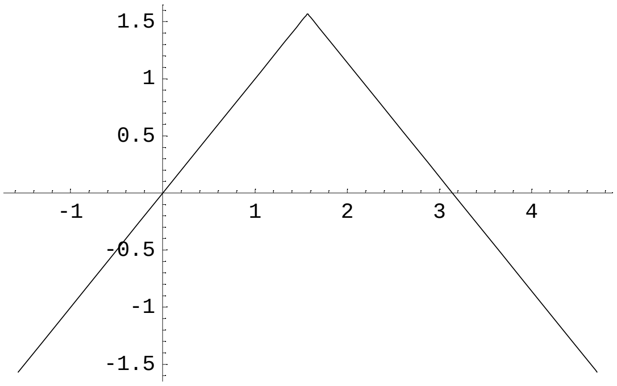
Lösungen

A. Zusammengesetzte Funktionen

```
In[1]:= Remove["Global`*"]

In[2]:= f[t_] := t /; (-Pi / 2 ≤ t && t ≤ Pi / 2);
        f[1]

Out[3]= 1

In[4]:= f[t_] := Pi - t /; (Pi / 2 < t && t ≤ 3 Pi / 2);

In[5]:= Plot[f[t], {t, -Pi / 2, 3 Pi / 2}];
```



B. Anwendung von eingebauten Funktionen

```
In[6]:= Remove["Global`*"]

In[7]:= ? *Four*
```

System`

```
Fourier              FourierParameters    FourierTransform   InverseFourierCosTransfor
FourierCosTransform  FourierSinTransform  InverseFourier     InverseFourierSinTransfor
```

```
In[8]:= << Calculus`FourierTransform`;

In[9]:= ?*Four*
```

System`

```
Fourier                FourierParameters    FourierTransform  InverseFourierCosTransfor
FourierCosTransform    FourierSinTransform  InverseFourier    InverseFourierSinTransfor
```

Calculus`FourierTransform`

```
DTFourierTransform            FourierOverallConstant         InverseFourierCoefficient
FourierCoefficient            FourierSample                  NDTFourierTransform
FourierCosCoefficient         FourierSeries                  NFourierCoefficient
FourierCosSeriesCoefficient   FourierSinCoefficient          NFourierCosCoefficient
FourierExpSeries              FourierSinSeriesCoefficient    NFourierCosSeriesCoefficie
FourierExpSeriesCoefficient   FourierTrigSeries              NFourierCosTransform
FourierFrequencyConstant      InverseDTFourierTransform      NFourierExpSeries
```

Beispiel 1

```
In[10]:= f[t_] := t;

In[11]:= FourierTrigSeries[f[t], t, 10]
```

Out[11]= $\frac{\mathrm{Sin}[2\pi t]}{\pi} - \frac{\mathrm{Sin}[4\pi t]}{2\pi} + \frac{\mathrm{Sin}[6\pi t]}{3\pi} - \frac{\mathrm{Sin}[8\pi t]}{4\pi} + \frac{\mathrm{Sin}[10\pi t]}{5\pi} - \frac{\mathrm{Sin}[12\pi t]}{6\pi} + \frac{\mathrm{Sin}[14\pi t]}{7\pi} - \frac{\mathrm{Sin}[16\pi t]}{8\pi} + \frac{\mathrm{Sin}[18\pi t]}{9\pi} - \frac{\mathrm{Sin}[20\pi t]}{10\pi}$

```
In[12]:= FourierSinCoefficient[f[t], t, 10]
```

Out[12]= $-\frac{1}{10\pi}$

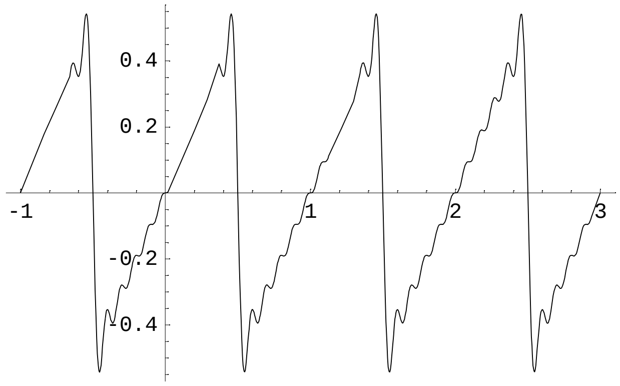
```
In[13]:= FourierCosCoefficient[f[t], t, 10]
```

Out[13]= 0

```
In[14]:= FourierSeries[f[t], t, 10]
```

Out[14]= $\frac{i e^{-2 i \pi t}}{2\pi} - \frac{i e^{2 i \pi t}}{2\pi} - \frac{i e^{-4 i \pi t}}{4\pi} + \frac{i e^{4 i \pi t}}{4\pi} + \frac{i e^{-6 i \pi t}}{6\pi} - \frac{i e^{6 i \pi t}}{6\pi} - \frac{i e^{-8 i \pi t}}{8\pi} + \frac{i e^{8 i \pi t}}{8\pi} + \frac{i e^{-10 i \pi t}}{10\pi} - \frac{i e^{10 i \pi t}}{10\pi} - \frac{i e^{-12 i \pi t}}{12\pi} + \frac{i e^{12 i \pi t}}{12\pi} + \frac{i e^{-14 i \pi t}}{14\pi} - \frac{i e^{14 i \pi t}}{14\pi} - \frac{i e^{-16 i \pi t}}{16\pi} + \frac{i e^{16 i \pi t}}{16\pi} + \frac{i e^{-18 i \pi t}}{18\pi} - \frac{i e^{18 i \pi t}}{18\pi} - \frac{i e^{-20 i \pi t}}{20\pi} + \frac{i e^{20 i \pi t}}{20\pi}$

```
In[15]:= Plot[Evaluate[FourierTrigSeries[f[t], t, 10]], {t, -1, 3}];
```



Beispiel 2

```
In[16]:= Remove["Global`*"]

In[17]:= << Calculus`FourierTransform`;

In[18]:= f[t_] := t;

In[19]:= f1[u_] = 2 Pi FourierTrigSeries[f[t], t, 10] /. t → u / (2 Pi) // Expand;
         Print[f1[u]];
```

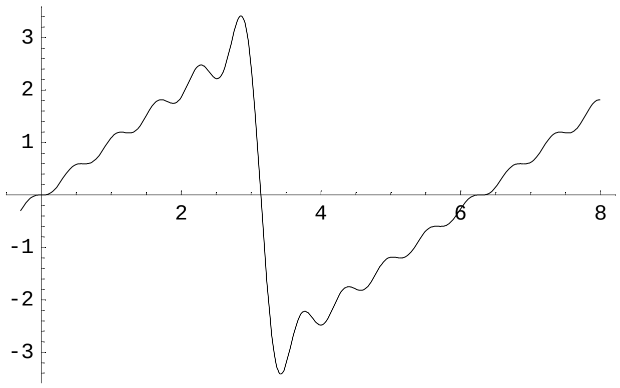
$$2\,\mathrm{Sin}[u] - \mathrm{Sin}[2\,u] + \frac{2}{3}\,\mathrm{Sin}[3\,u] - \frac{1}{2}\,\mathrm{Sin}[4\,u] + \frac{2}{5}\,\mathrm{Sin}[5\,u] - \frac{1}{3}\,\mathrm{Sin}[6\,u] + \frac{2}{7}\,\mathrm{Sin}[7\,u] - \frac{1}{4}\,\mathrm{Sin}[8\,u] + \frac{2}{9}\,\mathrm{Sin}[9\,u] - \frac{1}{5}\,\mathrm{Sin}[10\,u]$$

```
In[21]:= f2[u_] = 2 Pi (FourierSeries[f[t], t, 10] /. t → u / (2 Pi)) // Expand
```

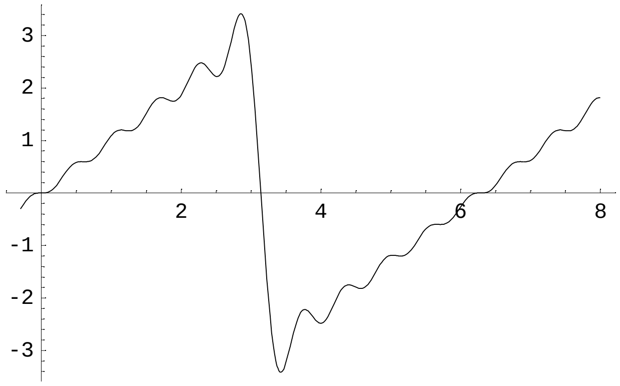
Out[21]=

$$i\,e^{-i u} - i\,e^{i u} - \frac{1}{2}\,i\,e^{-2 i u} + \frac{1}{2}\,i\,e^{2 i u} + \frac{1}{3}\,i\,e^{-3 i u} - \frac{1}{3}\,i\,e^{3 i u} - \frac{1}{4}\,i\,e^{-4 i u} + \frac{1}{4}\,i\,e^{4 i u} + \frac{1}{5}\,i\,e^{-5 i u} - \frac{1}{5}\,i\,e^{5 i u} - \frac{1}{6}\,i\,e^{-6 i u} + \frac{1}{6}\,i\,e^{6 i u} + \frac{1}{7}\,i\,e^{-7 i u} - \frac{1}{7}\,i\,e^{7 i u} - \frac{1}{8}\,i\,e^{-8 i u} + \frac{1}{8}\,i\,e^{8 i u} + \frac{1}{9}\,i\,e^{-9 i u} - \frac{1}{9}\,i\,e^{9 i u} - \frac{1}{10}\,i\,e^{-10 i u} + \frac{1}{10}\,i\,e^{10 i u}$$

```
In[22]:= Plot[f1[u], {u, -0.3, 8}];
```



```
In[23]:= Plot[Evaluate[Re[f2[u]]], {u, -0.3, 8}];
```



C. Konstruktion eines Moduls

```
In[24]:= Remove["Global`*"];
```

Beispiel zur Handhabung einer wählbaren Funktion mit Variablen in einem Modul

```
In[25]:= ser1[f_, s_, ss_] := Module[{},
            Print[f];
            Print[f /. t → s];
            Print[f /. x → s];
            Print[u[x]];
            w[m_] := Function[f[#]][m]; Print["w = ", w[ss]];
           ];
         u[t_] := Sin[t];
         u1[t_] := t^3; ser1[u1, j, k]

u1

u1

u1

Sin[x]

w = k^3
```

```
In[28]:= ser2[f_, s_] := Module[{w, m},
            w[m_] := Function[v[#]][m]; Print[w[s]];
           ];
         v[t_] := Cos[t]; ser2[v[t], xX]

Cos[xX]
```

Modul (in Zusatzfenster betreiben!)

```
In[30]:= Remove["Global`*"];
```

In[31]:=
```
four[fkt_, var_, perT_, start0Int_, n_, druck_] :=
  Module[{fktInt, tInt, nInt, znInt}, Print[" "]; Print["Output:"]; Print[" "];
   Print["Ausgabe:  ω, fktInt[var], a[0], a[k], b[k], c[k],
      Fourierreihen ff[var,n], ff[var], ffExp[var], ffKomplexTrig[
      var,n], ffKomplexExp[var,n], ffKomplex[var], Plot"];
   ω = 2 Pi / perT;   If[druck == 1,  Print["ω = ", ω], " "];
   fktInt[tInt_] := Function[fkt[#]][tInt];
   If[druck == 1, Print["Funktion[", var, "]  = ", fktInt[var]], " "];
   a[0] = 2 / T Integrate[fktInt[var], {var, start0Int, start0Int + perT}];
   If[druck == 1, Print["a[0] = ", a[0]], " "];
   a[k_] := 2 / T Integrate[Cos[k ω var] fktInt[var], {var, start0Int,
       start0Int + perT}];   If[druck == 1, Print["a[k] = ", a[k]], " "];
   b[k_] := 2 / T Integrate[Sin[k ω var] fktInt[var], {var, start0Int,
       start0Int + perT}];   If[druck == 1, Print["b[k] = ", b[k]], " "];
   c[k_] := 1 / T Integrate[fktInt[var] E^(-I k ω var), {var, start0Int,
       start0Int + perT}];  If[druck == 1, Print["c[k] = ", c[k]], " "];
   ff[tInt_, znInt_] := a[0] / 2 + Sum[a[nInt] Cos[nInt ω tInt] +
       b[nInt] Sin[nInt ω tInt], {nInt, 1, znInt}];
    If[druck == 1, Print["Fourierreihe[", var, ", ", n, "]  = ", ff[var, n]], " "];
   ff[tInt_] := a[0] / 2 +
     Sum[a[nInt] Cos[nInt ω tInt] + b[nInt] Sin[nInt ω tInt], {nInt, 1, Infinity}];
    If[druck == 1, Print["Unendliche Fourierreihe[", var, "]  = ", ff[var]], " "];
   ffExp[tInt_] := ExpToTrig[a[0] / 2 +
      Sum[a[nInt] Cos[nInt ω tInt] + b[nInt] Sin[nInt ω tInt], {nInt, 1, Infinity}]];
    If[druck == 1, Print["Unendliche Fourierreihe komplex[",
     var, "]  = ", ffExp[var]], " "];
   ffKomplexTrig[tInt_, znInt_] := ExpToTrig[
     Sum[c[nInt] E^(I nInt ω  tInt), {nInt, -znInt, znInt}]];
    If[druck == 1, Print["Komplexe Fourierreihe wieder trigonometrisch[",
     var, ", ", n, "]  = ", ffKomplexTrig[var, n]], " "];
   ffKomplexExp[tInt_, znInt_] := TrigToExp[
     Sum[c[nInt] E^(I nInt ω  tInt), {nInt, -znInt, znInt}]];
    If[druck == 1, Print["Komplexe Fourierreihe[", var, ", ",
     n, "]  = ", ffKomplexExp[var, n]], " "];
   ffKomplex[tInt_] := Sum[c[nInt] E^(I nInt ω  tInt), {nInt, -Infinity, Infinity}];
    If[druck == 1,
     Print["Komplexe Fourierreihe[", var, "]  = ", ffKomplex[var]], " "];
   If[druck == 1, Print["Plot"];
    Plot[Evaluate[{fktInt[var], ff[var, n]}],
     {var, start0Int, start0Int + perT}];, " "]
  ];
```

In[32]:=
```
(* Beispiel *)

f[t_] := t;
T = 2 Pi;
t0 = -Pi;
four[f, t, T, t0, 6, 1]
```

Output:

Ausgabe: ω, fktInt[var], a[0], a[k], b[k], c[k], Fourierreihen ff[var,n], ff[var], ffExp[var], ffKomplexTrig[var,n], ffKomplexExp[var,n], ffKomplex[var], Plot

$\omega = 1$

Funktion[t] = t

a[0] = 0

a[k] = 0

b[k] = $\frac{-2\,k\,\pi\,\text{Cos}[k\,\pi] + 2\,\text{Sin}[k\,\pi]}{k^2\,\pi}$

c[k] = $\frac{i\,(k\,\pi\,\text{Cos}[k\,\pi] - \text{Sin}[k\,\pi])}{k^2\,\pi}$

Fourierreihe[t, 6] = $2\,\text{Sin}[t] - \text{Sin}[2\,t] + \frac{2}{3}\,\text{Sin}[3\,t] - \frac{1}{2}\,\text{Sin}[4\,t] + \frac{2}{5}\,\text{Sin}[5\,t] - \frac{1}{3}\,\text{Sin}[6\,t]$

Unendliche Fourierreihe[t] = $\frac{1}{2\,\pi}$
$(i\,(2\,\pi\,\text{Log}[1 + e^{-i\,t}] - 2\,\pi\,\text{Log}[1 + e^{i\,t}] - i\,\text{PolyLog}[2, -e^{-i\,t}] - i\,\text{PolyLog}[2, -e^{i\,t}] + i\,\text{PolyLog}[2, e^{i\,(\pi+t)}] + i\,\text{PolyLog}[2, e^{2\,i\,\pi - i\,(\pi+t)}]))$

Unendliche Fourierreihe komplex[t] = $i\,\text{Log}[1 + \text{Cos}[t] - i\,\text{Sin}[t]] - i\,\text{Log}[1 + \text{Cos}[t] + i\,\text{Sin}[t]]$

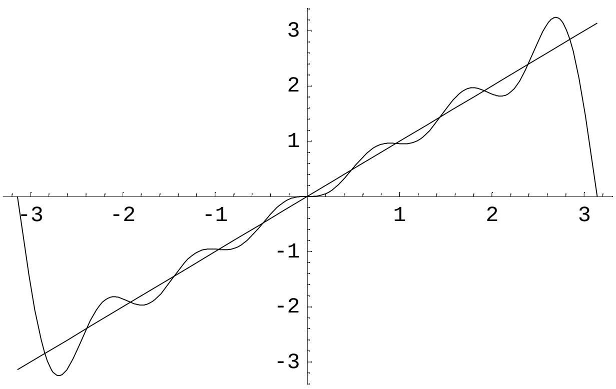
Komplexe Fourierreihe wieder trigonometrisch[t, 6] =
$2\,\text{Sin}[t] - \text{Sin}[2\,t] + \frac{2}{3}\,\text{Sin}[3\,t] - \frac{1}{2}\,\text{Sin}[4\,t] + \frac{2}{5}\,\text{Sin}[5\,t] - \frac{1}{3}\,\text{Sin}[6\,t]$

Komplexe Fourierreihe[t, 6] = $i\,e^{-i\,t} - i\,e^{i\,t} - \frac{1}{2}\,i\,e^{-2\,i\,t} + \frac{1}{2}\,i\,e^{2\,i\,t} + \frac{1}{3}\,i\,e^{-3\,i\,t} - \frac{1}{3}\,i\,e^{3\,i\,t} - \frac{1}{4}\,i\,e^{-4\,i\,t} + \frac{1}{4}\,i\,e^{4\,i\,t} + \frac{1}{5}\,i\,e^{-5\,i\,t} - \frac{1}{5}\,i\,e^{5\,i\,t} - \frac{1}{6}\,i\,e^{-6\,i\,t} + \frac{1}{6}\,i\,e^{6\,i\,t}$

Power::infy : Infinite expression $\frac{1}{0}$ encountered. Mehr…

Komplexe Fourierreihe[t] = ComplexInfinity

Plot



D. Lösungen

Aufgaben unter dem folgenden URL: http://rowicus.ch/Wir/ProblemsSolutBachelor/TM2Ana_10_FS_01.pdf

1

Modul aktivieren

```
In[36]:= Remove["Global`*"];
```

```
In[37]:= four[fkt_, var_, perT_, start0Int_, n_, druck_] :=
   Module[{fktInt, tInt, nInt, znInt}, Print[" "]; Print["Output:"]; Print[" "];
    Print["Ausgabe:  ω, fktInt[var], a[0], a[k], b[k], c[k],
       Fourierreihen ff[var,n], ff[var], ffExp[var], ffKomplexTrig[
       var,n], ffKomplexExp[var,n], ffKomplex[var], Plot: z.B. Plot[
       Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]"];
    ω = 2 Pi / perT;  If[druck == 1,  Print["ω = ", ω], " "];
    fktInt[tInt_] := Function[fkt[#]][tInt];
    If[druck == 1, Print["Funktion[", var, "]  = ", fktInt[var]], " "];
    a[0] = 2 / T Integrate[fktInt[var], {var, start0Int, start0Int + perT}];
    If[druck == 1, Print["a[0] = ", a[0]], " "];
    a[k_] := 2 / T Integrate[Cos[k ω var] fktInt[var], {var, start0Int,
        start0Int + perT}];   If[druck == 1, Print["a[k] = ", a[k]], " "];
    b[k_] := 2 / T Integrate[Sin[k ω var] fktInt[var], {var, start0Int,
        start0Int + perT}];   If[druck == 1, Print["b[k] = ", b[k]], " "];
    c[k_] := 1 / T Integrate[fktInt[var] E^(-I k ω var), {var, start0Int,
        start0Int + perT}];  If[druck == 1, Print["c[k] = ", c[k]], " "];
    ff[tInt_, znInt_] := a[0] / 2 + Sum[a[nInt] Cos[nInt ω tInt] +
        b[nInt] Sin[nInt ω tInt], {nInt, 1, znInt}];
     If[druck == 1, Print["Fourierreihe[", var, ", ", n, "]  = ", ff[var, n]], " "];
    ff[tInt_] := a[0] / 2 +
      Sum[a[nInt] Cos[nInt ω tInt] + b[nInt] Sin[nInt ω tInt], {nInt, 1, Infinity}];
     If[druck == 1, Print["Unendliche Fourierreihe[", var, "]  = ", ff[var]], " "];
    ffExp[tInt_] := ExpToTrig[a[0] / 2 +
       Sum[a[nInt] Cos[nInt ω tInt] + b[nInt] Sin[nInt ω tInt], {nInt, 1, Infinity}]];
     If[druck == 1, Print["Unendliche Fourierreihe komplex[",
      var, "]  = ", ffExp[var]], " "];
    ffKomplexTrig[tInt_, znInt_] := ExpToTrig[
      Sum[c[nInt] E^(I nInt ω tInt), {nInt, -znInt, znInt}]];
     If[druck == 1, Print["Komplexe Fourierreihe wieder trigonometrisch[",
      var, ", ", n, "]  = ", ffKomplexTrig[var, n]], " "];
    ffKomplexExp[tInt_, znInt_] := TrigToExp[
      Sum[c[nInt] E^(I nInt ω tInt), {nInt, -znInt, znInt}]];
     If[druck == 1, Print["Komplexe Fourierreihe[", var, ", ",
      n, "]  = ", ffKomplexExp[var, n]], " "];
    ffKomplex[tInt_] := Sum[c[nInt] E^(I nInt ω tInt), {nInt, -Infinity, Infinity}];
     If[druck == 1,
     Print["Komplexe Fourierreihe[", var, "]  = ", ffKomplex[var]], " "];
    If[druck == 1, Print["Plot"];
     Plot[Evaluate[{fktInt[var], ff[var, n]}],
      {var, start0Int, start0Int + perT}];, " "]
   ];
  four[f, t, T, t0, 6, 0]
```

```
Output:

Ausgabe:  ω, fktInt[var], a[0], a[k], b[k], c[k], Fourierreihen ff[var,n],
  ff[var], ffExp[var], ffKomplexTrig[var,n], ffKomplexExp[var,n], ffKomplex[
  var], Plot: z.B. Plot[Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]
```

Out[38]=

```
In[39]:= f[t_] := -t;
         T = 2 Pi;
         t0 = -Pi;
         (*  four[fkt_,var_,perT_,start0Int_,n_,druck_] *)
         four[f, t, T, t0, 6, 0]
```

Output:

Ausgabe: ω, fktInt[var], a[0], a[k], b[k], c[k], Fourierreihen ff[var,n], ff[var], ffExp[var], ffKomplexTrig[var,n], ffKomplexExp[var,n], ffKomplex[var], Plot: z.B. Plot[Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]

Out[42]=

a

In[43]:= **ff[t, 6]**

Out[43]= $-2\,\mathrm{Sin}[t] + \mathrm{Sin}[2\,t] - \frac{2}{3}\,\mathrm{Sin}[3\,t] + \frac{1}{2}\,\mathrm{Sin}[4\,t] - \frac{2}{5}\,\mathrm{Sin}[5\,t] + \frac{1}{3}\,\mathrm{Sin}[6\,t]$

b

In[44]:= **res1 = Integrate[f[t]^2, {t, t0, t0 + T}]**

Out[44]= $\frac{2\,\pi^3}{3}$

In[45]:= **res1 // N**

Out[45]= 20.6709

In[46]:= **res2 = Integrate[ff[t, 6]^2, {t, t0, t0 + T}]**

Out[46]= $\frac{5369\,\pi}{900}$

In[47]:= **res2 // N**

Out[47]= 18.7413

In[48]:= **res1 - res2 // N**

Out[48]= 1.92951

c

Sinusreihe

d

$$2 \sum_{k=1}^{\infty} \frac{(-1)^k}{k} \text{Sin}[k\,t]$$

e

In[49]:= **Pi**

Out[49]= π

$$f\left[\frac{\pi}{2}\right] == -\frac{\pi}{2} == -2 \sum_{k=1}^{\infty} \frac{1}{k} \text{Sin}\left[k\,\frac{\pi}{2}\right] == -2\left(\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...\right)$$

$$=== >> \quad -2 \sum_{k=1}^{\infty} \frac{1}{k} \text{Sin}[k\,\text{Pi}/2] = \frac{\pi}{2}$$

In[50]:= $-\text{Pi}/2 == -2 \sum_{k=1}^{\infty} \frac{1}{k} \text{Sin}[k\,\text{Pi}/2]$

N::meprec : Internal precision limit $MaxExtraPrecision = 49.99999999999999` reached while evaluating $-\frac{\pi}{2} + i\,(\text{Log}[1 - i] - \text{Log}[1 + i])$. Mehr…

Out[50]= $-\frac{\pi}{2} == -i\,(\text{Log}[1 - i] - \text{Log}[1 + i])$

In[51]:= $-2 \sum_{k=1}^{\infty} \frac{1}{k} \text{Sin}\left[k\,\frac{\pi}{2}\right]$

Out[51]= $-i\,(\text{Log}[1 - i] - \text{Log}[1 + i])$

f

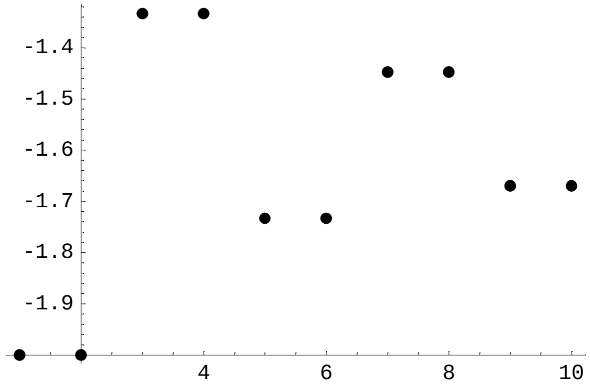
In[52]:= **Table[ff[Pi / 2, n], {n, 1, 10}]**

Out[52]= $\left\{-2, -2, -\frac{4}{3}, -\frac{4}{3}, -\frac{26}{15}, -\frac{26}{15}, -\frac{152}{105}, -\frac{152}{105}, -\frac{526}{315}, -\frac{526}{315}\right\}$

In[53]:= **ta1 = Table[ff[Pi / 2, n], {n, 1, 10}] // N**

Out[53]= {-2., -2., -1.33333, -1.33333, -1.73333, -1.73333, -1.44762, -1.44762, -1.66984, -1.66984}

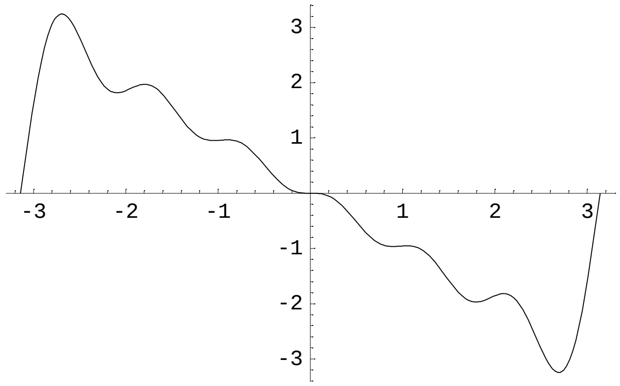
```
In[54]:= ta2 = Table[{k, ta1[[k]]}, {k, 1, Length[ta1]}];
         ListPlot[ta2, PlotStyle → PointSize[0.02]];
```



Das oszilliert!

g

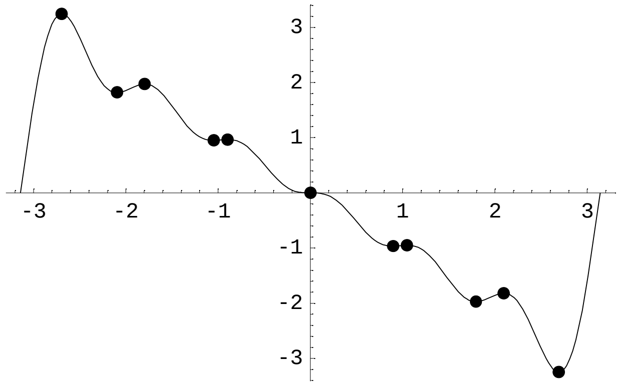
```
In[56]:= Plot[Evaluate[ff[t, 6]], {t, -Pi, Pi}, PlotPoints → 50];
```



h

```
In[57]:= D[Evaluate[ff[t, 6]], t]

Out[57]= -2 Cos[t] + 2 Cos[2 t] - 2 Cos[3 t] + 2 Cos[4 t] - 2 Cos[5 t] + 2 Cos[6 t]

In[58]:= solv = Solve[D[Evaluate[ff[t, 6]], t] == 0, {t}] // N // Chop

    Solve::ifun : Inverse functions are being used by Solve, so some
       solutions may not be found; use Reduce for complete solution information. Mehr…

Out[58]= {{t → 0}, {t → -2.0944}, {t → -1.0472}, {t → 1.0472}, {t → 2.0944}, {t → -0.897598},
          {t → 0.897598}, {t → -2.69279}, {t → 2.69279}, {t → -1.7952}, {t → 1.7952}}

In[59]:= xKoord = Table[t /. solv[[k]], {k, 1, Length[solv]}]

Out[59]= {0, -2.0944, -1.0472, 1.0472, 2.0944,
          -0.897598, 0.897598, -2.69279, 2.69279, -1.7952, 1.7952}
```

```
In[60]:= pkt = Union[Table[{xKoord[[k]], ff[xKoord[[k]], 6]}, {k, 1, Length[xKoord]}]]

Out[60]= {{-2.69279, 3.24438}, {-2.0944, 1.81865}, {-1.7952, 1.97013},
  {-1.0472, 0.952628}, {-0.897598, 0.965572}, {0, 0}, {0.897598, -0.965572},
  {1.0472, -0.952628}, {1.7952, -1.97013}, {2.0944, -1.81865}, {2.69279, -3.24438}}

In[61]:= Plot[Evaluate[ff[t, 6]], {t, -Pi, Pi},
  PlotPoints → 50, Epilog → {PointSize[0.02], Map[Point, pkt]}];
```



11 Stellen mit horizontaler Tangente

i

```
In[62]:= ffKomplexExp[t, 6]
```

Out[62]= $-i e^{-it} + i e^{it} + \frac{1}{2} i e^{-2it} - \frac{1}{2} i e^{2it} - \frac{1}{3} i e^{-3it} + \frac{1}{3} i e^{3it} + \frac{1}{4} i e^{-4it} - \frac{1}{4} i e^{4it} - \frac{1}{5} i e^{-5it} + \frac{1}{5} i e^{5it} + \frac{1}{6} i e^{-6it} - \frac{1}{6} i e^{6it}$

2

```
In[63]:= Remove["Global`*"];
```

In[64]:=
```
four[fkt_, var_, perT_, start0Int_, n_, druck_] :=
  Module[{fktInt, tInt, nInt, znInt}, Print[" "]; Print["Output:"]; Print[" "];
   Print["Ausgabe:  ω, fktInt[var], a[0], a[k], b[k], c[k],
      Fourierreihen ff[var,n], ff[var], ffExp[var], ffKomplexTrig[
      var,n], ffKomplexExp[var,n], ffKomplex[var], Plot: z.B. Plot[
      Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]"];
   ω = 2 Pi / perT;  If[druck == 1, Print["ω = ", ω], " "];
   fktInt[tInt_] := Function[fkt[#]][tInt];
   If[druck == 1, Print["Funktion[", var, "]  = ", fktInt[var]], " "];
   a[0] = 2 / T Integrate[fktInt[var], {var, start0Int, start0Int + perT}];
   If[druck == 1, Print["a[0] = ", a[0]], " "];
   a[k_] := 2 / T Integrate[Cos[k ω var] fktInt[var], {var, start0Int,
       start0Int + perT}];   If[druck == 1, Print["a[k] = ", a[k]], " "];
   b[k_] := 2 / T Integrate[Sin[k ω var] fktInt[var], {var, start0Int,
       start0Int + perT}];   If[druck == 1, Print["b[k] = ", b[k]], " "];
   c[k_] := 1 / T Integrate[fktInt[var] E^(-I k ω var), {var, start0Int,
       start0Int + perT}];  If[druck == 1, Print["c[k] = ", c[k]], " "];
   ff[tInt_, znInt_] := a[0] / 2 + Sum[a[nInt] Cos[nInt ω tInt] +
       b[nInt] Sin[nInt ω tInt], {nInt, 1, znInt}];
    If[druck == 1, Print["Fourierreihe[", var, ", ", n, "]  = ", ff[var, n]], " "];
   ff[tInt_] := a[0] / 2 +
     Sum[a[nInt] Cos[nInt ω tInt] + b[nInt] Sin[nInt ω tInt], {nInt, 1, Infinity}];
    If[druck == 1, Print["Unendliche Fourierreihe[", var, "]  = ", ff[var]], " "];
   ffExp[tInt_] := ExpToTrig[a[0] / 2 +
      Sum[a[nInt] Cos[nInt ω tInt] + b[nInt] Sin[nInt ω tInt], {nInt, 1, Infinity}]];
    If[druck == 1, Print["Unendliche Fourierreihe komplex[",
     var, "]  = ", ffExp[var]], " "];
   ffKomplexTrig[tInt_, znInt_] := ExpToTrig[
     Sum[c[nInt] E^(I nInt ω tInt), {nInt, -znInt, znInt}]];
    If[druck == 1, Print["Komplexe Fourierreihe wieder trigonometrisch[",
     var, ", ", n, "]  = ", ffKomplexTrig[var, n]], " "];
   ffKomplexExp[tInt_, znInt_] := TrigToExp[
     Sum[c[nInt] E^(I nInt ω tInt), {nInt, -znInt, znInt}]];
    If[druck == 1, Print["Komplexe Fourierreihe[", var, ", ",
     n, "]  = ", ffKomplexExp[var, n]], " "];
   ffKomplex[tInt_] := Sum[c[nInt] E^(I nInt ω tInt), {nInt, -Infinity, Infinity}];
    If[druck == 1,
     Print["Komplexe Fourierreihe[", var, "]  = ", ffKomplex[var]], " "];
   If[druck == 1, Print["Plot"];
    Plot[Evaluate[{fktInt[var], ff[var, n]}],
     {var, start0Int, start0Int + perT}];, " "]
  ];
four[f, t, T, t0, 6, 0]
```

```
Output:

Ausgabe:  ω, fktInt[var], a[0], a[k], b[k], c[k], Fourierreihen ff[var,n],
  ff[var], ffExp[var], ffKomplexTrig[var,n], ffKomplexExp[var,n], ffKomplex[
  var], Plot: z.B. Plot[Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]
```

Out[65]=

In[66]:=
```
f[t_] := -t;
T = 1;
t0 = -1 / 2;
(*  four[fkt_,var_,perT_,start0Int_,n_,druck_] *)
four[f, t, T, t0, 6, 0]
```

Output:

Ausgabe: ω, fktInt[var], a[0], a[k], b[k], c[k], Fourierreihen ff[var,n], ff[var], ffExp[var], ffKomplexTrig[var,n], ffKomplexExp[var,n], ffKomplex[var], Plot: z.B. Plot[Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]

Out[69]=

a

In[70]:= `ff[t, 6]`

Out[70]= $-\frac{\text{Sin}[2\pi t]}{\pi}+\frac{\text{Sin}[4\pi t]}{2\pi}-\frac{\text{Sin}[6\pi t]}{3\pi}+\frac{\text{Sin}[8\pi t]}{4\pi}-\frac{\text{Sin}[10\pi t]}{5\pi}+\frac{\text{Sin}[12\pi t]}{6\pi}$

b

$$2\sum_{k=1}^{\infty}\frac{(-1)^k}{k\pi}\,\text{Sin}[k\pi t]$$

c

In[71]:= `f[1 / 4]`

Out[71]= $-\frac{1}{4}$

In[72]:= `ff[1 / 4, 6]`

Out[72]= $-\frac{13}{15\pi}$

In[73]:= `% // N`

Out[73]= -0.275869

In[74]:= `f[1 / 4] - ff[1 / 4, 6]`

Out[74]= $-\frac{1}{4}+\frac{13}{15\pi}$

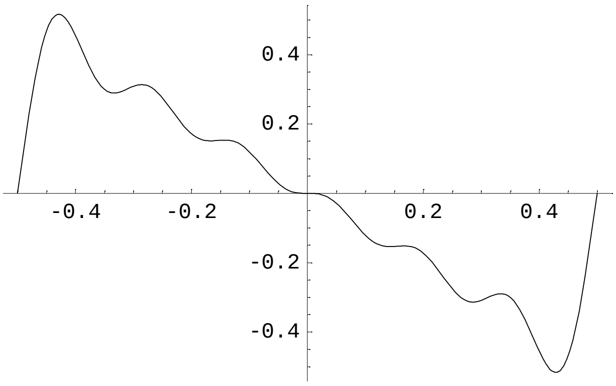
In[75]:= `% // Together`

Out[75]= $\frac{52-15\pi}{60\pi}$

```
In[76]:= % // N

Out[76]= 0.0258686
```

d

```
In[77]:= Plot[Evaluate[ff[t, 6]], {t, -1/2, 1/2}, PlotPoints → 50];
```



e

```
In[78]:= ffKomplexExp[t, 6]
```

Out[78]= $-\frac{i e^{-2 i \pi t}}{2 \pi}+\frac{i e^{2 i \pi t}}{2 \pi}+\frac{i e^{-4 i \pi t}}{4 \pi}-\frac{i e^{4 i \pi t}}{4 \pi}-\frac{i e^{-6 i \pi t}}{6 \pi}+\frac{i e^{6 i \pi t}}{6 \pi}+\frac{i e^{-8 i \pi t}}{8 \pi}-\frac{i e^{8 i \pi t}}{8 \pi}-\frac{i e^{-10 i \pi t}}{10 \pi}+\frac{i e^{10 i \pi t}}{10 \pi}+\frac{i e^{-12 i \pi t}}{12 \pi}-\frac{i e^{12 i \pi t}}{12 \pi}$

3

```
In[79]:= Remove["Global`*"];
```

```
In[80]:= four[fkt_, var_, perT_, start0Int_, n_, druck_] :=
    Module[{fktInt, tInt, nInt, znInt}, Print[" "]; Print["Output:"]; Print[" "];
     Print["Ausgabe:  ω, fktInt[var], a[0], a[k], b[k], c[k],
        Fourierreihen ff[var,n], ff[var], ffExp[var], ffKomplexTrig[
        var,n], ffKomplexExp[var,n], ffKomplex[var], Plot: z.B. Plot[
        Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]"];
     ω = 2 Pi / perT;  If[druck == 1, Print["ω = ", ω], " "];
     fktInt[tInt_] := Function[fkt[#]][tInt];
     If[druck == 1, Print["Funktion[", var, "]  = ", fktInt[var]], " "];
     a[0] = 2 / T Integrate[fktInt[var], {var, start0Int, start0Int + perT}];
     If[druck == 1, Print["a[0] = ", a[0]], " "];
     a[k_] := 2 / T Integrate[Cos[k ω var] fktInt[var], {var, start0Int,
         start0Int + perT}];   If[druck == 1, Print["a[k] = ", a[k]], " "];
     b[k_] := 2 / T Integrate[Sin[k ω var] fktInt[var], {var, start0Int,
         start0Int + perT}];   If[druck == 1, Print["b[k] = ", b[k]], " "];
     c[k_] := 1 / T Integrate[fktInt[var] E^(-I k ω var), {var, start0Int,
         start0Int + perT}];  If[druck == 1, Print["c[k] = ", c[k]], " "];
     ff[tInt_, znInt_] := a[0] / 2 + Sum[a[nInt] Cos[nInt ω tInt] +
         b[nInt] Sin[nInt ω tInt], {nInt, 1, znInt}];
      If[druck == 1, Print["Fourierreihe[", var, ", ", n, "]  = ", ff[var, n]], " "];
     ff[tInt_] := a[0] / 2 +
       Sum[a[nInt] Cos[nInt ω tInt] + b[nInt] Sin[nInt ω tInt], {nInt, 1, Infinity}];
      If[druck == 1, Print["Unendliche Fourierreihe[", var, "]  = ", ff[var]], " "];
     ffExp[tInt_] := ExpToTrig[a[0] / 2 +
        Sum[a[nInt] Cos[nInt ω tInt] + b[nInt] Sin[nInt ω tInt], {nInt, 1, Infinity}]];
      If[druck == 1, Print["Unendliche Fourierreihe komplex[",
       var, "]  = ", ffExp[var]], " "];
     ffKomplexTrig[tInt_, znInt_] := ExpToTrig[
       Sum[c[nInt] E^(I nInt ω tInt), {nInt, -znInt, znInt}]];
      If[druck == 1, Print["Komplexe Fourierreihe wieder trigonometrisch[",
       var, ", ", n, "]  = ", ffKomplexTrig[var, n]], " "];
     ffKomplexExp[tInt_, znInt_] := TrigToExp[
       Sum[c[nInt] E^(I nInt ω tInt), {nInt, -znInt, znInt}]];
      If[druck == 1, Print["Komplexe Fourierreihe[", var, ", ",
       n, "]  = ", ffKomplexExp[var, n]], " "];
     ffKomplex[tInt_] := Sum[c[nInt] E^(I nInt ω tInt), {nInt, -Infinity, Infinity}];
      If[druck == 1,
       Print["Komplexe Fourierreihe[", var, "]  = ", ffKomplex[var]], " "];
     If[druck == 1, Print["Plot"];
      Plot[Evaluate[{fktInt[var], ff[var, n]}],
       {var, start0Int, start0Int + perT}];, " "]
    ];
   four[f, t, T, t0, 6, 0]
```

```
Output:

Ausgabe:  ω, fktInt[var], a[0], a[k], b[k], c[k], Fourierreihen ff[var,n],
  ff[var], ffExp[var], ffKomplexTrig[var,n], ffKomplexExp[var,n], ffKomplex[
  var], Plot: z.B. Plot[Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]

Out[81]=
```

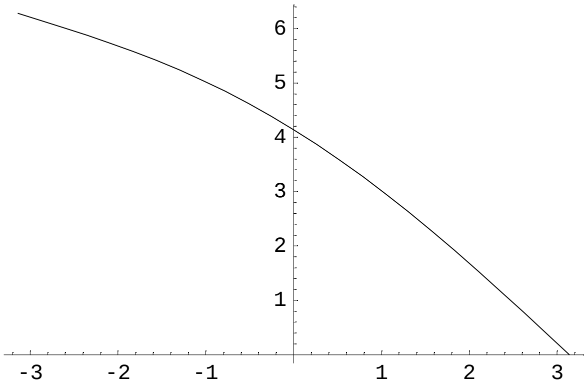
In[82]:=
```
f[t_] := Abs[-t + Pi] + Cos[t / 2];
T = 2 Pi;
t0 = -Pi;
(*  four[fkt_,var_,perT_,start0Int_,n_,druck_] *)
four[f, t, T, t0, 6, 0]
```

Output:

Ausgabe: ω, fktInt[var], a[0], a[k], b[k], c[k], Fourierreihen ff[var,n], ff[var], ffExp[var], ffKomplexTrig[var,n], ffKomplexExp[var,n], ffKomplex[var], Plot: z.B. Plot[Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]

Out[85]=

In[86]:=
```
Plot[f[t], {t, -Pi, Pi}];
```



a

In[87]:=
```
ff[t, 4]
```

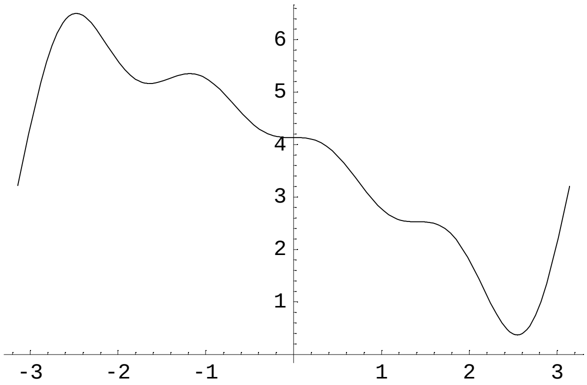
Out[87]=
$$\frac{2+\pi^2}{\pi} + \frac{4\,\text{Cos}[t]}{3\pi} - \frac{4\,\text{Cos}[2\,t]}{15\pi} + \frac{4\,\text{Cos}[3\,t]}{35\pi} - \frac{4\,\text{Cos}[4\,t]}{63\pi} - 2\,\text{Sin}[t] + \text{Sin}[2\,t] - \frac{2}{3}\,\text{Sin}[3\,t] + \frac{1}{2}\,\text{Sin}[4\,t]$$

In[88]:=
```
N[%]
```

Out[88]= 3.77821 + 0.424413 Cos[t] - 0.0848826 Cos[2. t] + 0.0363783 Cos[3. t] - 0.0202102 Cos[4. t] - 2. Sin[t] + Sin[2. t] - 0.666667 Sin[3. t] + 0.5 Sin[4. t]

b

```
In[89]:= Plot[Evaluate[ff[t, 4]], {t, -Pi, Pi}, PlotPoints → 50];
```



4

```
In[90]:= Remove["Global`*"];
```

In[91]:=
```
four[fkt_, var_, perT_, start0Int_, n_, druck_] :=
  Module[{fktInt, tInt, nInt, znInt}, Print[" "]; Print["Output:"]; Print[" "];
   Print["Ausgabe:  ω, fktInt[var], a[0], a[k], b[k], c[k],
      Fourierreihen ff[var,n], ff[var], ffExp[var], ffKomplexTrig[
      var,n], ffKomplexExp[var,n], ffKomplex[var], Plot: z.B. Plot[
      Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]"];
   ω = 2 Pi / perT;   If[druck == 1,  Print["ω = ", ω], " "];
   fktInt[tInt_] := Function[fkt[#]][tInt];
   If[druck == 1, Print["Funktion[", var, "]  = ", fktInt[var]], " "];
   a[0] = 2 / T Integrate[fktInt[var], {var, start0Int, start0Int + perT}];
   If[druck == 1, Print["a[0] = ", a[0]], " "];
   a[k_] := 2 / T Integrate[Cos[k ω var] fktInt[var], {var, start0Int,
       start0Int + perT}];   If[druck == 1, Print["a[k] = ", a[k]], " "];
   b[k_] := 2 / T Integrate[Sin[k ω var] fktInt[var], {var, start0Int,
       start0Int + perT}];   If[druck == 1, Print["b[k] = ", b[k]], " "];
   c[k_] := 1 / T Integrate[fktInt[var] E^(-I k ω var), {var, start0Int,
       start0Int + perT}];  If[druck == 1, Print["c[k] = ", c[k]], " "];
   ff[tInt_, znInt_] := a[0] / 2 + Sum[a[nInt] Cos[nInt ω tInt] +
       b[nInt] Sin[nInt ω tInt], {nInt, 1, znInt}];
    If[druck == 1, Print["Fourierreihe[", var, ", ", n, "]  = ", ff[var, n]], " "];
   ff[tInt_] := a[0] / 2 +
     Sum[a[nInt] Cos[nInt ω tInt] + b[nInt] Sin[nInt ω tInt], {nInt, 1, Infinity}];
    If[druck == 1, Print["Unendliche Fourierreihe[", var, "]  = ", ff[var]], " "];
   ffExp[tInt_] := ExpToTrig[a[0] / 2 +
      Sum[a[nInt] Cos[nInt ω tInt] + b[nInt] Sin[nInt ω tInt], {nInt, 1, Infinity}]];
    If[druck == 1, Print["Unendliche Fourierreihe komplex[",
     var, "]  = ", ffExp[var]], " "];
   ffKomplexTrig[tInt_, znInt_] := ExpToTrig[
     Sum[c[nInt] E^(I nInt ω tInt), {nInt, -znInt, znInt}]];
    If[druck == 1, Print["Komplexe Fourierreihe wieder trigonometrisch[",
     var, ", ", n, "]  = ", ffKomplexTrig[var, n]], " "];
   ffKomplexExp[tInt_, znInt_] := TrigToExp[
     Sum[c[nInt] E^(I nInt ω tInt), {nInt, -znInt, znInt}]];
    If[druck == 1, Print["Komplexe Fourierreihe[", var, ", ",
     n, "]  = ", ffKomplexExp[var, n]], " "];
   ffKomplex[tInt_] := Sum[c[nInt] E^(I nInt ω tInt), {nInt, -Infinity, Infinity}];
    If[druck == 1,
     Print["Komplexe Fourierreihe[", var, "]  = ", ffKomplex[var]], " "];
   If[druck == 1, Print["Plot"];
    Plot[Evaluate[{fktInt[var], ff[var, n]}],
     {var, start0Int, start0Int + perT}];, " "]
  ];
four[f, t, T, t0, 6, 0]
```

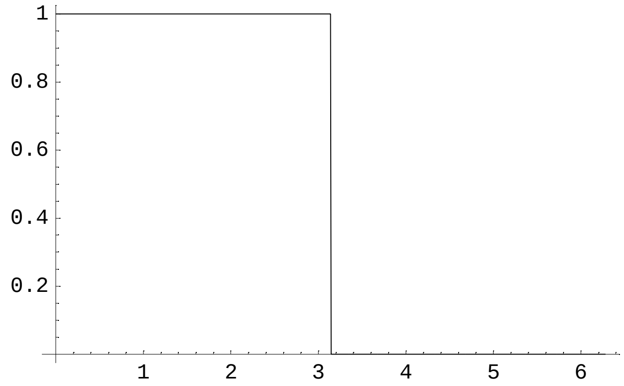
```
Output:
```

```
Ausgabe:  ω, fktInt[var], a[0], a[k], b[k], c[k], Fourierreihen ff[var,n],
  ff[var], ffExp[var], ffKomplexTrig[var,n], ffKomplexExp[var,n], ffKomplex[
  var], Plot: z.B. Plot[Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]
```

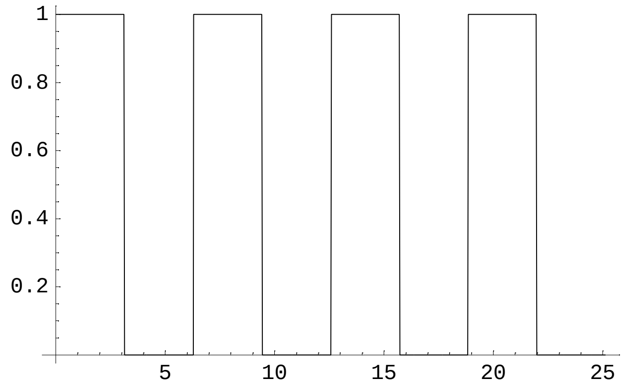
Out[92]=

Funktion

```
In[93]:= f[t_] := 1 /; (0 ≤  t && t ≤ Pi);
         f[t_] := 0 /; (Pi <  t && t ≤ 2 Pi);
         Plot[f[t], {t, 0, 2 Pi}];
```



```
In[96]:= Remove[f];
         f[t_] := 1 / 2 ((-1) ^Floor[t / (Pi)] + 1);
         Plot[f[t], {t, 0, 8 Pi}];
```



```
In[99]:= T = 2 Pi;
         t0 = 0;
         (*  four[fkt_,var_,perT_,start0Int_,n_,druck_] *)
         four[f, t, T, t0, 6, 0]
```

```
Output:

Ausgabe:  ω, fktInt[var], a[0], a[k], b[k], c[k], Fourierreihen ff[var,n],
  ff[var], ffExp[var], ffKomplexTrig[var,n], ffKomplexExp[var,n], ffKomplex[
  var], Plot: z.B. Plot[Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]

Out[101]=
```

a

In[102]:=

```
ff[t, 6]
```

Out[102]=

$$\frac{1}{2}+\frac{2\,\text{Sin}[t]}{\pi}+\frac{2\,\text{Sin}[3\,t]}{3\,\pi}+\frac{2\,\text{Sin}[5\,t]}{5\,\pi}$$

In[103]:=

```
ff[t, 10]
```

Out[103]=

$$\frac{1}{2}+\frac{2\,\text{Sin}[t]}{\pi}+\frac{2\,\text{Sin}[3\,t]}{3\,\pi}+\frac{2\,\text{Sin}[5\,t]}{5\,\pi}+\frac{2\,\text{Sin}[7\,t]}{7\,\pi}+\frac{2\,\text{Sin}[9\,t]}{9\,\pi}$$

$$\frac{1}{2}+2\sum_{k=1}^{\infty}\frac{\text{Sin}[(2\,k-1)\,t]}{(2\,k-1)\,\pi}$$

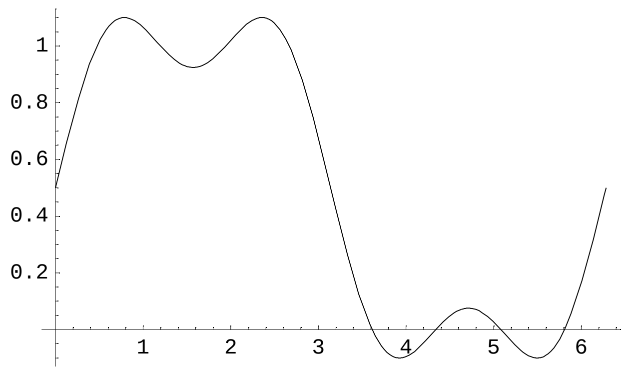
b

Die Spannung ist eine unendliche Ueberlagerung (Superposition) von harmonischen Schwingungen

c

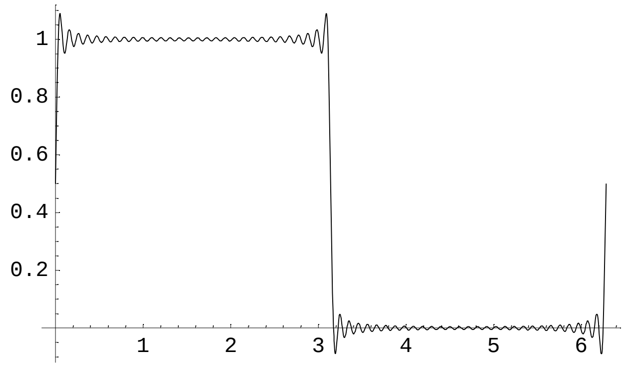
In[104]:=

```
Plot[Evaluate[ff[t, 4]], {t, 0, 2 Pi}, PlotPoints → 50];
```



In[105]:=

```
Plot[Evaluate[ff[t, 60]], {t, 0, 2 Pi}, PlotPoints → 50];
```



Gibbs kommt zur Wirkung.

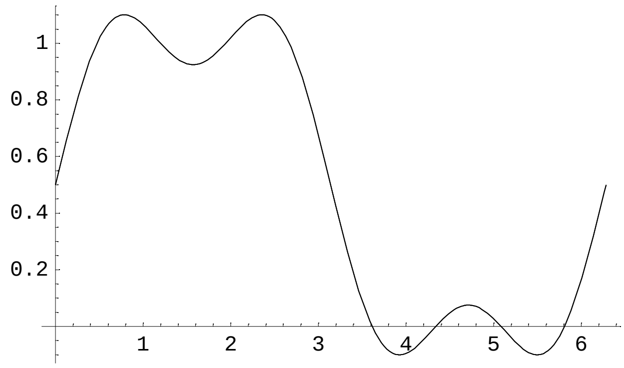
d

In[106]:=

$$g[t\_, n\_] := \frac{1}{2} + 2 \sum_{k=1}^{n} \frac{\text{Sin}[(2k-1)t]}{(2k-1)\pi}$$

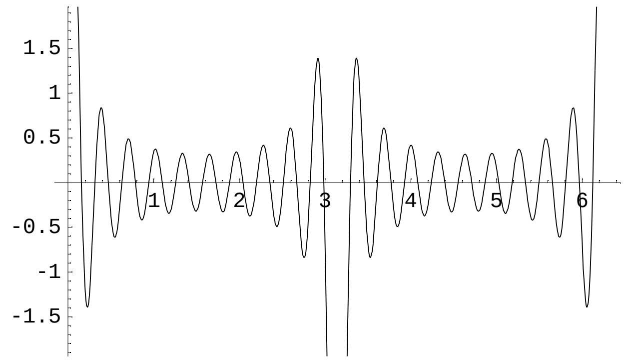
In[107]:=

```
Plot[Evaluate[{ff[t, 4], g[t, 2]}], {t, 0, 2 Pi}, PlotPoints → 50];
```

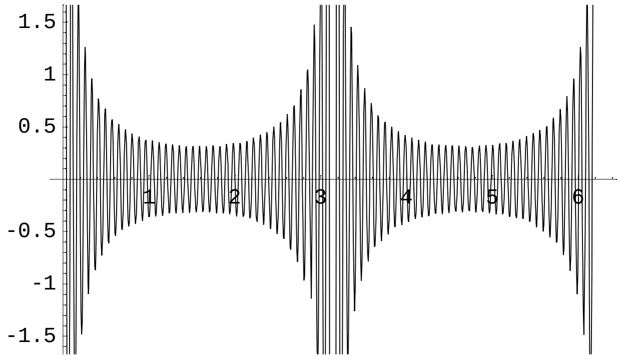


In[108]:=

```
Plot[Evaluate[D[g[t, 10], t]], {t, 0, 2 Pi}];
```



In[109]:=
```
Plot[Evaluate[D[g[t, 40], t]], {t, 0, 2 Pi}];
```



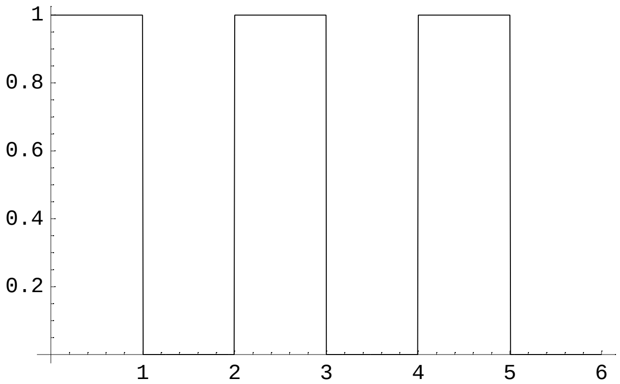
Ausser an den Sprungstellen sollte das Resultat der Ableitung 0 sein. Das ist keineswegs der Fall.

e

In[110]:=
```
Remove[g];
Solve[(c x == t /. {t → 2 Pi, x → 2}), {c}]
```

Out[111]=
$\{\{c \to \pi\}\}$

In[112]:=
```
g[x_] := f[Pi x];
Plot[g[x], {x, 0, 6}];
```



f

In[114]:=
```
Remove["Global`*"];
```

In[115]:=

```
four[fkt_, var_, perT_, start0Int_, n_, druck_] :=
  Module[{fktInt, tInt, nInt, znInt}, Print[" "]; Print["Output:"]; Print[" "];
   Print["Ausgabe:  ω, fktInt[var], a[0], a[k], b[k], c[k],
      Fourierreihen ff[var,n], ff[var], ffExp[var], ffKomplexTrig[
      var,n], ffKomplexExp[var,n], ffKomplex[var], Plot: z.B. Plot[
      Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]"];
   ω = 2 Pi / perT;  If[druck == 1, Print["ω = ", ω], " "];
   fktInt[tInt_] := Function[fkt[#]][tInt];
   If[druck == 1, Print["Funktion[", var, "]  = ", fktInt[var]], " "];
   a[0] = 2 / T Integrate[fktInt[var], {var, start0Int, start0Int + perT}];
   If[druck == 1, Print["a[0] = ", a[0]], " "];
   a[k_] := 2 / T Integrate[Cos[k ω var] fktInt[var], {var, start0Int,
       start0Int + perT}];   If[druck == 1, Print["a[k] = ", a[k]], " "];
   b[k_] := 2 / T Integrate[Sin[k ω var] fktInt[var], {var, start0Int,
       start0Int + perT}];   If[druck == 1, Print["b[k] = ", b[k]], " "];
   c[k_] := 1 / T Integrate[fktInt[var] E^(-I k ω var), {var, start0Int,
       start0Int + perT}];  If[druck == 1, Print["c[k] = ", c[k]], " "];
   ff[tInt_, znInt_] := a[0] / 2 + Sum[a[nInt] Cos[nInt ω tInt] +
       b[nInt] Sin[nInt ω tInt], {nInt, 1, znInt}];
    If[druck == 1, Print["Fourierreihe[", var, ", ", n, "]  = ", ff[var, n]], " "];
   ff[tInt_] := a[0] / 2 +
     Sum[a[nInt] Cos[nInt ω tInt] + b[nInt] Sin[nInt ω tInt], {nInt, 1, Infinity}];
    If[druck == 1, Print["Unendliche Fourierreihe[", var, "]  = ", ff[var]], " "];
   ffExp[tInt_] := ExpToTrig[a[0] / 2 +
      Sum[a[nInt] Cos[nInt ω tInt] + b[nInt] Sin[nInt ω tInt], {nInt, 1, Infinity}]];
    If[druck == 1, Print["Unendliche Fourierreihe komplex[",
     var, "]  = ", ffExp[var]], " "];
   ffKomplexTrig[tInt_, znInt_] := ExpToTrig[
     Sum[c[nInt] E^(I nInt ω tInt), {nInt, -znInt, znInt}]];
    If[druck == 1, Print["Komplexe Fourierreihe wieder trigonometrisch[",
     var, ", ", n, "]  = ", ffKomplexTrig[var, n]], " "];
   ffKomplexExp[tInt_, znInt_] := TrigToExp[
     Sum[c[nInt] E^(I nInt ω tInt), {nInt, -znInt, znInt}]];
    If[druck == 1, Print["Komplexe Fourierreihe[", var, ", ",
     n, "]  = ", ffKomplexExp[var, n]], " "];
   ffKomplex[tInt_] := Sum[c[nInt] E^(I nInt ω tInt), {nInt, -Infinity, Infinity}];
    If[druck == 1,
     Print["Komplexe Fourierreihe[", var, "]  = ", ffKomplex[var]], " "];
   If[druck == 1, Print["Plot"];
    Plot[Evaluate[{fktInt[var], ff[var, n]}],
     {var, start0Int, start0Int + perT}];, " "]
  ];
four[h, t, T, t0, 6, 0]
```

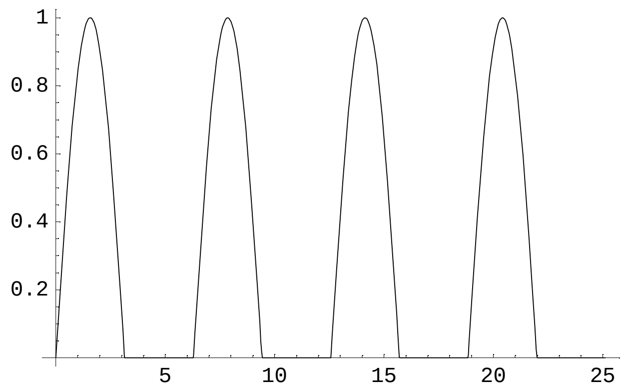
Output:

Ausgabe: ω, fktInt[var], a[0], a[k], b[k], c[k], Fourierreihen ff[var,n],
ff[var], ffExp[var], ffKomplexTrig[var,n], ffKomplexExp[var,n], ffKomplex[
var], Plot: z.B. Plot[Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]

Out[116]=

In[117]:=

```
f[t_] := 1 / 2 ((-1) ^Floor[t / (Pi)] + 1);
h[t_] := f[t] Sin[t];
Plot[h[t], {t, 0, 8 Pi}];
T = 2 Pi;
t0 = 0;
(*  four[fkt_,var_,perT_,start0Int_,n_,druck_] *)
four[h, t, T, t0, 6, 0]
```



Output:

Ausgabe: ω, fktInt[var], a[0], a[k], b[k], c[k], Fourierreihen ff[var,n], ff[var], ffExp[var], ffKomplexTrig[var,n], ffKomplexExp[var,n], ffKomplex[var], Plot: z.B. Plot[Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]

Out[122]=

In[123]:=

```
ff[t, 16]
```

Out[123]=

$$\frac{1}{\pi} - \frac{2\,\mathrm{Cos}[2\,t]}{3\pi} - \frac{2\,\mathrm{Cos}[4\,t]}{15\pi} - \frac{2\,\mathrm{Cos}[6\,t]}{35\pi} - \frac{2\,\mathrm{Cos}[8\,t]}{63\pi} - \frac{2\,\mathrm{Cos}[10\,t]}{99\pi} - \frac{2\,\mathrm{Cos}[12\,t]}{143\pi} - \frac{2\,\mathrm{Cos}[14\,t]}{195\pi} - \frac{2\,\mathrm{Cos}[16\,t]}{255\pi} + \frac{\mathrm{Sin}[t]}{2}$$

In[124]:=

```
N[%]
```

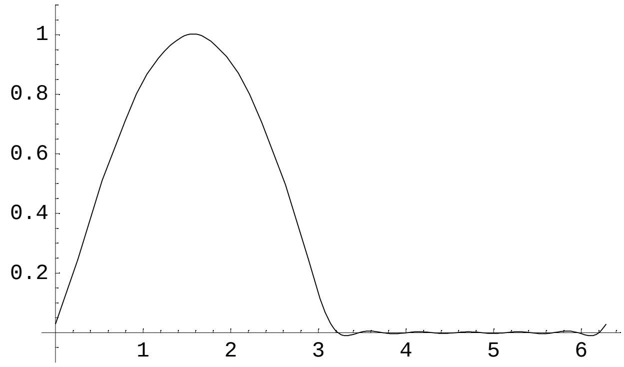
Out[124]=

0.31831 − 0.212207 Cos[2. t] − 0.0424413 Cos[4. t] − 0.0181891 Cos[6. t] − 0.0101051 Cos[8. t] − 0.0064305 Cos[10. t] − 0.00445189 Cos[12. t] − 0.00326472 Cos[14. t] − 0.00249655 Cos[16. t] + 0.5 Sin[t]

$$\frac{1}{\pi} + \frac{\mathrm{Sin}[t]}{2} - 2\sum_{k=1}^{\infty} \frac{\mathrm{Cos}[2\,k\,t]}{(2\,k-1)\,(2\,k+1)\,\pi}$$

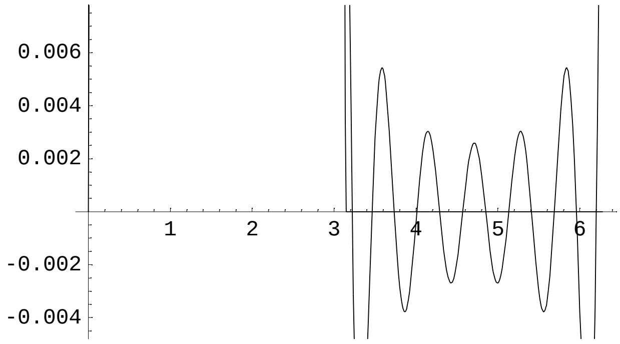
In[125]:=

```
Plot[Evaluate[{ff[t, 10]}], {t, 0, 2 Pi}, PlotRange → {-0.1, 1.1}];
```



In[126]:=

```
Plot[Evaluate[{ff[t, 10], h[t]}], {t, 0, 2 Pi}];
```



5

Uebliche Normierung

In[127]:=

```
yVec = {0, 1, 3};
r = E^(-I 2 Pi / 3);
W = {{r^0, r^0, r^0}, {r^0, r^1, r^2}, {r^0, r^2, r^4}}; W // MatrixForm
```

Out[129]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & e^{-\frac{2 i \pi}{3}} & e^{\frac{2 i \pi}{3}} \\ 1 & e^{\frac{2 i \pi}{3}} & e^{-\frac{2 i \pi}{3}} \end{pmatrix}$$

In[130]:=

```
Inverse[W] // Simplify
```

Out[130]=

$$\left\{\left\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right\}, \left\{\frac{1}{3}, -\frac{1}{3\,(1+(-1)^{2/3})}, -\frac{(-1)^{2/3}}{3\,(1+(-1)^{2/3})}\right\}, \left\{\frac{1}{3}, -\frac{(-1)^{2/3}}{3\,(1+(-1)^{2/3})}, -\frac{1}{3\,(1+(-1)^{2/3})}\right\}\right\}$$

In[131]:=

```
% // N // MatrixForm
```

Out[131]//MatrixForm=

$$\begin{pmatrix} 0.333333 & 0.333333 & 0.333333 \\ 0.333333 & -0.166667 + 0.288675\,i & -0.166667 - 0.288675\,i \\ 0.333333 & -0.166667 - 0.288675\,i & -0.166667 + 0.288675\,i \end{pmatrix}$$

In[132]:=

```
1 / 3 Conjugate[W]
```

Out[132]=

$$\left\{\left\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right\}, \left\{\frac{1}{3}, \frac{1}{3} e^{\frac{2 i \pi}{3}}, \frac{1}{3} e^{-\frac{2 i \pi}{3}}\right\}, \left\{\frac{1}{3}, \frac{1}{3} e^{-\frac{2 i \pi}{3}}, \frac{1}{3} e^{\frac{2 i \pi}{3}}\right\}\right\}$$

In[133]:=

```
% // N // MatrixForm
```

Out[133]//MatrixForm=

$$\begin{pmatrix} 0.333333 & 0.333333 & 0.333333 \\ 0.333333 & -0.166667 + 0.288675\,i & -0.166667 - 0.288675\,i \\ 0.333333 & -0.166667 - 0.288675\,i & -0.166667 + 0.288675\,i \end{pmatrix}$$

In[134]:=

```
cVec = Inverse[W].yVec // Simplify
```

```
General::spell1 :
 Possible spelling error: new symbol name "cVec" is similar to existing symbol "yVec". Mehr…
```

Out[134]=

$$\left\{\frac{4}{3}, -\frac{1 + 3 (-1)^{2/3}}{3 (1 + (-1)^{2/3})}, -\frac{3 + (-1)^{2/3}}{3 (1 + (-1)^{2/3})}\right\}$$

In[135]:=

```
% // N
```

Out[135]=

$\{1.33333, -0.666667 - 0.57735\,i, -0.666667 + 0.57735\,i\}$

In[136]:=

```
cVec = 1 / 3 Conjugate[W].yVec // Simplify
```

Out[136]=

$$\left\{\frac{4}{3}, -\frac{2}{3} - \frac{i}{\sqrt{3}}, \frac{1}{3} i \left(2 i + \sqrt{3}\right)\right\}$$

In[137]:=

```
% // N
```

Out[137]=

$\{1.33333, -0.666667 - 0.57735\,i, -0.666667 + 0.57735\,i\}$

Andere Normierung

In[138]:=

```
cVec = Sqrt[3] Inverse[W].yVec // Simplify
```

Out[138]=

$$\left\{\frac{4}{\sqrt{3}}, \frac{9 + i \sqrt{3}}{3 i - 3 \sqrt{3}}, -\frac{3 + (-1)^{2/3}}{\sqrt{3} (1 + (-1)^{2/3})}\right\}$$

In[139]:=
```
% // N
```
Out[139]=
```
{2.3094, -1.1547 - 1. i, -1.1547 + 1. i}
```
In[140]:=
```
cVec = 1 / Sqrt[3] Conjugate[W].yVec // Simplify
```
Out[140]=

$$\left\{\frac{4}{\sqrt{3}}, -i - \frac{2}{\sqrt{3}}, i - \frac{2}{\sqrt{3}}\right\}$$

In[141]:=
```
% // N
```
Out[141]=
```
{2.3094, -1.1547 - 1. i, -1.1547 + 1. i}
```

6

In[142]:=
```
Remove["Global`*"];
```

a

In[143]:=
```
f1[x_] := E^(-x^2);
fTransf1[Ω_] := 1 / (2 Pi) Integrate[f1[λ] E^(-I λ Ω), {λ, -Infinity, Infinity}];
fTransf1[Ω]
```
Out[145]=

$$\frac{e^{-\frac{\Omega^2}{4}}}{2\sqrt{\pi}}$$

In[146]:=
```
f2[x_] := f1[x] / Sqrt[2 Pi];
fTransf2[Ω_] := 1 / Sqrt[2 Pi] Integrate[f2[λ] E^(-I λ Ω), {λ, -Infinity, Infinity}];
fTransf2[Ω]
```
Out[148]=

$$\frac{e^{-\frac{\Omega^2}{4}}}{2\sqrt{\pi}}$$

In[149]:=
```
1 / Sqrt[2 Pi] Integrate[Evaluate[fTransf2[Ω] E^(I Ω x)], {Ω, -Infinity, Infinity}]
```
Out[149]=

$$\frac{e^{-x^2}}{\sqrt{2\pi}}$$

In[150]:=

```
1 / Sqrt[2 Pi]
  Integrate[Evaluate[fTransf2[Ω] E^(I Ω x)], {Ω, -Infinity, Infinity}] == f2[x]
```

Out[150]=

```
True
```

In[151]:=

```
? FourierTransform
```

FourierTransform[expr, t, ω] gives the symbolic Fourier transform of expr. FourierTransform[expr, {t1, t2, ... }, {ω1, ω2, ... }] gives the multidimensional Fourier transform of expr. Mehr…

In[152]:=

```
FourierTransform[f1[t], t, Ω]
```

Out[152]=

$$\frac{e^{-\frac{\Omega^2}{4}}}{\sqrt{2}}$$

In[153]:=

```
FourierTransform[f2[t], t, Ω]
```

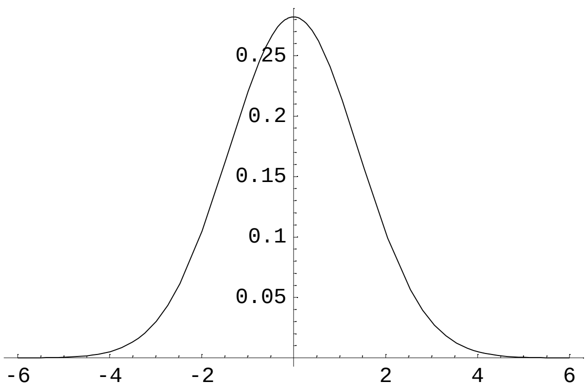
Out[153]=

$$\frac{e^{-\frac{\Omega^2}{4}}}{2\sqrt{\pi}}$$

b

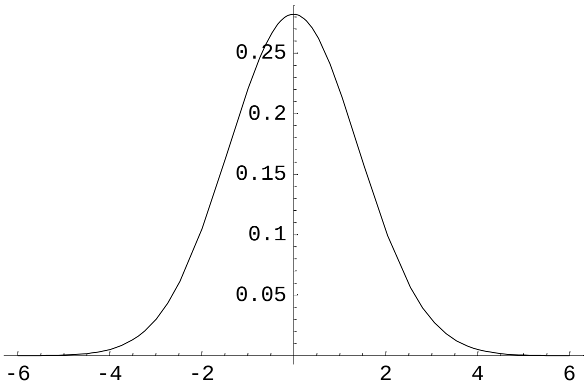
In[154]:=

```
Plot[Evaluate[fTransf1[Ω]], {Ω, -6, 6}];
```



In[155]:=

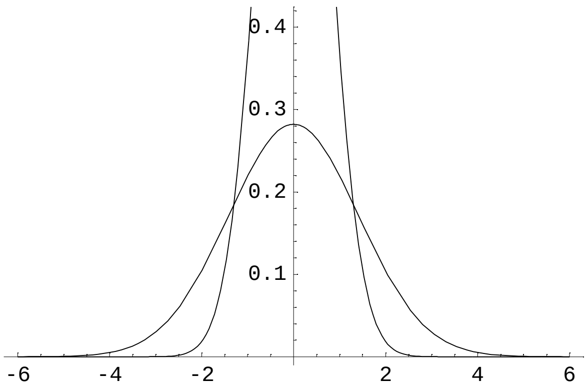
```
Plot[Evaluate[fTransf2[Ω]], {Ω, -6, 6}];
```



c

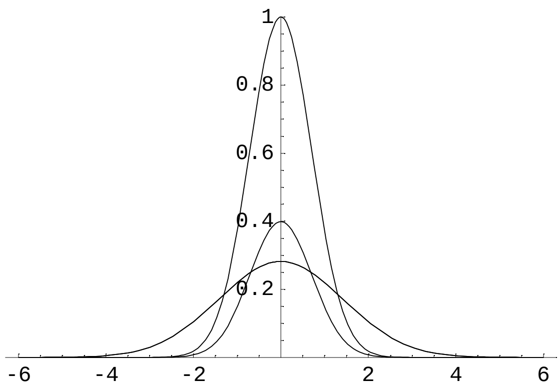
In[156]:=

```
Plot[Evaluate[{f1[x], fTransf1[x]}], {x, -6, 6}];
```



In[157]:=

```
Plot[Evaluate[{f1[x], f2[x], fTransf1[x], fTransf2[x]}],
  {x, -6, 6}, PlotRange → {0, 1}];
```



In[158]:=

```
{f1[x], f2[x], fTransf1[x] , fTransf2[x]}
```

Out[158]=

$$\left\{e^{-x^2},\ \frac{e^{-x^2}}{\sqrt{2\pi}},\ \frac{e^{-\frac{x^2}{4}}}{2\sqrt{\pi}},\ \frac{e^{-\frac{x^2}{4}}}{2\sqrt{\pi}}\right\}$$

In[159]:=

```
fTransf1[x] == fTransf2[x]
```

Out[159]=

```
True
```

Bemerkenswert: Alles Gauss-Glocken

7

Eine brauchbare Funktion

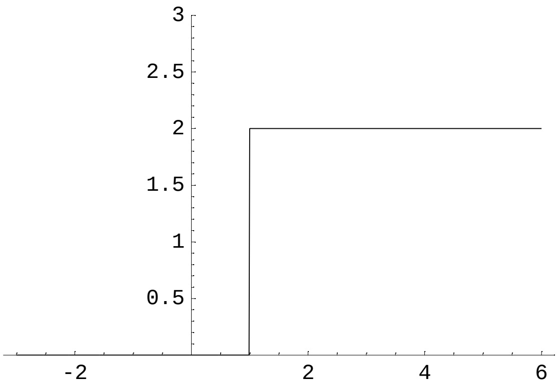
In[160]:=

```
?UnitStep
```

UnitStep[x] represents the unit step function, equal to 0 for x < 0 and 1 for x ≥ 0. UnitStep[x1, x2, ...] represents the multidimensional unit step function which is 1 only if none of the xi are negative. Mehr…

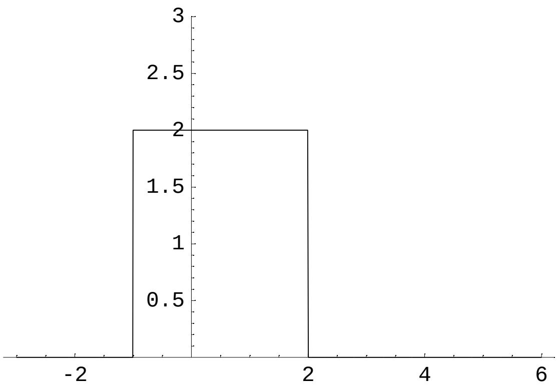
In[161]:=

```
u[a_, b_, x_] = b UnitStep[(x - a)];
Plot[u[1, 2, x], {x, -3, 6}, PlotRange → {0, 3}];
```

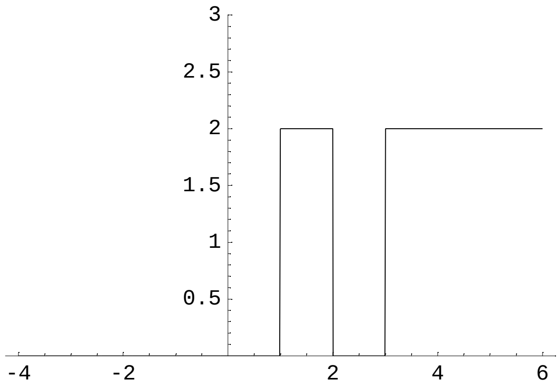


In[163]:=

```
u[a_, b_, c_, x_] = c UnitStep[-(x - a) (x - b)];
Plot[u[-1, 2, 2, x], {x, -3, 6}, PlotRange → {0, 3}];
```



In[165]:=
```
u[a_, b_, c_, d_, x_] = d UnitStep[(x - a) (x - b) (x - c)];
Plot[u[1, 2, 3, 2, x], {x, -4, 6}, PlotRange → {0, 3}];
```



a

In[167]:=
```
fTransf1[Ω_] := 1 / (2 Pi) Integrate[u[1, 2, 3, λ] E^(-I λ Ω), {λ, -Infinity, Infinity}];
fTransf1[Ω]
```

Out[168]=
$$-\frac{3\, i\, e^{-2 i \Omega} (-1 + e^{i \Omega})}{2 \pi \Omega}$$

In[169]:=
```
fTransf1[Ω] // Expand
```

Out[169]=
$$-\frac{3\, i\, e^{-i \Omega}}{2 \pi \Omega} + \frac{3\, i\, e^{-2 i \Omega}}{2 \pi \Omega}$$

In[170]:=
```
fTransf2[Ω_] := 1 / (2 Pi) Integrate[u[4, 5, 6, λ] E^(-I λ Ω), {λ, -Infinity, Infinity}];
fTransf2[Ω]
```

Out[171]=
$$-\frac{3\, i\, e^{-5 i \Omega} (-1 + e^{i \Omega})}{\pi \Omega}$$

In[172]:=
```
fTransf2[Ω] // Expand
```

Out[172]=
$$-\frac{3\, i\, e^{-4 i \Omega}}{\pi \Omega} + \frac{3\, i\, e^{-5 i \Omega}}{\pi \Omega}$$

a1

In[173]:=
```
FourierTransform[u[1, 2, 3, t], t, Ω]
```

Out[173]=
$$-\frac{3\, i\, e^{i \Omega} (-1 + e^{i \Omega})}{\sqrt{2 \pi}\, \Omega}$$

b

In[174]:=

```
fTransf3[Ω_] := 1/(2 Pi)
   Integrate[(u[1, 2, 3, λ] + u[4, 5, 6, λ]) E^(-I λ Ω), {λ, -Infinity, Infinity}];
fTransf3[
 Ω]
```

Out[175]=

$$-\frac{3\, i\, e^{-5 i \Omega} \left(-1+e^{i \Omega}\right) \left(2+e^{3 i \Omega}\right)}{2 \pi \Omega}$$

In[176]:=

```
fTransf1[Ω] + fTransf2[Ω] // Simplify
```

Out[176]=

$$-\frac{3\, i\, e^{-5 i \Omega} \left(-2+2 e^{i \Omega}-e^{3 i \Omega}+e^{4 i \Omega}\right)}{2 \pi \Omega}$$

In[177]:=

```
Expand[fTransf3[Ω]]
```

Out[177]=

$$-\frac{3\, i\, e^{-i \Omega}}{2 \pi \Omega}+\frac{3\, i\, e^{-2 i \Omega}}{2 \pi \Omega}-\frac{3\, i\, e^{-4 i \Omega}}{\pi \Omega}+\frac{3\, i\, e^{-5 i \Omega}}{\pi \Omega}$$

In[178]:=

```
Expand[fTransf3[Ω]] == Expand[fTransf1[Ω] + fTransf2[Ω]]
```

Out[178]=

True

---

work

In[180]:=

```
<< Calculus`FourierTransform`
```

In[182]:=

```
f[t_] := -t
```

In[183]:=

```
FourierTrigSeries[f[t], t, 10]
```

Out[183]=

$$-\frac{\text{Sin}[2 \pi t]}{\pi}+\frac{\text{Sin}[4 \pi t]}{2 \pi}-\frac{\text{Sin}[6 \pi t]}{3 \pi}+\frac{\text{Sin}[8 \pi t]}{4 \pi}-\frac{\text{Sin}[10 \pi t]}{5 \pi}+\frac{\text{Sin}[12 \pi t]}{6 \pi}-\frac{\text{Sin}[14 \pi t]}{7 \pi}+\frac{\text{Sin}[16 \pi t]}{8 \pi}-\frac{\text{Sin}[18 \pi t]}{9 \pi}+\frac{\text{Sin}[20 \pi t]}{10 \pi}$$

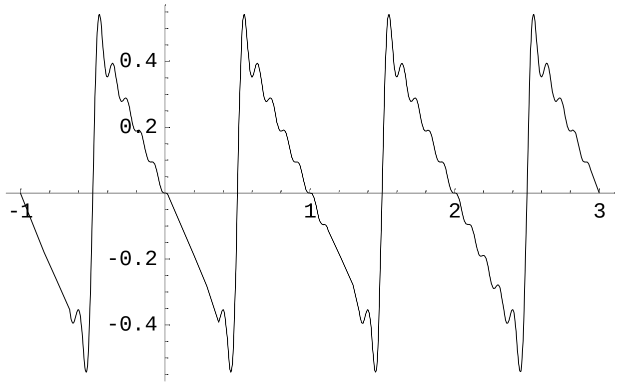
In[184]:=

```
f1[u_] = 2 Pi FourierTrigSeries[f[t], t, 10] /. t → u / (2 Pi) // Expand;
Print[f1[u]];
```

$$-2\,\text{Sin}[u]+\text{Sin}[2 u]-\frac{2}{3}\text{Sin}[3 u]+\frac{1}{2}\text{Sin}[4 u]-\frac{2}{5}\text{Sin}[5 u]+\frac{1}{3}\text{Sin}[6 u]-\frac{2}{7}\text{Sin}[7 u]+\frac{1}{4}\text{Sin}[8 u]-\frac{2}{9}\text{Sin}[9 u]+\frac{1}{5}\text{Sin}[10 u]$$

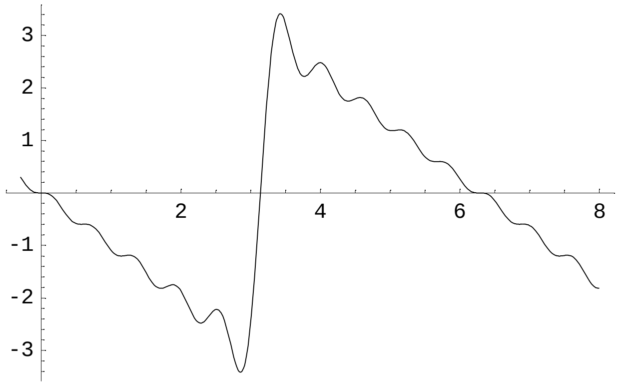
In[186]:=

```
Plot[Evaluate[FourierTrigSeries[f[t], t, 10]], {t, -1, 3}];
```



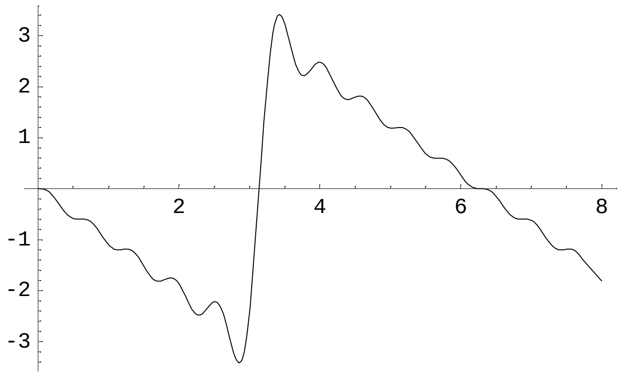
In[187]:=

```
Plot[f1[u], {u, -0.3, 8}];
```



In[188]:=

```
Plot[Evaluate[2 Pi FourierTrigSeries[f[t], t, 10] /. t → u / (2 Pi)], {u, 0, 8}];
```



In[189]:=

```
FourierSinCoefficient[f[t], t, 10]
```

Out[189]=

$$\frac{1}{10\,\pi}$$

In[190]:=

```
FourierSinCoefficient[f[t], t, 10] // TeXForm
```

Out[190]//TeXForm=

```
\frac{1}{10 \pi }
```

In[191]:=

```
FourierCosCoefficient[f[t], t, 10]
```

Out[191]=

0

In[192]:=

```
FourierSeries[f[t], t, 4]
```

Out[192]=

$$-\frac{i\, e^{-2 i \pi t}}{2\pi}+\frac{i\, e^{2 i \pi t}}{2\pi}+\frac{i\, e^{-4 i \pi t}}{4\pi}-\frac{i\, e^{4 i \pi t}}{4\pi}-\frac{i\, e^{-6 i \pi t}}{6\pi}+\frac{i\, e^{6 i \pi t}}{6\pi}+\frac{i\, e^{-8 i \pi t}}{8\pi}-\frac{i\, e^{8 i \pi t}}{8\pi}$$

In[193]:=

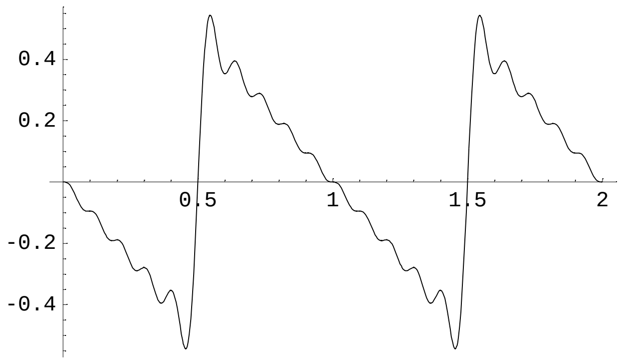
```
FourierSeries[f[t], t, 10]
```

Out[193]=

$$-\frac{i\, e^{-2 i \pi t}}{2\pi}+\frac{i\, e^{2 i \pi t}}{2\pi}+\frac{i\, e^{-4 i \pi t}}{4\pi}-\frac{i\, e^{4 i \pi t}}{4\pi}-\frac{i\, e^{-6 i \pi t}}{6\pi}+\frac{i\, e^{6 i \pi t}}{6\pi}+$$
$$\frac{i\, e^{-8 i \pi t}}{8\pi}-\frac{i\, e^{8 i \pi t}}{8\pi}-\frac{i\, e^{-10 i \pi t}}{10\pi}+\frac{i\, e^{10 i \pi t}}{10\pi}+\frac{i\, e^{-12 i \pi t}}{12\pi}-\frac{i\, e^{12 i \pi t}}{12\pi}-\frac{i\, e^{-14 i \pi t}}{14\pi}+$$
$$\frac{i\, e^{14 i \pi t}}{14\pi}+\frac{i\, e^{-16 i \pi t}}{16\pi}-\frac{i\, e^{16 i \pi t}}{16\pi}-\frac{i\, e^{-18 i \pi t}}{18\pi}+\frac{i\, e^{18 i \pi t}}{18\pi}+\frac{i\, e^{-20 i \pi t}}{20\pi}-\frac{i\, e^{20 i \pi t}}{20\pi}$$

In[194]:=

```
Plot[Evaluate[Re[FourierSeries[f[t], t, 10]] /. t → u], {u, -0, 2}, PlotPoints → 100];
```



In[195]:=

```
FourierTrigSeries[f[t], t, 6]
```

Out[195]=

$$-\frac{\mathrm{Sin}[2\pi t]}{\pi}+\frac{\mathrm{Sin}[4\pi t]}{2\pi}-\frac{\mathrm{Sin}[6\pi t]}{3\pi}+\frac{\mathrm{Sin}[8\pi t]}{4\pi}-\frac{\mathrm{Sin}[10\pi t]}{5\pi}+\frac{\mathrm{Sin}[12\pi t]}{6\pi}$$

In[196]:=

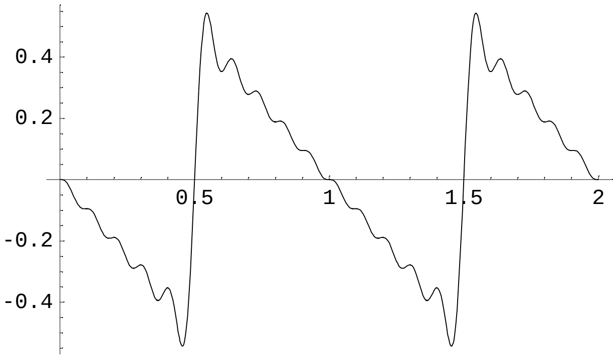
```
(FourierTrigSeries[f[t], t, 6] /. t → 1 / 4) - 1 / 4 // N
```

Out[196]=

-0.525869

In[197]:=

```
Plot[Evaluate[FourierTrigSeries[f[t], t, 10] /. t → u], {u, -0, 2}, PlotPoints → 100];
```



In[198]:=

```
2 Pi (FourierSeries[f[t], t, 10] /. t → u / (2 Pi)) // Expand
```

Out[198]=

$$-i e^{-i u} + i e^{i u} + \frac{1}{2} i e^{-2 i u} - \frac{1}{2} i e^{2 i u} - \frac{1}{3} i e^{-3 i u} + \frac{1}{3} i e^{3 i u} + \frac{1}{4} i e^{-4 i u} - \frac{1}{4} i e^{4 i u} - \frac{1}{5} i e^{-5 i u} + \frac{1}{5} i e^{5 i u} + \frac{1}{6} i e^{-6 i u} - \frac{1}{6} i e^{6 i u} - \frac{1}{7} i e^{-7 i u} + \frac{1}{7} i e^{7 i u} + \frac{1}{8} i e^{-8 i u} - \frac{1}{8} i e^{8 i u} - \frac{1}{9} i e^{-9 i u} + \frac{1}{9} i e^{9 i u} + \frac{1}{10} i e^{-10 i u} - \frac{1}{10} i e^{10 i u}$$

In[199]:=

```
Re[FourierSeries[f[t], t, 10]]
```

Out[199]=

$$\mathrm{Re}\left[-\frac{i e^{-2 i \pi t}}{2 \pi} + \frac{i e^{2 i \pi t}}{2 \pi} + \frac{i e^{-4 i \pi t}}{4 \pi} - \frac{i e^{4 i \pi t}}{4 \pi} - \frac{i e^{-6 i \pi t}}{6 \pi} + \frac{i e^{6 i \pi t}}{6 \pi} + \frac{i e^{-8 i \pi t}}{8 \pi} - \frac{i e^{8 i \pi t}}{8 \pi} - \frac{i e^{-10 i \pi t}}{10 \pi} + \frac{i e^{10 i \pi t}}{10 \pi} + \frac{i e^{-12 i \pi t}}{12 \pi} - \frac{i e^{12 i \pi t}}{12 \pi} - \frac{i e^{-14 i \pi t}}{14 \pi} + \frac{i e^{14 i \pi t}}{14 \pi} + \frac{i e^{-16 i \pi t}}{16 \pi} - \frac{i e^{16 i \pi t}}{16 \pi} - \frac{i e^{-18 i \pi t}}{18 \pi} + \frac{i e^{18 i \pi t}}{18 \pi} + \frac{i e^{-20 i \pi t}}{20 \pi} - \frac{i e^{20 i \pi t}}{20 \pi}\right]$$