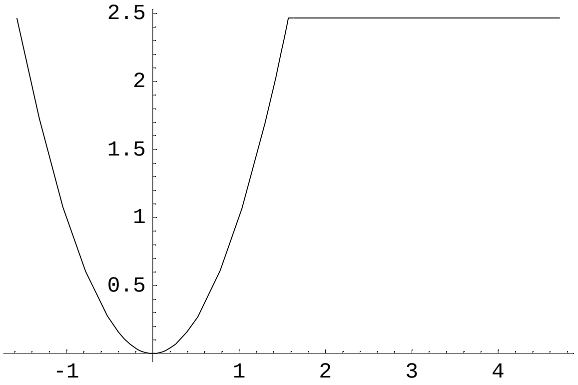
Lösungen

1

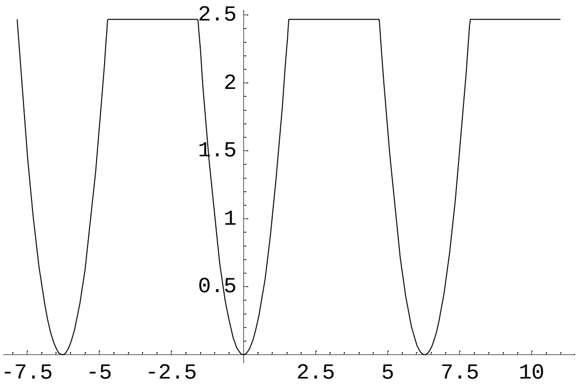
```
Remove["Global`*"]
```

```
f[t_] := t^2 /; (-Pi/2 ≤ t && t ≤ Pi/2);
f[t_] := f[Pi/2] /; (Pi/2 < t && t ≤ 3 Pi/2);
f[t_] := f[t + 2 Pi] /; (-5 Pi/2 ≤ t && t < -Pi/2);
f[t_] := f[t - 2 Pi] /; (3 Pi/2 < t && t ≤ 7 Pi/2);
f1[t_] := t^2;
f2[t_] := f1[Pi/2];
```

```
Plot[f[t], {t, -Pi/2, 3 Pi/2}];
```




```
Plot[f[t], {t, -5 Pi/2, 7 Pi/2}];
```



a Koeffizienten

```
T = 2 Pi;
cc = -Pi / 2;
ω = 2 Pi / T;
a[0] := 2 / T NIntegrate[f[t], {t, cc, cc + T}];
a[k_] := 2 / T NIntegrate[f[t] Cos[k ω t], {t, cc, cc + T}];
b[k_] := 2 / T NIntegrate[f[t] Sin[k ω t], {t, cc, cc + T}];
(* c[k_]:=1/T Integrate[f[t] E^(-I k ω t),{t,cc,cc+T}]; *)
ff[t_] := a[0] / 2 + Sum[a[n] Cos[n ω t] + b[n] Sin[n ω t], {n, 1, Infinity}];
ff[t_, h_] := a[0] / 2 + Sum[a[n] Cos[n ω t] + b[n] Sin[n ω t], {n, 1, h}] // Chop;
(* ffk[t_]:=Sum[c[n] E^(I n ω t),{n,-Infinity,Infinity}]; *)
(* ffk[t_,h_]:=Sum[c[n] E^(I n ω t),{n,-h,h}]; *)
g[t_] := (ff[u, 4] /. u → t) // Simplify; g[t]
```

1.64493 - 1.27324 Cos[t] - 0.5 Cos[2 t] + 0.047157 Cos[3 t] + 0.125 Cos[4 t]

```
a1[0] :=
  2 / T (Integrate[f1[t], {t, cc, cc + T / 2}] + Integrate[f2[t], {t, cc + T / 2, cc + T}]);
a1[k_] := 2 / T (Integrate[f1[t] Cos[k ω t], {t, cc, cc + T / 2}] +
     Integrate[f2[t] Cos[k ω t], {t, cc + T / 2, cc + T}]);
b1[k_] := 2 / T ( Integrate[f1[t] Sin[k ω t], {t, cc, cc + T / 2}] +
     Integrate[f2[t] Sin[k ω t], {t, cc + T / 2, cc + T}]);

ff1[u_, h_] := a1[0] / 2 + Sum[a1[n] Cos[n ω u] + b1[n] Sin[n ω u], {n, 1, h}] // Chop;
g1[t_] := (ff1[u, 4] /. u → t);
g1[t] // Simplify
```

$$\frac{\pi^2}{6} - \frac{4 \operatorname{Cos}[t]}{\pi} - \frac{1}{2} \operatorname{Cos}[2 t] + \frac{4 \operatorname{Cos}[3 t]}{27 \pi} + \frac{1}{8} \operatorname{Cos}[4 t]$$

```
g1[t_] := (ff1[u, 4] /. u → t) // N;
g1[t]
```

1.64493 - 1.27324 Cos[t] - 0.5 Cos[2. t] + 0.047157 Cos[3. t] + 0.125 Cos[4. t]

```
a1[0]
```

$$\frac{\pi^2}{3}$$

```
(* a0/2, a0 *)  {1.6449340668482262`, 2 1.6449340668482262`}
```

{1.64493, 3.28987}

```
(* ak *) {-1.2732395447351628, -0.5`, +0.047157020175376325`, 0.125`}
```

{-1.27324, -0.5, 0.047157, 0.125}

```
(* bk *) {0, 0, 0, 0}
```

{0, 0, 0, 0}

b

```
Abs[g[Pi / 2] - f[Pi / 2]]
```

0.197467

```
Abs[g1[Pi / 2] - f[Pi / 2]]
```

0.197467

```
Abs[g[3 Pi / 2] - f[3 Pi / 2]]
```

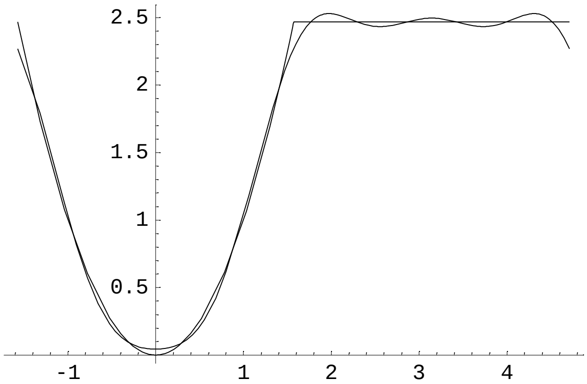
0.197467

```
Abs[g1[3 Pi / 2] - f[3 Pi / 2]]
```

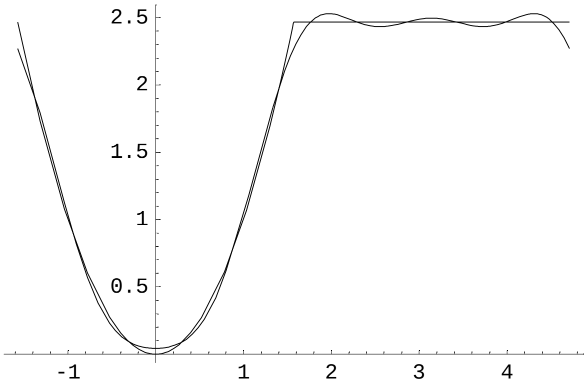
0.197467

c: Gute Näherung schon mit wenigen Koeffizienten

```
Plot[Evaluate[{f[t], g[t]}], {t, -Pi / 2, 3 Pi / 2}];
```




```
Plot[Evaluate[{f[t], g1[t]}], {t, -Pi / 2, 3 Pi / 2}];
```



d

```
f[Pi / 2]
```

$\frac{\pi^2}{4}$

```
N[%]
```

2.4674

```
a1[0]
```

$\frac{\pi^2}{3}$

```
{Cos[Pi / 2], Cos[2 Pi / 2], Cos[3 Pi / 2], Cos[4 Pi / 2]}
```

{0, -1, 0, 1}

In $\frac{\pi^2}{4} == \frac{\pi^2}{6} + \frac{\left(-\frac{\pi^2}{2} + \frac{1}{2}\,(-8+\pi^2)\right) * 0}{\pi} - \frac{1}{2} * (-1) + \frac{\left(\frac{\pi^2}{6} + \frac{1}{54}\,(8 - 9\,\pi^2)\right) * 0}{\pi} + \frac{1}{8} * 1 + ...$
lässt sich π auf eine Seite der Gleichung bringen und so isolieren, also berechnen

```
ff1[u, Infinity]
```

$\frac{\pi^2}{6} + \frac{1}{2\pi}\left(\pi\,\text{PolyLog}[2, -i\,e^{-iu}] + \pi\,\text{PolyLog}[2, i\,e^{-iu}] + \pi\,\text{PolyLog}[2, -i\,e^{iu}] + \pi\,\text{PolyLog}[2, i\,e^{iu}] - 2\,i\,\text{PolyLog}[3, -i\,e^{-iu}] + 2\,i\,\text{PolyLog}[3, i\,e^{-iu}] - 2\,i\,\text{PolyLog}[3, -i\,e^{iu}] + 2\,i\,\text{PolyLog}[3, i\,e^{iu}]\right)$

2

```
Remove["Global`*"]
```

```
g1[t_] := -t;
```

a Koeffizienten

```
T = 2;
cc = -1;
ω = 2 Pi / T;
a[0] := 2 / T Integrate[g1[t], {t, cc, cc + T}];
a[k_] := 2 / T Integrate[g1[t] Cos[k ω t], {t, cc, cc + T}];
b[k_] := 2 / T Integrate[g1[t] Sin[k ω t], {t, cc, cc + T}];
(* c[k_]:=1/T Integrate[g1[t] E^(-I k ω t),{t,cc,cc+T}]; *)
gg[t_] := a[0] / 2 + Sum[a[n] Cos[n ω t] + b[n] Sin[n ω t], {n, 1, Infinity}];
gg[t_, h_] := a[0] / 2 + Sum[a[n] Cos[n ω t] + b[n] Sin[n ω t], {n, 1, h}] // Chop;
(* ffk[t_]:=Sum[c[n] E^(I n ω t),{n,-Infinity,Infinity}]; *)
(* ffk[t_,h_]:=Sum[c[n] E^(I n ω t),{n,-h,h}]; *)
h4[t_] := (gg[u, 4] /. u → t) // Simplify; ExpandAll[h4[t]]
```

$-\frac{2\,\text{Sin}[\pi\,t]}{\pi} + \frac{\text{Sin}[2\,\pi\,t]}{\pi} - \frac{2\,\text{Sin}[3\,\pi\,t]}{3\,\pi} + \frac{\text{Sin}[4\,\pi\,t]}{2\,\pi}$

```
h20[t_] := (gg[u, 20] /. u → t) // Simplify;
```

```
ω = 2 Pi / T
```

π

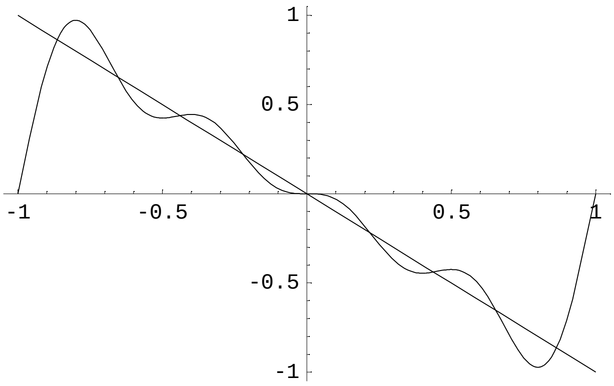
```
h4N[t_] := h4[t] // N // Simplify; h4N[t]
```

−0.63662 Sin[3.14159 t] + 0.31831 Sin[6.28319 t] −
0.212207 Sin[9.42478 t] + 0.159155 Sin[12.5664 t]

```
{−0.6366197723675814`, +0.3183098861837907`,
 −0.2122065907891938` , +0.15915494309189535`}
```

{−0.63662, 0.31831, −0.212207, 0.159155}

```
Plot[Evaluate[{g1[t], h4N[t]}], {t, −1, 1}];
```



b

```
3 h4[t] + 2 // ExpandAll
```

$$2 - \frac{6 \text{Sin}[\pi t]}{\pi} + \frac{3 \text{Sin}[2 \pi t]}{\pi} - \frac{2 \text{Sin}[3 \pi t]}{\pi} + \frac{3 \text{Sin}[4 \pi t]}{2 \pi}$$

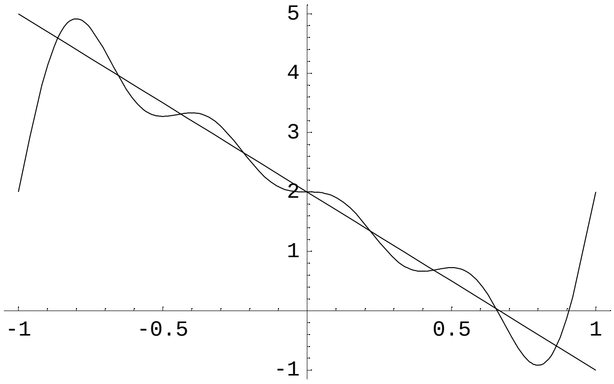
```
g2[t_] := 3 g1[t] + 2;
h2[t_] := 3 h4N[t] + 2 // Simplify;  {g2[t], h2[t]}
```

{2 − 3 t, 2. − 1.90986 Sin[3.14159 t] +
0.95493 Sin[6.28319 t] − 0.63662 Sin[9.42478 t] + 0.477465 Sin[12.5664 t]}

```
{2.`, −1.909859317102744`, +0.954929658551372`,
 −0.6366197723675814`, +0.477464829275686`}
```

{2., −1.90986, 0.95493, −0.63662, 0.477465}

```
Plot[Evaluate[{g2[t], h2[t]}], {t, -1, 1}];
```



c Integration der Fourierreihe!

```
Expand[h4[u]]
```

$$-\frac{2\,\mathrm{Sin}[\pi\,u]}{\pi}+\frac{\mathrm{Sin}[2\,\pi\,u]}{\pi}-\frac{2\,\mathrm{Sin}[3\,\pi\,u]}{3\,\pi}+\frac{\mathrm{Sin}[4\,\pi\,u]}{2\,\pi}$$

```
Integrate[Evaluate[Expand[h4[u]]], {u, 0, t}] // TrigReduce // Expand
```

$$-\frac{115}{72\,\pi^2}+\frac{2\,\mathrm{Cos}[\pi\,t]}{\pi^2}-\frac{\mathrm{Cos}[2\,\pi\,t]}{2\,\pi^2}+\frac{2\,\mathrm{Cos}[3\,\pi\,t]}{9\,\pi^2}-\frac{\mathrm{Cos}[4\,\pi\,t]}{8\,\pi^2}$$

```
g3[t_] := Integrate[g1[u], {u, 0, t}]; g3[t]
```

$$-\frac{t^2}{2}$$

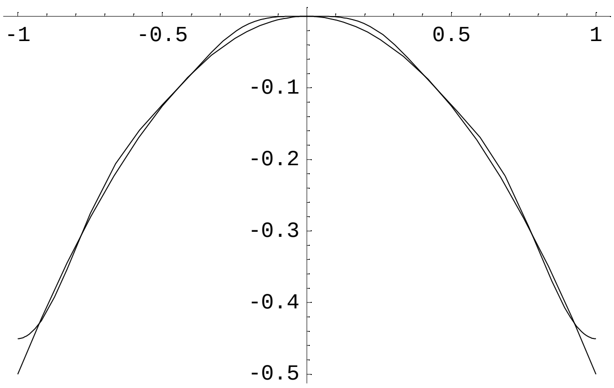
```
g31[t_] := Integrate[h4N[u], {u, 0, t}]; g31[t]
```

−0.161832 + 0.202642 Cos[3.14159 t] − 0.0506606 Cos[6.28319 t] +
0.0225158 Cos[9.42478 t] − 0.0126651 Cos[12.5664 t]

```
{-0.16183244609540065`, +0.20264236728467558`,
 -0.050660591821168895`, +0.022515818587186175`, -0.012665147955292224`}
```

{−0.161832, 0.202642, −0.0506606, 0.0225158, −0.0126651}

```
Plot[Evaluate[{g3[t], g31[t]}], {t, -1, 1}];
```



d Differentiation führt zu keiner Konstante: Oszillation, Gibbs!

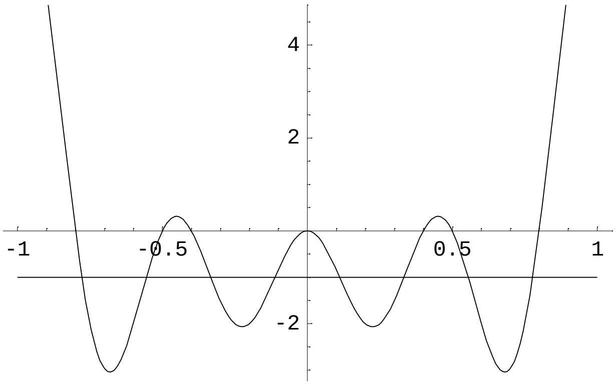
```
g1'[t]
```

```
-1
```

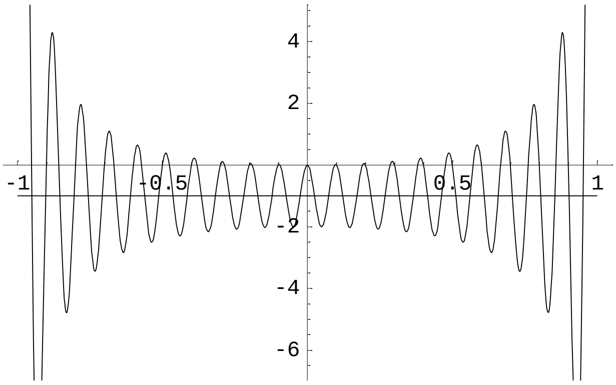
```
h4N'[t]
```

```
-2. Cos[3.14159 t] + 2. Cos[6.28319 t] - 2. Cos[9.42478 t] + 2. Cos[12.5664 t]
```

```
Plot[Evaluate[{ g1'[t], h4N'[t]}], {t, -1, 1}];
```




```
Plot[Evaluate[{ g1'[t], h20'[t]}], {t, -1, 1}];
```



3

```
Remove["Global`*"];
```

a

Wir verwenden zuerst die Skalierung nach der Periode 2 Pi. Das vereinfacht die Rechung etwas.

```
n=6; w = 2 Pi/n;
{x[0],y[0]}={0 w,2};
{x[1],y[1]}={1 w,2};
{x[2],y[2]}={2 w,3};
{x[3],y[3]}={3 w,2};
{x[4],y[4]}={4 w,3.5};
{x[5],y[5]}={5 w,7};
{x[6],y[6]}={6 w,2};
{x[-1],y[-1]}={-1 w,7};
{x[-2],y[-2]}={-2 w,3.5};
{x[-3],y[-3]}={-3 w,2};
{x[-4],y[-4]}={-4 w,3};
{x[-5],y[-5]}={-5 w,2};
{x[-6],y[-6]}={-6 w,2};
```

```
p[k_]:= {x[k],y[k]};
Table[p[k],{k,-(n-1),(n-1)}]
```

$\left\{\left\{-\frac{5\pi}{3}, 2\right\}, \left\{-\frac{4\pi}{3}, 3\right\}, \{-\pi, 2\}, \left\{-\frac{2\pi}{3}, 3.5\right\}, \left\{-\frac{\pi}{3}, 7\right\}, \{0, 2\}, \left\{\frac{\pi}{3}, 2\right\}, \left\{\frac{2\pi}{3}, 3\right\}, \{\pi, 2\}, \left\{\frac{4\pi}{3}, 3.5\right\}, \left\{\frac{5\pi}{3}, 7\right\}\right\}$

```
epi=Prepend[Map[Point,Table[p[k],{k,0,n}]],PointSize[0.03]]
```

$\left\{\text{PointSize}[0.03], \text{Point}[\{0, 2\}], \text{Point}\left[\left\{\frac{\pi}{3}, 2\right\}\right], \text{Point}\left[\left\{\frac{2\pi}{3}, 3\right\}\right], \text{Point}[\{\pi, 2\}], \text{Point}\left[\left\{\frac{4\pi}{3}, 3.5\right\}\right], \text{Point}\left[\left\{\frac{5\pi}{3}, 7\right\}\right], \text{Point}[\{2\pi, 2\}]\right\}$

```
epi1 = Prepend[Map[Point, Table[{k, y[k]}, {k, 0, n}]], PointSize[0.03]]
```

```
{PointSize[0.03], Point[{0, 2}], Point[{1, 2}], Point[{2, 3}],
 Point[{3, 2}], Point[{4, 3.5}], Point[{5, 7}], Point[{6, 2}]}
```

```
r = E^(-I 2 Pi/n);
c[s_]:= 1/n Sum[y[k] r^(s k),{k,-Floor[(n-1)/2],n-1-Floor[(n-1)/2]}];
Table[c[s],{s,0,10}]//N
```

```
{3.25, 0.208333 + 0.793857 i, -0.625 + 0.649519 i,
 -0.416667, -0.625 - 0.649519 i, 0.208333 - 0.793857 i, 3.25,
 0.208333 + 0.793857 i, -0.625 + 0.649519 i, -0.416667, -0.625 - 0.649519 i}
```

a c[s]

```
{3.25`, 0.20833333333333373` + 0.7938566201357355` i,
 -0.6249999999999993` + 0.649519052838329` i,
 -0.41666666666666663`, -0.6249999999999993` - 0.649519052838329` i,
 0.20833333333333373` - 0.7938566201357355` i,
 3.25`, 0.20833333333333373` + 0.7938566201357355` i,
 -0.6249999999999993` + 0.649519052838329` i,
 -0.41666666666666663`, -0.6249999999999993` - 0.649519052838329` i}
```

```
{3.25, 0.208333 + 0.793857 i, -0.625 + 0.649519 i,
 -0.416667, -0.625 - 0.649519 i, 0.208333 - 0.793857 i, 3.25,
 0.208333 + 0.793857 i, -0.625 + 0.649519 i, -0.416667, -0.625 - 0.649519 i}
```

```
fS[t_]:=Sum[c[k] E^(I k t),{k,-Floor[(n-1)/2],n-1-Floor[(n-1)/2]}];
fS[t]
```

$3.25 + (0.208333 - 0.793857\, i)\, e^{-i t} + (0.208333 + 0.793857\, i)\, e^{i t} - (0.625 + 0.649519\, i)\, e^{-2 i t} - (0.625 - 0.649519\, i)\, e^{2 i t} - 0.416667\, e^{3 i t}$

```
% // ExpandAll
```

$3.25 + (0.208333 - 0.793857\, i)\, e^{-i t} + (0.208333 + 0.793857\, i)\, e^{i t} - (0.625 + 0.649519\, i)\, e^{-2 i t} - (0.625 - 0.649519\, i)\, e^{2 i t} - 0.416667\, e^{3 i t}$

```
fS1[s_] := fS[s 2 Pi / n];
fS1[s]
```

$3.25 + (0.208333 - 0.793857\, i)\, e^{-\frac{1}{3} i \pi s} + (0.208333 + 0.793857\, i)\, e^{\frac{i \pi s}{3}} - (0.625 + 0.649519\, i)\, e^{-\frac{2}{3} i \pi s} - (0.625 - 0.649519\, i)\, e^{\frac{2 i \pi s}{3}} - 0.416667\, e^{i \pi s}$

```
% // ExpandAll
```

$3.25 + (0.208333 - 0.793857\, i)\, e^{-\frac{1}{3} i \pi s} + (0.208333 + 0.793857\, i)\, e^{\frac{i \pi s}{3}} - (0.625 + 0.649519\, i)\, e^{-\frac{2}{3} i \pi s} - (0.625 - 0.649519\, i)\, e^{\frac{2 i \pi s}{3}} - 0.416667\, e^{i \pi s}$

```
% // N // Simplify
```

$3.25 + (0.208333 - 0.793857\, i)\, e^{-1.0472\, i s} + (0.208333 + 0.793857\, i)\, e^{1.0472\, i s} - (0.625 + 0.649519\, i)\, e^{-2.0944\, i s} - (0.625 - 0.649519\, i)\, e^{2.0944\, i s} - 0.416667\, e^{3.14159\, i s}$

```
fS[t]//ExpToTrig
```

3.25 + (0.416667 + 0. i) Cos[t] - (1.25 + 0. i) Cos[2 t] - 0.416667 Cos[3 t] - (1.58771 + 0. i) Sin[t] - (1.29904 + 0. i) Sin[2 t] - 0.416667 i Sin[3 t]

```
% // ExpandAll
```

3.25 + (0.416667 + 0. i) Cos[t] - (1.25 + 0. i) Cos[2 t] - 0.416667 Cos[3 t] - (1.58771 + 0. i) Sin[t] - (1.29904 + 0. i) Sin[2 t] - 0.416667 i Sin[3 t]

```
% // N
```

3.25 + (0.416667 + 0. i) Cos[t] - (1.25 + 0. i) Cos[2. t] - 0.416667 Cos[3. t] - (1.58771 + 0. i) Sin[t] - (1.29904 + 0. i) Sin[2. t] - (0. + 0.416667 i) Sin[3. t]

```
fS1[s] // ExpToTrig
```

$3.25 + (0.416667 + 0.\, i)\, \mathrm{Cos}\left[\frac{\pi s}{3}\right] - (1.25 + 0.\, i)\, \mathrm{Cos}\left[\frac{2 \pi s}{3}\right] - 0.416667\, \mathrm{Cos}[\pi s] - (1.58771 + 0.\, i)\, \mathrm{Sin}\left[\frac{\pi s}{3}\right] - (1.29904 + 0.\, i)\, \mathrm{Sin}\left[\frac{2 \pi s}{3}\right] - 0.416667\, i\, \mathrm{Sin}[\pi s]$

```
% // ExpandAll
```

$3.25 + (0.416667 + 0.\, i)\, \mathrm{Cos}\left[\frac{\pi s}{3}\right] - (1.25 + 0.\, i)\, \mathrm{Cos}\left[\frac{2 \pi s}{3}\right] - 0.416667\, \mathrm{Cos}[\pi s] - (1.58771 + 0.\, i)\, \mathrm{Sin}\left[\frac{\pi s}{3}\right] - (1.29904 + 0.\, i)\, \mathrm{Sin}\left[\frac{2 \pi s}{3}\right] - 0.416667\, i\, \mathrm{Sin}[\pi s]$

```
% // N
```

3.25 + (0.416667 + 0. i) Cos[1.0472 s] - (1.25 + 0. i) Cos[2.0944 s] - 0.416667 Cos[3.14159 s] - (1.58771 + 0. i) Sin[1.0472 s] - (1.29904 + 0. i) Sin[2.0944 s] - (0. + 0.416667 i) Sin[3.14159 s]

```
fS1[s] // ExpToTrig // Chop
```

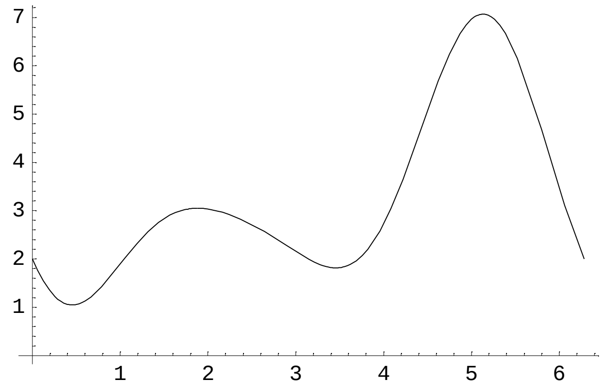
$3.25 + 0.416667\,\mathrm{Cos}\left[\frac{\pi s}{3}\right] - 1.25\,\mathrm{Cos}\left[\frac{2\pi s}{3}\right] - 0.416667\,\mathrm{Cos}[\pi s] - 1.58771\,\mathrm{Sin}\left[\frac{\pi s}{3}\right] - 1.29904\,\mathrm{Sin}\left[\frac{2\pi s}{3}\right] - 0.416667\,i\,\mathrm{Sin}[\pi s]$

```
{3.25`, {0.41666666666666746`, -1.2499999999999987`},
 {-0.41666666666666663`, -1.587713240271471`, -1.299038105676658`},
 {-0.41666666666666663`}}
```

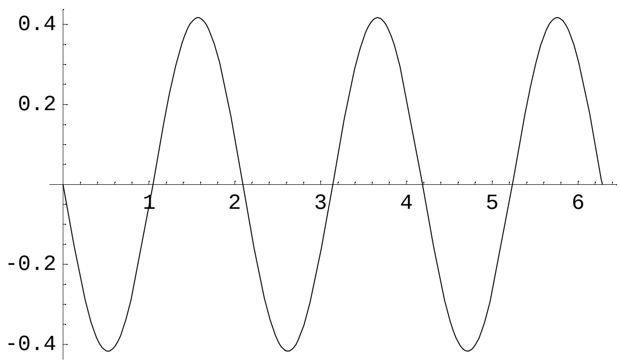
```
{3.25, {0.416667, -1.25}, {-0.416667, -1.58771, -1.29904}, {-0.416667}}
```

b

```
Plot[Re[fS[t]],{t,0,2Pi}];
```

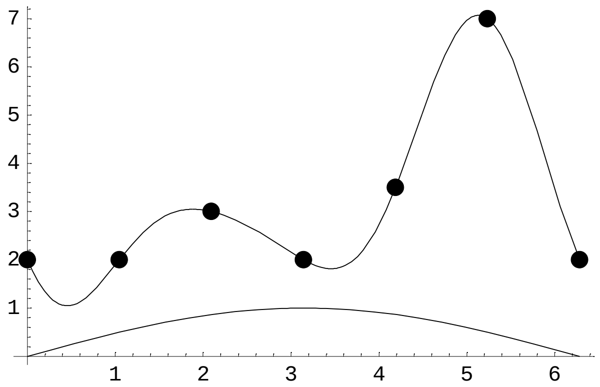



```
Plot[Im[fS[t]],{t,0,2Pi}];
```



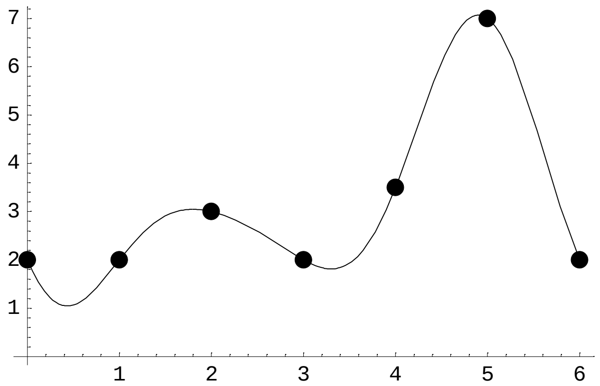
Man beachte im letzten Plot die Grösse der Amplitude.

```
Plot[{Re[fS[t]],Sin[t/2]},{t,0,2Pi},Epilog->epi];
```



Wie man sieht, liegen die verwendeten Punkte auf der Linie von Sin[t/2]. Der Fehler (z.B. grosser Imaginäranteil stammt vermutlich davon, dass so nur wenige Koeffizienten berechnet werden können.)

```
Plot[{Re[fS1[s]]}, {s, 0, n}, Epilog → epi1];
```



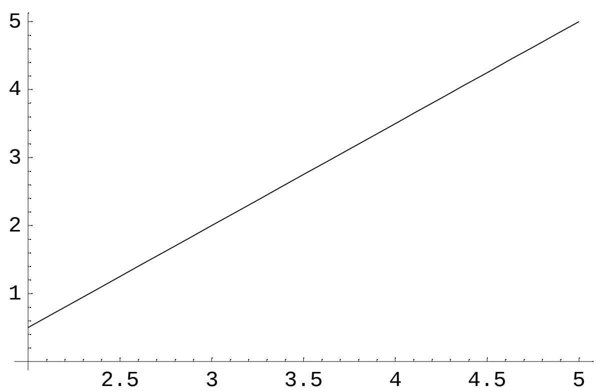
c

```
Re[fS1[3.5]]
```

```
1.93301
```

d

```
linInt[t_] := 2 + (t - 3) (3.5 - 2);
Plot[linInt[t], {t, 2, 5}];
```



```
linInt[3.5] - Re[fS1[3.5]]
```

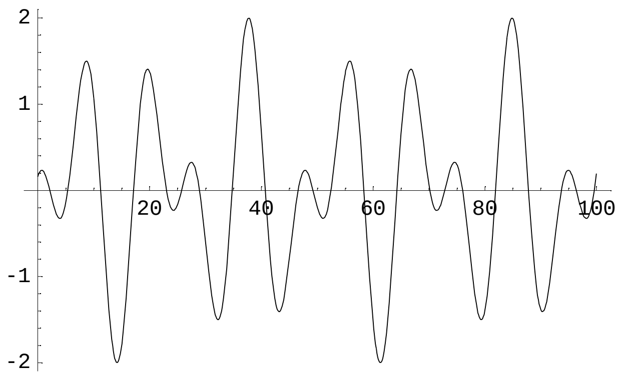
0.816987

4

```
Remove["Global`*"];
```

a Skizze

```
f[t_] := Cos[2 t / 3] + Sin[0.4 t - 1];
Plot[f[t], {t, 0, 100}];
```



b

```
{2 t / 3, 2 t / 3 + 2 Pi}
```

$\left\{\frac{2\,t}{3},\ 2\,\pi + \frac{2\,t}{3}\right\}$

```
{2 t / 3, 2 t / 3 + 2 Pi} 3 / 2 // ExpandAll
```

$\{t,\ 3\,\pi + t\}$

```
{4 / 10 t, 4 / 10 t + 2 Pi}
```

$\left\{\frac{2\,t}{5},\ 2\,\pi + \frac{2\,t}{5}\right\}$

```
{4 / 10 t, 4 / 10 t + 2 Pi} 5 / 2 // ExpandAll
```

$\{t,\ 5\,\pi + t\}$

```
T = LCM[5 , 3 ] π
```

$15\,\pi$

```
? GCD
```

GCD[n1, n2, ...] gives the greatest common divisor of the integers ni. Mehr…

```
0 == Simplify[f[t + 15 Pi] - f[t]] // Chop
```

True

```
T = 15 Pi;
```

c

```
cc = 0;
```

```
ω = 2 Pi / T;
a[0] := 2 / T Integrate[f[t], {t, cc, cc + T}];
a[k_] := 2 / T Integrate[f[t] Cos[k ω t], {t, cc, cc + T}];
b[k_] := 2 / T Integrate[f[t] Sin[k ω t], {t, cc, cc + T}];
c[k_] := 1 / T Integrate[f[t] E^(-I k ω t), {t, cc, cc + T}];
ff[t_] := a[0] / 2 + Sum[a[n] Cos[n ω t] + b[n] Sin[n ω t], {n, 1, Infinity}];
ff[t_, h_] := a[0] / 2 + Sum[a[n] Cos[n ω t] + b[n] Sin[n ω t], {n, 1, h}];
ffk[t_] := Sum[c[n] E^(I n ω t), {n, -Infinity, Infinity}];
ffk[t_, h_] := Sum[c[n] E^(I n ω t), {n, -h, h}];
ff[t, 6] // Chop
```

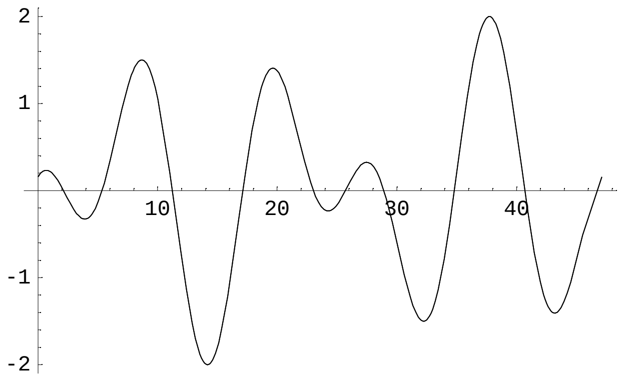
$-0.841471 \text{Cos}\left[\frac{2 t}{5}\right] + 1. \text{Cos}\left[\frac{2 t}{3}\right] + 0.540302 \text{Sin}\left[\frac{2 t}{5}\right]$

```
Abs[f[10] - ff[10, 6]]
```

1.11022×10^{-15}

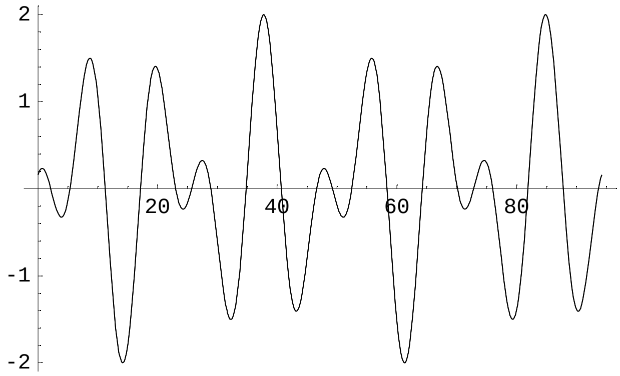
d

```
Plot[Evaluate[{f[t], ff[t, 6] // Chop}], {t, 0, T}];
```



Die Genauigkeit der beiden Funktionen f und ff6 ist anhand der Skizze nicht mehr unterscheidbar.

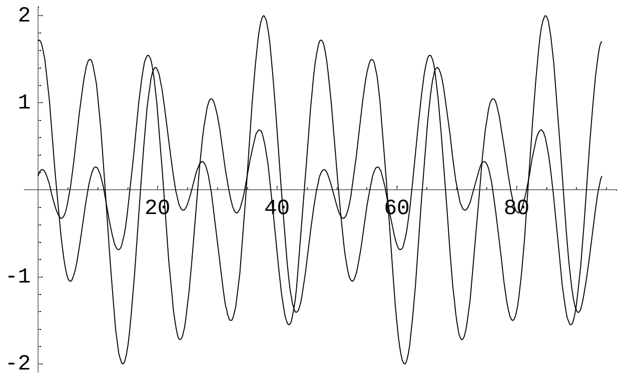
```
Plot[Evaluate[{f[t], ff[t, 6] // Chop}], {t, 0, 2 T}];
```



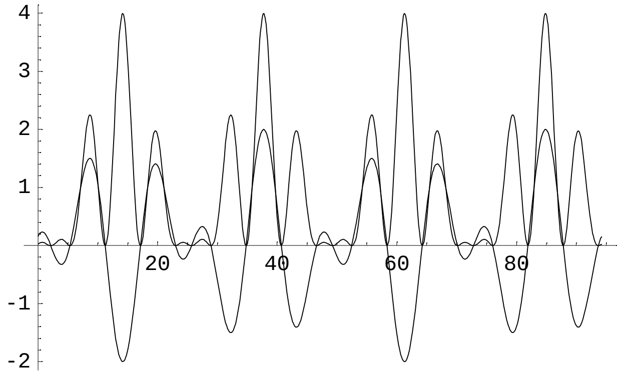
e

```
ff2[t_, h_] := a[0]^2/2 + Sum[a[n]^2 Cos[n ω t] + b[n]^2 Sin[n ω t], {n, 1, h}];
```

```
Plot[Evaluate[{f[t], ff2[t, 6] // Chop}], {t, 0, 30 Pi}];
```



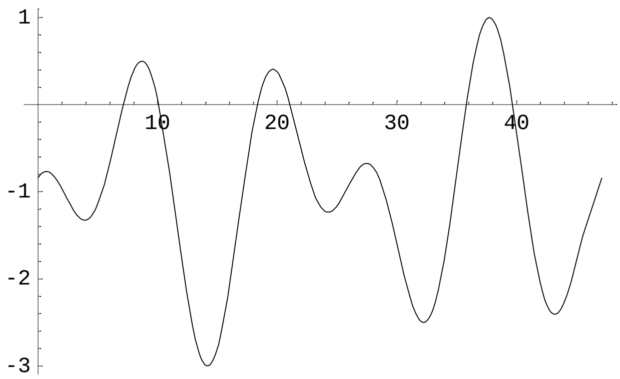
```
Plot[Evaluate[{f[t]^2, ff[t, 6] // Chop}], {t, 0, 30 Pi}];
```



```
Evaluate[(f[t]^2 - ff[t, 6]) / ff[t, 6] // Chop] /. t → 15.
```

```
-2.798
```

```
Plot[Evaluate[(f[t]^2 - ff[t, 6]) / ff[t, 6] // Chop], {t, 0, T}];
```



Der Ausdruck schwanktje nach t. Die maximale absolute relative Differenz ist etwa 3.

5 Varianten!

a

```
Remove["Global`*"];
```

```
f[x_] := Cos[2 x] + I Sin[2 x]
```

```
FourierTransform[f[x], x, ω] // Simplify
```

$\sqrt{2\pi}$ DiracDelta[2 + ω]

```
1/ √(2 π) FourierTransform[f[x], x, ω] // Simplify
```

DiracDelta[2 + ω]

```
√(2 π) FourierTransform[f[x], x, ω] // Simplify
```

2π DiracDelta[2 + ω]

b

```
Remove["Global`*"];
```

```
f[x_] := Sin[2 x] + I Cos[2 x]
```

```
FourierTransform[f[x], x, ω] // Simplify
```

$i\sqrt{2\pi}$ DiracDelta[-2 + ω]

```
1/ √(2 π) FourierTransform[f[x], x, ω] // Simplify
```

i DiracDelta[-2 + ω]

```
√(2 π) FourierTransform[f[x], x, ω] // Simplify
```

```
2 ⅈ π DiracDelta[-2 + ω]
```

c

```
Remove["Global`*"];
```

```
fHat[x_] := Cos[2 ω] + I Sin[2 ω]
```

```
InverseFourierTransform[fHat[x], ω, x] // Simplify
```

```
√(2 π) DiracDelta[-2 + x]
```

```
√(2 π) InverseFourierTransform[fHat[x], ω, x] // Simplify
```

```
2 π DiracDelta[-2 + x]
```

```
1/√(2 π) InverseFourierTransform[fHat[x], ω, x] // Simplify
```

```
DiracDelta[-2 + x]
```

```
1/ √(2 π) FourierTransform[√(2 π) InverseFourierTransform[fHat[x], ω, x], x, ω] //
 Simplify
```

$e^{2 i \omega}$

```
ⅇ^(2 ⅈ ω) // ExpToTrig
```

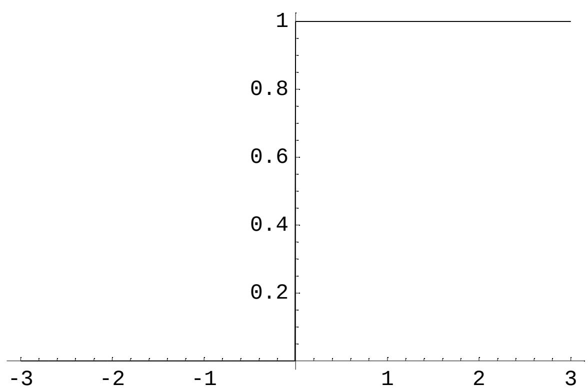
```
Cos[2 ω] + ⅈ Sin[2 ω]
```

6 Varianten!

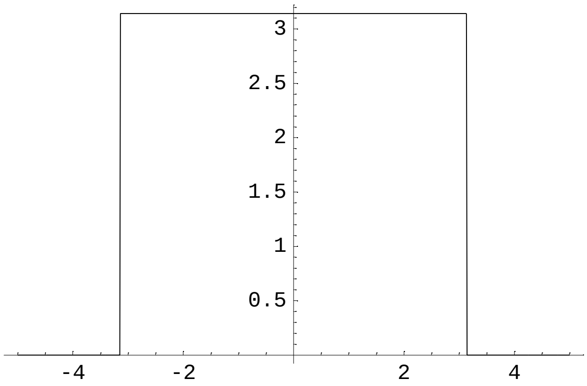
```
Remove["Global`*"];
```

a

```
H[x_] := UnitStep[x];
Plot[H[x], {x, -3, 3}];
```



```
f[x_] := Pi (H[x + Pi] - H[x - Pi]);
Plot[f[x], {x, -5, 5}];
```




```
FourierTransform[f[x], x, ω] // Simplify
```

$$\frac{\sqrt{2\pi}\,\mathrm{Sin}[\pi\,\omega]}{\omega}$$

```
1/√(2π) FourierTransform[f[x], x, ω] // Simplify
```

$$\frac{\mathrm{Sin}[\pi\,\omega]}{\omega}$$

```
√(2π) FourierTransform[f[x], x, ω] // Simplify
```

$$\frac{2\,\pi\,\mathrm{Sin}[\pi\,\omega]}{\omega}$$