Lösungen

1

Abgabe

2

In[360]:=
```
Remove["Global`*"]
```

a

In[361]:=
```
(LaplaceTransform[y''[t] + a y'[t] + b y[t], t, s] /.
    {y[0] → y0, y'[0] → yS0, LaplaceTransform[y[t], t, s] → Y[s]}) ==
 (LaplaceTransform[f[t], t, s] /. {LaplaceTransform[f[t], t, s] → F[s]})
```

Out[361]=
$$-s\,y0 - yS0 + b\,Y[s] + s^2\,Y[s] + a\,(-y0 + s\,Y[s]) == F[s]$$

In[362]:=
```
Solve[-s y0 - yS0 + b Y[s] + s^2 Y[s] + a (-y0 + s Y[s]) == F[s], {Y[s]}]
```

Out[362]=
$$\left\{\left\{Y[s] \to \frac{a\,y0 + s\,y0 + yS0 + F[s]}{b + a\,s + s^2}\right\}\right\}$$

In[363]:=
```
((((DSolve[{y''[t] + a y'[t] + b y[t] == 0, y[0] == y0, y'[0] == yS0}, y, t] // Flatten))) //
  Simplify)
```

Out[363]=
$$\left\{y \to \text{Function}\left[\{t\},\ \frac{1}{2\sqrt{a^2 - 4\,b}}\right.\right.$$
$$\left(-a\,e^{\frac{1}{2}\left(-a-\sqrt{a^2-4\,b}\right)t}\,y0 + \sqrt{a^2 - 4\,b}\;e^{\frac{1}{2}\left(-a-\sqrt{a^2-4\,b}\right)t}\,y0 + a\,e^{\frac{1}{2}\left(-a+\sqrt{a^2-4\,b}\right)t}\,y0 +\right.$$
$$\left.\left.\left.\sqrt{a^2 - 4\,b}\;e^{\frac{1}{2}\left(-a+\sqrt{a^2-4\,b}\right)t}\,y0 - 2\,e^{\frac{1}{2}\left(-a-\sqrt{a^2-4\,b}\right)t}\,yS0 + 2\,e^{\frac{1}{2}\left(-a+\sqrt{a^2-4\,b}\right)t}\,yS0\right)\right]\right\}$$

In[364]:=
```
((((DSolve[{y''[t] + a y'[t] + b y[t] == 0, y[0] == y0, y'[0] == yS0}, y, t] // Flatten) //.
    {Sqrt[a^2 - 4 b] → k}) //. 1/(2 Sqrt[a^2 - 4 b]) → 1/(2 k)) // Simplify)
```

Out[364]=
$$\left\{y \to \text{Function}\left[\{t\},\ \frac{-a\,e^{\frac{1}{2}(-a-k)t}\,y0 + k\,e^{\frac{1}{2}(-a-k)t}\,y0 + a\,e^{\frac{1}{2}(-a+k)t}\,y0 + k\,e^{\frac{1}{2}(-a+k)t}\,y0 - 2\,e^{\frac{1}{2}(-a-k)t}\,yS0 + 2\,e^{\frac{1}{2}(-a+k)t}\,yS0}{2\,k}\right]\right\}$$

b

In[365]:=
```
a = 1; b = 1; F[s_] := 0; y0 = 1; yS0 = 0;
Y[s_] := (a y0 + s y0 + yS0 + F[s])/(b + a s + s^2); Y[s]
```

Out[366]=
$$\frac{1 + s}{1 + s + s^2}$$

In[367]:=
```
InverseLaplaceTransform[Y[s], s, t]
```

Out[367]=
$$\frac{e^{-t/2}\left(\sqrt{3}\,\text{Cos}\left[\frac{\sqrt{3}\,t}{2}\right]+\text{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]\right)}{\sqrt{3}}$$

In[368]:=
```
% // Expand
```

Out[368]=
$$e^{-t/2}\,\text{Cos}\left[\frac{\sqrt{3}\,t}{2}\right]+\frac{e^{-t/2}\,\text{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]}{\sqrt{3}}$$

In[369]:=
```
(% // N // Simplify) /. e^-0.5 t → N[1 / Sqrt[E], 10]^ t
```

Out[369]=
$$0.6065306597^{t}\,(\text{Cos}[0.866025\,t]+0.57735\,\text{Sin}[0.866025\,t])$$

In[370]:=
```
% // Expand // Simplify
```

Out[370]=
$$0.6065306597^{t}\,(\text{Cos}[0.866025\,t]+0.57735\,\text{Sin}[0.866025\,t])$$

- **c**

In[371]:=
```
a = 1; b = 1; F[s_] := 0; y0 = 0; yS0 = 1;
Y[s_] := (a y0 + s y0 + yS0 + F[s]) / (b + a s + s^2); Y[s]
```

Out[372]=
$$\frac{1}{1+s+s^2}$$

In[373]:=
```
InverseLaplaceTransform[Y[s], s, t]
```

Out[373]=
$$\frac{2\,e^{-t/2}\,\text{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]}{\sqrt{3}}$$

- **d**

In[374]:=
```
Remove[Y]
```

In[375]:=
```
a = -1; b = 1; f[t_] := Sin[t]; y0 = 0; yS0 = 1;
InverseLaplaceTransform[
 Y[s] /. Flatten[Solve[(LaplaceTransform[y''[t] + a y'[t] + b y[t], t, s] /.
       {y[0] → y0, y'[0] → yS0, LaplaceTransform[y[t], t, s] → Y[s]}) ==
     LaplaceTransform[f[t], t, s], {Y[s]}]], s, t]
```

Out[376]=
$$\text{Cos}[t]-e^{t/2}\,\text{Cos}\left[\frac{\sqrt{3}\,t}{2}\right]+\sqrt{3}\,e^{t/2}\,\text{Sin}\left[\frac{\sqrt{3}\,t}{2}\right]$$

- **e**

In[377]:=
```
Remove[Y]
```

In[378]:=
```
a = -2; b = 1; f[t_] := Sin[t]; y0 = 0; yS0 = 1;
InverseLaplaceTransform[
 Y[s] /. Flatten[Solve[(LaplaceTransform[y''[t] + a y'[t] + b y[t], t, s] /.
       {y[0] → y0, y'[0] → yS0, LaplaceTransform[y[t], t, s] → Y[s]}) ==
     LaplaceTransform[f[t], t, s], {Y[s]}]], s, t]
```

Out[379]=
$$\frac{1}{2}\left(-e^{t}+3\,e^{t}\,t+\text{Cos}[t]\right)$$

■ f

In[380]:=
```
a = 0; b = -1; f[t_] := DiracDelta[t]; y0 = 0; yS0 = 0;
Flatten[Solve[(LaplaceTransform[y''[t] + a y'[t] + b y[t], t, s] /. {y[0] → y0, y'[0] → yS0,
       LaplaceTransform[y[t], t, s] → Y[s]}) == LaplaceTransform[f[t], t, s], {Y[s]}]]
```

Out[380]= $\left\{Y[s] \to \frac{1}{-1+s^2}\right\}$

In[381]:=
```
a = 0; b = -1; f[t_] := DiracDelta[t]; y0 = 0; yS0 = 0;
InverseLaplaceTransform[
 Y[s] /. Flatten[Solve[(LaplaceTransform[y''[t] + a y'[t] + b y[t], t, s] /.
        {y[0] → y0, y'[0] → yS0, LaplaceTransform[y[t], t, s] → Y[s]}) ==
      LaplaceTransform[f[t], t, s], {Y[s]}]], s, t]
```

Out[382]= $\frac{1}{2} e^{-t}\left(-1+e^{2t}\right)$

In[383]:=
```
Expand[%]
```

Out[383]= $-\frac{e^{-t}}{2}+\frac{e^{t}}{2}$

3

In[384]:=
```
Remove["Global`*"]
```

In[385]:=
```
(LaplaceTransform[y'''[t] - 3 y''[t] + 3 y'[t] - y[t], t, s] /.
   {y[0] → y0, y'[0] → yS0, y''[0] → yS00, LaplaceTransform[y[t], t, s] → Y[s]}) ==
 (LaplaceTransform[f[t], t, s] /. {LaplaceTransform[f[t], t, s] → F[s]})
```

Out[385]= $-s^2\, y0 - s\, yS0 - yS00 - Y[s] + s^3\, Y[s] + 3\,(-y0 + s\, Y[s]) - 3\left(-s\, y0 - yS0 + s^2\, Y[s]\right) == F[s]$

In[386]:=
```
Solve[
 -s^2 y0 - s yS0 - yS00 - Y[s] + s^3 Y[s] + 3 (-y0 + s Y[s]) - 3 (-s y0 - yS0 + s^2 Y[s]) == F[s], {Y[s]}]
```

Out[386]= $\left\{\left\{Y[s] \to \frac{3\,y0 - 3\,s\,y0 + s^2\,y0 - 3\,yS0 + s\,yS0 + yS00 + F[s]}{(-1+s)^3}\right\}\right\}$

■ a

In[387]:=
```
y0 = 0; yS0 = 0;  yS00 = 0; F[s_] := LaplaceTransform[DiracDelta[t], t, s];
Y[s_] := (3 y0 - 3 s y0 + s^2 y0 - 3 yS0 + s yS0 + yS00 + F[s])/(-1 + s)^3; Y[s]
```

Out[387]= $\frac{1}{(-1+s)^3}$

In[388]:=
```
LaplaceTransform[DiracDelta[t], t, s]
```

Out[388]= 1

In[389]:=
```
InverseLaplaceTransform[1/(-1 + s)^3, s, t] // Expand
```

Out[389]= $\frac{e^{t}\,t^2}{2}$

In[390]:=
```
DSolve[
 {y'''[t] - 3 y''[t] + 3 y'[t] - y[t] == DiracDelta[0], y[0] == 0, y'[0] == 0, y''[0] == 0}, y, t]
```

Out[390]= $\left\{\left\{y \to \text{Function}\left[\{t\}, \frac{1}{2}\left(-2 + 2\,e^{t} - 2\,e^{t}\,t + e^{t}\,t^2\right)\text{DiracDelta}[0]\right]\right\}\right\}$

■ b

In[391]:=
```
y0 = 0; yS0 = 0; yS00 = 0; F[s_] := LaplaceTransform[E^(-t), t, s];
Y[s_] := (3 y0 - 3 s y0 + s^2 y0 - 3 yS0 + s yS0 + yS00 + F[s]) / (-1 + s)^3; Y[s] // Together
```

Out[391]=
$$\frac{1}{(-1+s)^3\,(1+s)}$$

In[392]:=
```
LaplaceTransform[E^(-t), t, s]
```

Out[392]=
$$\frac{1}{1+s}$$

In[393]:=
```
InverseLaplaceTransform[1 / ((-1 + s)^3 (1 + s)), s, t] // Expand
```

Out[393]=
$$-\frac{e^{-t}}{8}+\frac{e^{t}}{8}-\frac{e^{t}\,t}{4}+\frac{e^{t}\,t^2}{4}$$

In[394]:=
```
DSolve[{y'''[t] - 3 y''[t] + 3 y'[t] - y[t] == E^(-t), y[0] == 0, y'[0] == 0, y''[0] == 0}, y, t]
```

Out[394]=
$$\left\{\left\{y \to \text{Function}\left[\{t\}, \frac{1}{8} e^{-t}\left(-1+e^{2t}-2e^{2t}t+2e^{2t}t^2\right)\right]\right\}\right\}$$

■ c

In[395]:=
```
y0 = 1; yS0 = 0; yS00 = 0; F[s_] := LaplaceTransform[E^(-t), t, s];
Y[s_] := (3 y0 - 3 s y0 + s^2 y0 - 3 yS0 + s yS0 + yS00 + F[s]) / (-1 + s)^3; Y[s] // Together
```

Out[395]=
$$\frac{4-2s^2+s^3}{(-1+s)^3\,(1+s)}$$

In[396]:=
```
LaplaceTransform[E^(-t), t, s]
```

Out[396]=
$$\frac{1}{1+s}$$

In[397]:=
```
InverseLaplaceTransform[(4 - 2 s^2 + s^3) / ((-1 + s)^3 (1 + s)), s, t] // Expand
```

Out[397]=
$$-\frac{e^{-t}}{8}+\frac{9\,e^{t}}{8}-\frac{5\,e^{t}\,t}{4}+\frac{3\,e^{t}\,t^2}{4}$$

In[398]:=
```
DSolve[{y'''[t] - 3 y''[t] + 3 y'[t] - y[t] == E^(-t), y[0] == 1, y'[0] == 0, y''[0] == 0}, y, t]
```

Out[398]=
$$\left\{\left\{y \to \text{Function}\left[\{t\}, \frac{1}{8} e^{-t}\left(-1+9e^{2t}-10e^{2t}t+6e^{2t}t^2\right)\right]\right\}\right\}$$

4

In[399]:=
```
Remove["Global`*"]
```

In[400]:=
```
VK = 4 / 3 × 5^3 Pi
```

Out[400]=
$$\frac{500\,\pi}{3}$$

In[401]:=
```
VK // N
```

Out[401]=
```
523.599
```

In[402]:=
```
VZ = NIntegrate[1, {x, 0.5, 2.5}, {y, -Sqrt[1^2 - (x - 1.5)^2], Sqrt[1^2 - (x - 1.5)^2]},
  {z, -Sqrt[5^2 - x^2 - y^2], Sqrt[5^2 - x^2 - y^2]}]
```

Out[402]= 29.62

In[403]:=
```
V = VK - VZ
```

Out[403]= 493.979

5

In[404]:=
```
Remove["Global`*"]
```

In[405]:=
```
(*  <<Statistics`DescriptiveStatistics`  *)
```

In[406]:=
```
<< "StatisticalPlots`"
```

In[407]:=
```
 (*  <<Statistics`DataManipulation`  *)
```

In[408]:=
```
N[Pi, 50]
```

Out[408]= 3.14159265358979323846264338327950288419716939937751

In[409]:=
```
N[Pi, 50]
```

Out[409]= 3.14159265358979323846264338327950288419716939937751

In[410]:=
```
N[Pi, 100]
```

Out[410]= 3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862
8034825342117068

In[411]:=
```
M2 = Reverse[{3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3, 2, 3, 8, 4, 6, 2, 6,
   4, 3, 3, 8, 3, 2, 7, 9, 5, 0, 2, 8, 8, 4, 1, 9, 7, 1, 6, 9, 3, 9, 9, 3, 7, 5, 1}]
```

Out[411]= {1, 5, 7, 3, 9, 9, 3, 9, 6, 1, 7, 9, 1, 4, 8, 8, 2, 0, 5, 9, 7, 2, 3, 8,
 3, 3, 4, 6, 2, 6, 4, 8, 3, 2, 3, 9, 7, 9, 8, 5, 3, 5, 6, 2, 9, 5, 1, 4, 1, 3}

In[412]:=
```
M1 = Reverse[{0, 5, 8, 2, 0, 9, 7, 4, 9, 4, 4, 5, 9, 2, 3, 0, 7, 8, 1, 6, 4, 0, 6,
   2, 8, 6, 2, 0, 8, 9, 9, 8, 6, 2, 8, 0, 3, 4, 8, 2, 5, 3, 4, 2, 1, 1, 7, 0, 6, 8}]
```

Out[412]= {8, 6, 0, 7, 1, 1, 2, 4, 3, 5, 2, 8, 4, 3, 0, 8, 2, 6, 8, 9, 9, 8, 0, 2,
 6, 8, 2, 6, 0, 4, 6, 1, 8, 7, 0, 3, 2, 9, 5, 4, 4, 9, 4, 7, 9, 0, 2, 8, 5, 0}

■ a

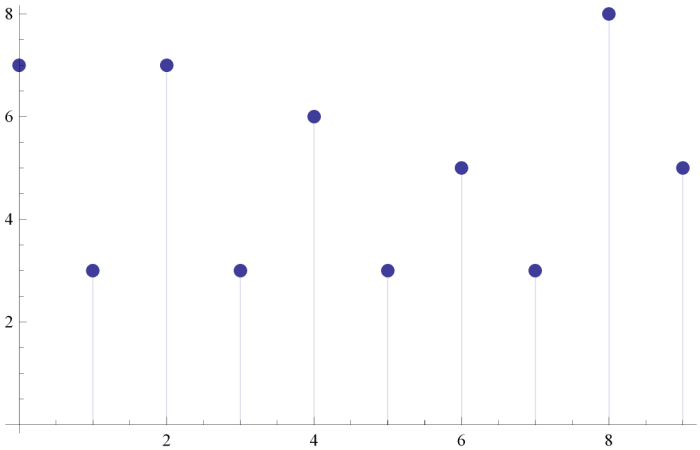
In[413]:=
```
Reverse[Sort[Tally[M1]], 2];
```

In[414]:=
```
Tally[M1] // Sort
```

Out[414]= {{0, 7}, {1, 3}, {2, 7}, {3, 3}, {4, 6}, {5, 3}, {6, 5}, {7, 3}, {8, 8}, {9, 5}}

In[415]:=
```
ListPlot[Tally[M1], Filling -> Axis, PlotStyle → {PointSize[Large]}]
```

Out[415]=



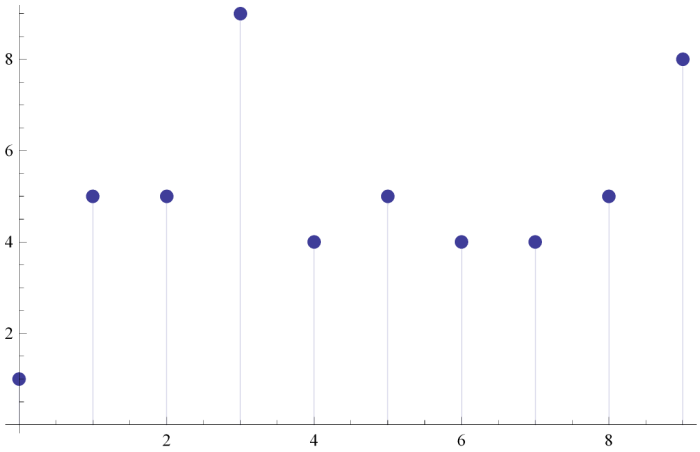
In[416]:=
```
Reverse[Sort[Tally[M2]], 2];
```

In[417]:=
```
Tally[M2] // Sort
```

Out[417]=
```
{{0, 1}, {1, 5}, {2, 5}, {3, 9}, {4, 4}, {5, 5}, {6, 4}, {7, 4}, {8, 5}, {9, 8}}
```

In[418]:=
```
ListPlot[Tally[M2], Filling -> Axis, PlotStyle → {PointSize[Large]}]
```

Out[418]=

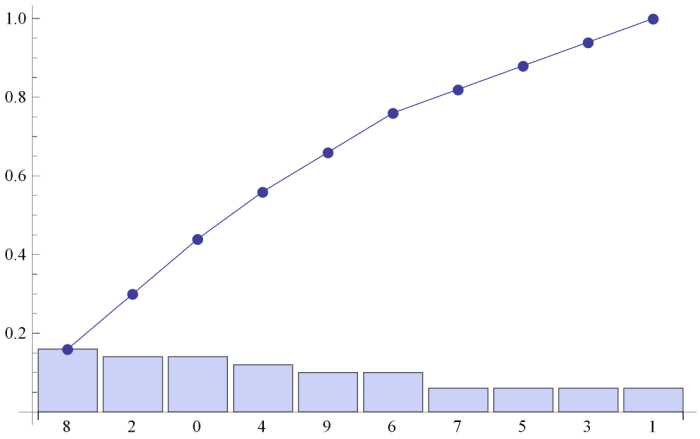


■ b

In[419]:=
```
Needs["StatisticalPlots`"]
```

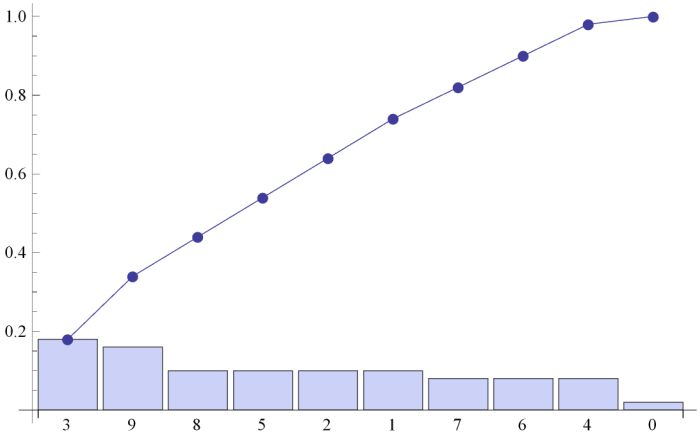
In[420]:=
```
ParetoPlot[M1]
```

Out[420]=



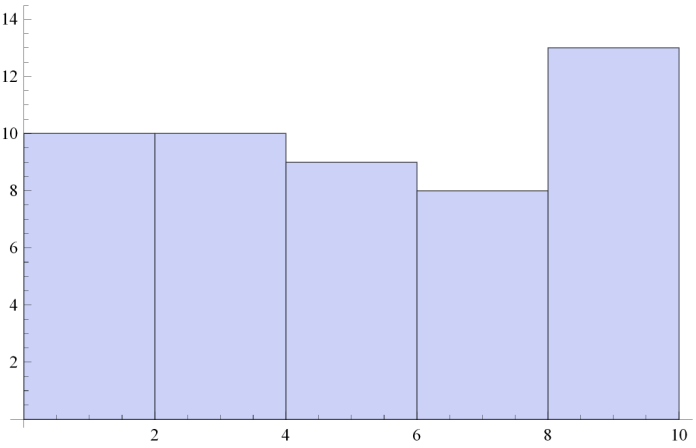
In[421]:=
```
ParetoPlot[M2]
```

Out[421]=



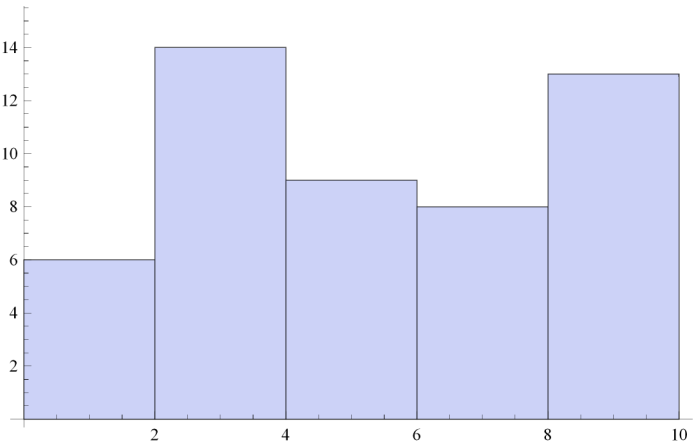
In[422]:= **Histogram[M1]**

Out[422]=



In[423]:= **Histogram[M2]**

Out[423]=



■ C

In[424]:= **Length[M1]**

Out[424]= 50

In[425]:= **report[x_] := {"Mean -> " Mean[x], "StandardDeviation -> " StandardDeviation[x]}**

In[426]:= **(* LocationReport[M1]//N *)**

In[427]:= **report[M1] // N**

Out[427]= {4.5 Mean -> , 3.06561 StandardDeviation -> }

In[428]:= **(* DispersionReport[M1]//N *)**

In[429]:= **Length[M2]**

Out[429]= 50

In[430]:= **(* LocationReport[M2]//N *)**

In[431]:= **(* DispersionReport[M2]//N *)**

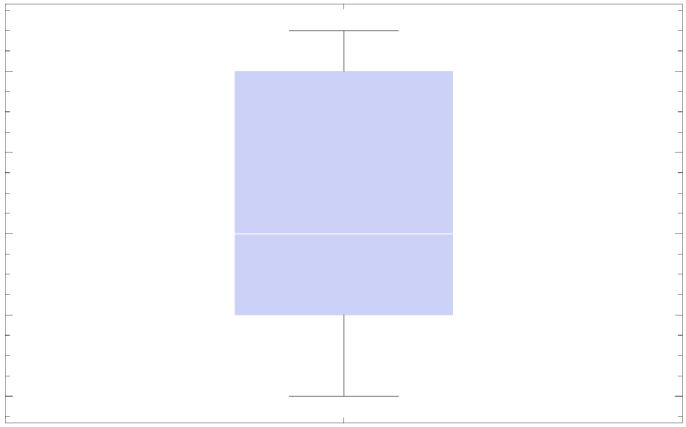
In[432]:= **report[M2] // N**

Out[432]= {4.94 Mean -> , 2.79511 StandardDeviation -> }

d

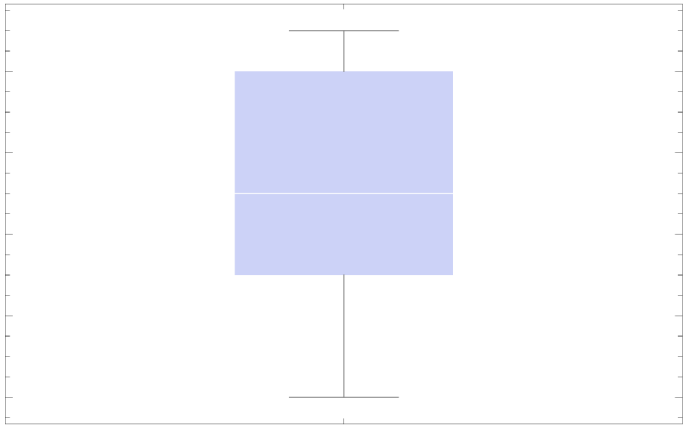
In[433]:=
```
BoxWhiskerChart[M1]
```

Out[433]=



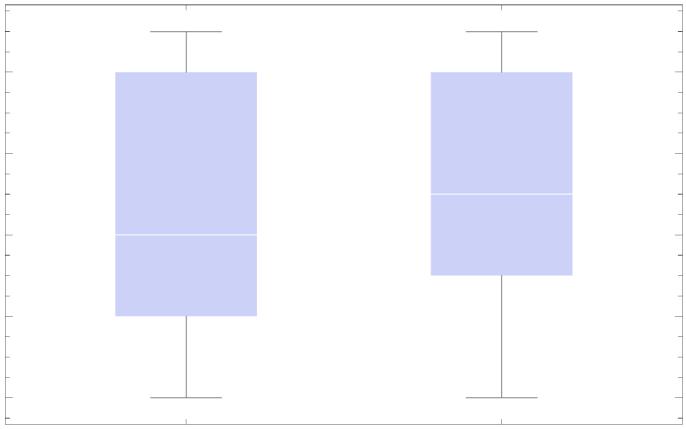
In[434]:=
```
BoxWhiskerChart[M2]
```

Out[434]=



In[435]:=
```
BoxWhiskerChart[{M1, M2}]
```

Out[435]=



e

Es handelt sich um je 50 Stellen der Dezimalpruchentwicklung von π.

In[436]:=
```
N[Pi, 150] - 314 159 / 100 000
```

Out[436]=
$$2.653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117067982148086513282306647093844609550582231725359408 13 \times 10^{-6}$$

In[437]:=
```
N[Pi, 150]
```

Out[437]=
```
3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862↘
 80348253421170679821480865132823066470938446095505822317253594081284811174503778
```

In[438]:=
```
2 653 589 793 238 462 643 383 279 502 884 197 169 399 375 105 820 974 944 592 307 816 406 286 208 998↘
 628 034 825 342
```

Out[438]=
```
2 653 589 793 238 462 643 383 279 502 884 197 169 399 375 105 820 974 944 592 307 816 406 286 208 998↘
 628 034 825 342
```

In[439]:=
```
3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862↘
  80348253421170679821480865132823066470938446095505822317253594081284811174503778 ×
 10^30 // N
```

Out[439]=
$$3.14159 \times 10^{30}$$

In[440]:=
```
N[Pi, 50]
```

Out[440]=
```
3.14159265358979323846264338327950288419716939937510
```

In[441]:=
```
N[Pi, 100]
```

Out[441]=
```
3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862↘
 8034825342117068
```

6

In[442]:=
```
Remove["Global`*"]
```

■ a

In[443]:=
```
10!
```

Out[443]=
```
3 628 800
```

In[444]:=
```
10! // N
```

Out[444]=
$$3.6288 \times 10^{6}$$

Bei Wiederholung:

In[445]:=
```
10^10
```

Out[445]=
```
10 000 000 000
```

In[446]:=
```
% // N
```

Out[446]=
$$1. \times 10^{10}$$

■ b

In[447]:=
```
49 / 7 // N
```

Out[447]=
```
7.
```

In[448]:=
```
Binomial[49, 7] Binomial[49 - 7, 7] Binomial[49 - 2 7, 7] Binomial[49 - 3 7, 7]
 Binomial[49 - 4 7, 7] Binomial[49 - 5 7, 7] Binomial[49 - 6 7, 7]
```

Out[448]=
```
7 363 615 666 157 189 603 982 585 462 030 336 000
```

In[449]:=
```
% // N
```

Out[449]=
$$7.36362 \times 10^{36}$$

In[450]:=
```
7 Product[Binomial[49 - k 7, 7], {k, 0, 6}]
```

Out[450]=
```
51 545 309 663 100 327 227 878 098 234 212 352 000
```

In[451]:=
```
% // N
```

Out[451]=
$$5.15453 \times 10^{37}$$