Lösungen

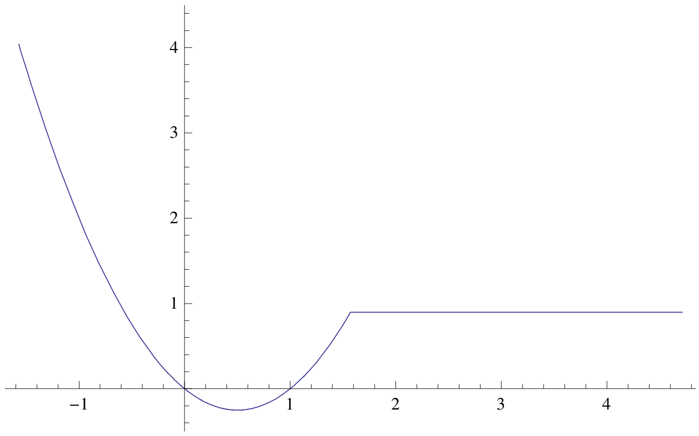
1

```
Remove["Global`*"]
```

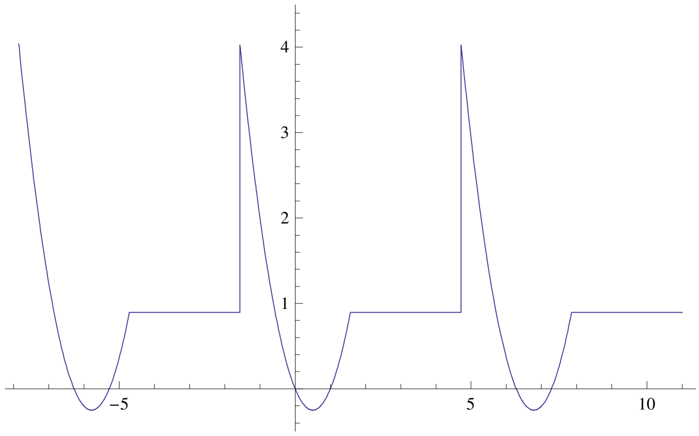
```
f[t_] := t^2 - t /; (-Pi / 2 ≤ t && t ≤ Pi / 2);
f[t_] := f[Pi / 2] /; (Pi / 2 < t && t ≤ 3 Pi / 2);
f[t_] := f[t + 2 Pi] /; (-5 Pi / 2 ≤ t && t < -Pi / 2);
f[t_] := f[t - 2 Pi] /; (3 Pi / 2 < t && t ≤ 7 Pi / 2);
f1[t_] := t^2 - t;
f2[t_] := f1[Pi / 2];
```

- **a Skizze**

```
Plot[f[t], {t, -Pi / 2, 3 Pi / 2}, PlotRange → {-0.5, 4.5}]
```




```
Plot[f[t], {t, -5 Pi / 2, 7 Pi / 2}, PlotRange → {-0.5, 4.5}]
```



▪ b Koeffizienten

```
T = 2 Pi;
cc = -Pi / 2;
ω = 2 Pi / T;
a[0] := 2 / T NIntegrate[f[t], {t, cc, cc + T}];
a[k_] := 2 / T NIntegrate[f[t] Cos[k ω t], {t, cc, cc + T}];
b[k_] := 2 / T NIntegrate[f[t] Sin[k ω t], {t, cc, cc + T}];
(* c[k_]:=1/T Integrate[f[t] E^(-I k ω t),{t,cc,cc+T}]; *)
ff[t_] := a[0] / 2 + Sum[a[n] Cos[n ω t] + b[n] Sin[n ω t], {n, 1, Infinity}];
ff[t_, h_] := a[0] / 2 + Sum[a[n] Cos[n ω t] + b[n] Sin[n ω t], {n, 1, h}] // Chop;
(* ffk[t_]:=Sum[c[n] E^(I n ω t),{n,-Infinity,Infinity}]; *)
(* ffk[t_,h_]:=Sum[c[n] E^(I n ω t),{n,-h,h}]; *)
g[t_] := (ff[u, 4] /. u → t) // Simplify; g[t]
```

NIntegrate::ncvb :
NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in t near {t} = {1.57379}. NIntegrate obtained 0.22222220216854388` and 5.505613236842656`*^-7 for the integral and error estimates. ≫

NIntegrate::ncvb :
NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in t near {t} = {1.57379}. NIntegrate obtained 0.392699101752458` and 5.503283727217613`*^-7 for the integral and error estimates. ≫

```
0.859536 - 0.27324 Cos[t] - 0.5 Cos[2 t] - 0.286176 Cos[3 t] + 0.125 Cos[4 t] -
 0.63662 Sin[t] - 1. Cos[t] Sin[t] + 0.0707355 Sin[3 t] + 0.25 Sin[4 t]
```

Infinity ist hier doch zu weit.

```
0.859536 - 0.27324 Cos[t] - 0.5 Cos[2 t] - 0.286176 Cos[3 t] + 0.125 Cos[4 t] -
 0.63662 Sin[t] - 1. Cos[t] Sin[t] + 0.0707355 Sin[3 t] + 0.25 Sin[4 t]
```

```
a1[0] :=
  2 / T (Integrate[f1[t], {t, cc, cc + T / 2}] + Integrate[f2[t], {t, cc + T / 2, cc + T}]);
a1[k_] := 2 / T (Integrate[f1[t] Cos[k ω t], {t, cc, cc + T / 2}] +
     Integrate[f2[t] Cos[k ω t], {t, cc + T / 2, cc + T}]);
b1[k_] := 2 / T ( Integrate[f1[t] Sin[k ω t], {t, cc, cc + T / 2}] +
     Integrate[f2[t] Sin[k ω t], {t, cc + T / 2, cc + T}]);
```

```
ff1[u_, h_] := a1[0] / 2 + Sum[a1[n] Cos[n ω u] + b1[n] Sin[n ω u], {n, 1, h}] // Chop;
g1[t_] := (ff1[u, 4] /. u → t);
g1[t] // Expand
```

$$-\frac{\pi}{4} + \frac{\pi^2}{6} + \mathrm{Cos}[t] - \frac{4\,\mathrm{Cos}[t]}{\pi} - \frac{1}{2}\mathrm{Cos}[2\,t] - \frac{1}{3}\mathrm{Cos}[3\,t] + \frac{4\,\mathrm{Cos}[3\,t]}{27\,\pi} + \frac{1}{8}\mathrm{Cos}[4\,t] - \frac{2\,\mathrm{Sin}[t]}{\pi} - \frac{1}{2}\mathrm{Sin}[2\,t] + \frac{2\,\mathrm{Sin}[3\,t]}{9\,\pi} + \frac{1}{4}\mathrm{Sin}[4\,t]$$

```
g1N[t_] := (ff1[u, 4] /. u → t) // N;
g1N[t]
```

```
0.859536 - 0.27324 Cos[t] - 0.5 Cos[2. t] - 0.286176 Cos[3. t] + 0.125 Cos[4. t] -
 0.63662 Sin[t] - 0.5 Sin[2. t] + 0.0707355 Sin[3. t] + 0.25 Sin[4. t]
```

```
g1N[t] // Expand
```

```
0.859536 - 0.27324 Cos[t] - 0.5 Cos[2. t] - 0.286176 Cos[3. t] + 0.125 Cos[4. t] -
 0.63662 Sin[t] - 0.5 Sin[2. t] + 0.0707355 Sin[3. t] + 0.25 Sin[4. t]
```

```
a1[0]
```

$$\frac{\frac{\pi^3}{12} + \pi\left(-\frac{\pi}{2} + \frac{\pi^2}{4}\right)}{\pi}$$

```
(* a0/2, a0 *)  {0.859535903450778`, 2 × 0.859535903450778`}
```

{0.859536, 1.71907}

```
(* ak *) {-0.27323954473516276`, -0.5` , -0.286176313157957` , +0.125` }
```

{-0.27324, -0.5, -0.286176, 0.125}

```
(* bk *) {0.6366197723675814`, -0.5` , +0.07073553026306459` , +0.25` }
```

{0.63662, -0.5, 0.0707355, 0.25}

c: Gute Näherung schon mit wenigen Koeffizienten

```
g1[Pi / 2]
```

$$\frac{5}{8} - \frac{20}{9\pi} + \frac{\frac{\pi^3}{12} + \pi\left(-\frac{\pi}{2} + \frac{\pi^2}{4}\right)}{2\pi}$$

```
f[Pi / 2]
```

$$-\frac{\pi}{2} + \frac{\pi^2}{4}$$

```
% // N
```

0.896605

```
f[3 Pi / 2]
```

$$-\frac{\pi}{2} + \frac{\pi^2}{4}$$

```
% // N
```

0.896605

```
Abs[g1[Pi / 2] - f[Pi / 2]]
```

$$-\frac{5}{8} + \frac{20}{9\pi} - \frac{\pi}{2} + \frac{\pi^2}{4} - \frac{\frac{\pi^3}{12} + \pi\left(-\frac{\pi}{2} + \frac{\pi^2}{4}\right)}{2\pi}$$

```
Abs[g1N[Pi / 2] - f[Pi / 2]]
```

0.119424

```
Abs[g1[3 Pi / 2] - f[Pi / 2]]
```

$$\frac{5}{8} + \frac{20}{9\pi} + \frac{\pi}{2} - \frac{\pi^2}{4} + \frac{\frac{\pi^3}{12} + \pi\left(-\frac{\pi}{2} + \frac{\pi^2}{4}\right)}{2\pi}$$

```
Abs[g1N[3 Pi / 2] - f[3 Pi / 2]]
```

1.29529

```
Abs[g1[3 Pi / 2] - f[3 Pi / 2]]
```

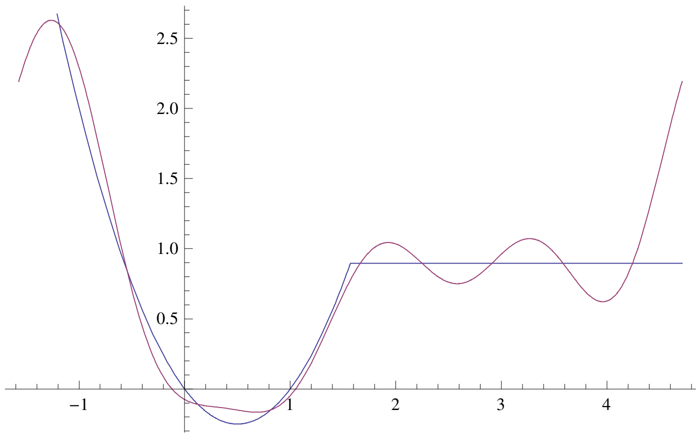
$$\frac{5}{8} + \frac{20}{9\pi} + \frac{\pi}{2} - \frac{\pi^2}{4} + \frac{\frac{\pi^3}{12} + \pi\left(-\frac{\pi}{2} + \frac{\pi^2}{4}\right)}{2\pi}$$

■ d:

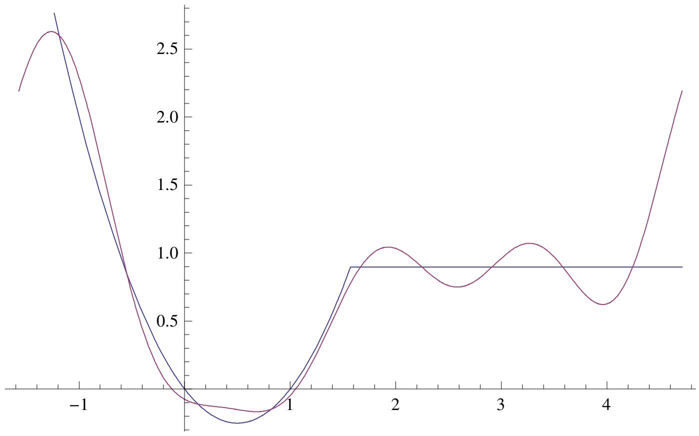
```
Plot[Evaluate[{f[t], g[t]}], {t, -Pi / 2, 3 Pi / 2}, PlotPoints → 60]
```

NIntegrate::ncvb :
NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in t near {t} = {1.57379}. NIntegrate obtained 0.22222220216854388` and 5.505613236842656`*^-7 for the integral and error estimates. ≫

NIntegrate::ncvb :
NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in t near {t} = {1.57379}. NIntegrate obtained 0.392699101752458` and 5.503283727217613`*^-7 for the integral and error estimates. ≫




```
Plot[Evaluate[{f[t], g1[t]}], {t, -Pi / 2, 3 Pi / 2}]
```



■ e

```
{f[Pi / 2], g1[Pi / 2]}
```

$$\left\{-\frac{\pi}{2}+\frac{\pi^2}{4},\ \frac{5}{8}-\frac{20}{9\pi}+\frac{\frac{\pi^3}{12}+\pi\left(-\frac{\pi}{2}+\frac{\pi^2}{4}\right)}{2\pi}\right\}$$

```
N[%]
```

{0.896605, 0.777181}

In $\frac{\pi}{2}+\frac{\pi^2}{4}$ = f[Pi / 2] = g1[Pi / 2]
lässt sich π auf eine Seite der Gleichung bringen und so isolieren, also berechnen.

2

▪ a

```
Remove["Global`*"];
```

```
T = Pi;
cc = -Pi / 2;
f[t_] := Abs[t] + t;
ω = 2 Pi / T;
a[0] := 2 / T Integrate[f[t], {t, cc, cc + T}];
a[k_] := 2 / T Integrate[f[t] Cos[k ω t], {t, cc, cc + T}];
b[k_] := 2 / T Integrate[f[t] Sin[k ω t], {t, cc, cc + T}];
ff[s_, h_] := a[0] / 2 + Sum[a[n] Cos[n ω s] + b[n] Sin[n ω s], {n, 1, h}];
ff[s, 10]
```

$$\frac{\pi}{4} - \frac{2\,\mathrm{Cos}[2\,s]}{\pi} - \frac{2\,\mathrm{Cos}[6\,s]}{9\,\pi} - \frac{2\,\mathrm{Cos}[10\,s]}{25\,\pi} - \frac{2\,\mathrm{Cos}[14\,s]}{49\,\pi} - \frac{2\,\mathrm{Cos}[18\,s]}{81\,\pi} + \mathrm{Sin}[2\,s] - \frac{1}{2}\,\mathrm{Sin}[4\,s] + \frac{1}{3}\,\mathrm{Sin}[6\,s] - \frac{1}{4}\,\mathrm{Sin}[8\,s] + \frac{1}{5}\,\mathrm{Sin}[10\,s] - \frac{1}{6}\,\mathrm{Sin}[12\,s] + \frac{1}{7}\,\mathrm{Sin}[14\,s] - \frac{1}{8}\,\mathrm{Sin}[16\,s] + \frac{1}{9}\,\mathrm{Sin}[18\,s] - \frac{1}{10}\,\mathrm{Sin}[20\,s]$$

```
N[%]
```

```
0.785398 - 0.63662 Cos[2. s] - 0.0707355 Cos[6. s] - 0.0254648 Cos[10. s] -
 0.0129922 Cos[14. s] - 0.0078595 Cos[18. s] + Sin[2. s] - 0.5 Sin[4. s] +
 0.333333 Sin[6. s] - 0.25 Sin[8. s] + 0.2 Sin[10. s] - 0.166667 Sin[12. s] +
 0.142857 Sin[14. s] - 0.125 Sin[16. s] + 0.111111 Sin[18. s] - 0.1 Sin[20. s]
```

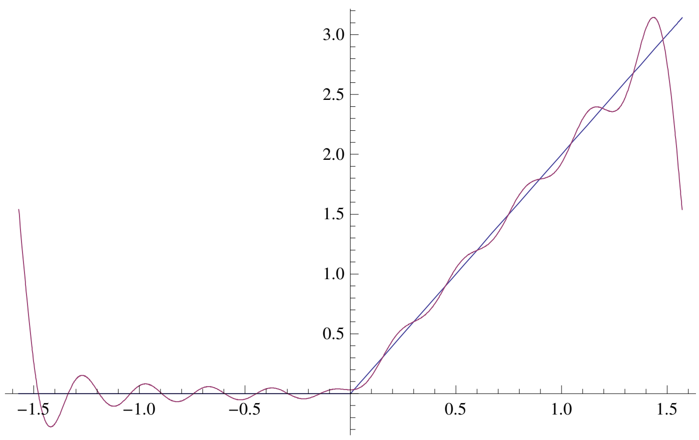
```
ff[4, 10]
```

$$\frac{\pi}{4} - \frac{2\,\mathrm{Cos}[8]}{\pi} - \frac{2\,\mathrm{Cos}[24]}{9\,\pi} - \frac{2\,\mathrm{Cos}[40]}{25\,\pi} - \frac{2\,\mathrm{Cos}[56]}{49\,\pi} - \frac{2\,\mathrm{Cos}[72]}{81\,\pi} + \mathrm{Sin}[8] - \frac{\mathrm{Sin}[16]}{2} + \frac{\mathrm{Sin}[24]}{3} - \frac{\mathrm{Sin}[32]}{4} + \frac{\mathrm{Sin}[40]}{5} - \frac{\mathrm{Sin}[48]}{6} + \frac{\mathrm{Sin}[56]}{7} - \frac{\mathrm{Sin}[64]}{8} + \frac{\mathrm{Sin}[72]}{9} - \frac{\mathrm{Sin}[80]}{10}$$

```
ff[s, 10] /. s → t
```

$$\frac{\pi}{4} - \frac{2\,\mathrm{Cos}[2\,t]}{\pi} - \frac{2\,\mathrm{Cos}[6\,t]}{9\,\pi} - \frac{2\,\mathrm{Cos}[10\,t]}{25\,\pi} - \frac{2\,\mathrm{Cos}[14\,t]}{49\,\pi} - \frac{2\,\mathrm{Cos}[18\,t]}{81\,\pi} + \mathrm{Sin}[2\,t] - \frac{1}{2}\,\mathrm{Sin}[4\,t] + \frac{1}{3}\,\mathrm{Sin}[6\,t] - \frac{1}{4}\,\mathrm{Sin}[8\,t] + \frac{1}{5}\,\mathrm{Sin}[10\,t] - \frac{1}{6}\,\mathrm{Sin}[12\,t] + \frac{1}{7}\,\mathrm{Sin}[14\,t] - \frac{1}{8}\,\mathrm{Sin}[16\,t] + \frac{1}{9}\,\mathrm{Sin}[18\,t] - \frac{1}{10}\,\mathrm{Sin}[20\,t]$$

```
Plot[Evaluate[{f[s], ff[s, 10]}], {s, -Pi / 2, Pi / 2}]
```



■ b

```
Remove["Global`*"];
```

Ersetze t durch - t und Abs[t]+t durch -(Abs[-t]-t). Schiebe dann die Funktion um 3.

```
f[t_] := Abs[t] + t;
```

```
f1[t_] := 3 - f[-t];  f1[t]
```

```
3 + t - Abs[t]
```

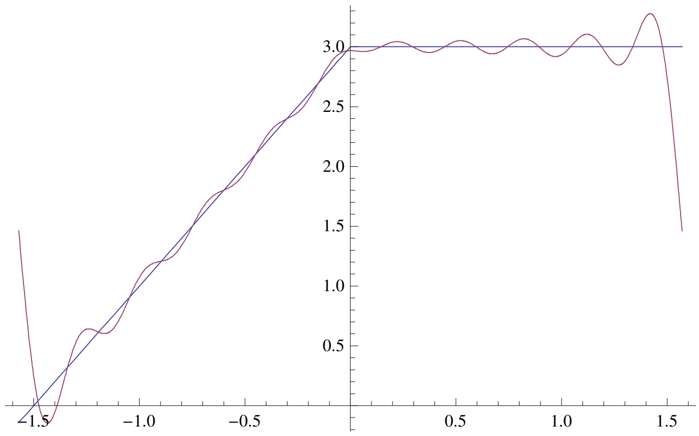
```
T = Pi;
cc  = -Pi / 2;
ω  =  2 Pi / T;
a[0] := 2 / T Integrate[f1[t], {t, cc, cc + T}];
a[k_] := 2 / T Integrate[f1[t] Cos[k ω t], {t, cc, cc + T}];
b[k_] := 2 / T Integrate[f1[t] Sin[k ω t], {t, cc, cc + T}];
ff1[s_, h_] := a[0] / 2 + Sum[a[n] Cos[n ω s] + b[n] Sin[n ω s], {n, 1, h}];
ff1[s, 10] // Simplify
```

$$3 - \frac{\pi}{4} + \frac{2\,\mathrm{Cos}[2\,s]}{\pi} + \frac{2\,\mathrm{Cos}[6\,s]}{9\,\pi} + \frac{2\,\mathrm{Cos}[10\,s]}{25\,\pi} + \frac{2\,\mathrm{Cos}[14\,s]}{49\,\pi} + \frac{2\,\mathrm{Cos}[18\,s]}{81\,\pi} + \mathrm{Sin}[2\,s] - \frac{1}{2}\,\mathrm{Sin}[4\,s] + \frac{1}{3}\,\mathrm{Sin}[6\,s] - \frac{1}{4}\,\mathrm{Sin}[8\,s] + \frac{1}{5}\,\mathrm{Sin}[10\,s] - \frac{1}{6}\,\mathrm{Sin}[12\,s] + \frac{1}{7}\,\mathrm{Sin}[14\,s] - \frac{1}{8}\,\mathrm{Sin}[16\,s] + \frac{1}{9}\,\mathrm{Sin}[18\,s] - \frac{1}{10}\,\mathrm{Sin}[20\,s]$$

```
N[%]
```

```
2.2146 + 0.63662 Cos[2. s] + 0.0707355 Cos[6. s] + 0.0254648 Cos[10. s] +
 0.0129922 Cos[14. s] + 0.0078595 Cos[18. s] + Sin[2. s] - 0.5 Sin[4. s] +
 0.333333 Sin[6. s] - 0.25 Sin[8. s] + 0.2 Sin[10. s] - 0.166667 Sin[12. s] +
 0.142857 Sin[14. s] - 0.125 Sin[16. s] + 0.111111 Sin[18. s] - 0.1 Sin[20. s]
```

```
Plot[Evaluate[{f1[s], ff1[s, 10]}], {s, -Pi / 2, Pi / 2}]
```



3 “Beidseitig”

```
Remove["Global`*"];
```

■ a

Wir verwenden zuerst die Skalierung nach der Periode 2 Pi. Das vereinfacht die Rechung etwas.

```
n=4; w = 2 Pi/n;
{x[0],y[0]}={0 w,2};
{x[1],y[1]}={1 w,2};
{x[2],y[2]}={2 w,3};
{x[3],y[3]}={3 w,3};
{x[-1],y[-1]}={-1 w,3};
{x[-2],y[-2]}={-2 w,3};
{x[-3],y[-3]}={-3 w,2};
{x[-4],y[-4]}={-4 w,2};
```

```
p[k_]:= {x[k],y[k]};
Table[p[k],{k,-(n-1),(n-1)}]
```

$\left\{\left\{-\frac{3\pi}{2}, 2\right\}, \{-\pi, 3\}, \left\{-\frac{\pi}{2}, 3\right\}, \{0, 2\}, \left\{\frac{\pi}{2}, 2\right\}, \{\pi, 3\}, \left\{\frac{3\pi}{2}, 3\right\}\right\}$

```
epi=Prepend[Map[Point,Table[p[k],{k,-n,n-1}]],PointSize[0.03]]
```

$\{\text{PointSize}[0.03], \text{Point}[\{-2\pi, 2\}], \text{Point}\left[\left\{-\frac{3\pi}{2}, 2\right\}\right], \text{Point}[\{-\pi, 3\}], \text{Point}\left[\left\{-\frac{\pi}{2}, 3\right\}\right], \text{Point}[\{0, 2\}], \text{Point}\left[\left\{\frac{\pi}{2}, 2\right\}\right], \text{Point}[\{\pi, 3\}], \text{Point}\left[\left\{\frac{3\pi}{2}, 3\right\}\right]\}$

```
epi1 = Prepend[Map[Point, Table[{k, y[k]}, {k, 0, n - 1}]], PointSize[0.03]]
```

{PointSize[0.03], Point[{0, 2}], Point[{1, 2}], Point[{2, 3}], Point[{3, 3}]}

```
r = E^(-I 2 Pi/n);
c[s_]:= 1/n Sum[y[k] r^(s k),{k,-Floor[(n-1)/2],n-1-Floor[(n-1)/2]}];
Table[c[s],{s,0,10}]//N
```

{2.5, −0.25 + 0.25 i, 0., −0.25 − 0.25 i, 2.5, −0.25 + 0.25 i, 0., −0.25 − 0.25 i, 2.5, −0.25 + 0.25 i, 0.}

```
fS[t_]:=Sum[c[k] E^(I k t),{k,-Floor[(n-1)/2],n-1-Floor[(n-1)/2]}];
fS[t]
```

$\frac{5}{2} - \left(\frac{1}{4} + \frac{i}{4}\right) e^{-i t} - \left(\frac{1}{4} - \frac{i}{4}\right) e^{i t}$

```
% // ExpandAll
```

$\frac{5}{2} - \left(\frac{1}{4} + \frac{i}{4}\right) e^{-i t} - \left(\frac{1}{4} - \frac{i}{4}\right) e^{i t}$

```
fS1[s_] := fS[s 2 Pi / n];
fS1[s]
```

$\frac{5}{2} - \left(\frac{1}{4} + \frac{i}{4}\right) e^{-\frac{1}{2} i \pi s} - \left(\frac{1}{4} - \frac{i}{4}\right) e^{\frac{i \pi s}{2}}$

```
% // ExpandAll
```

$\frac{5}{2} - \left(\frac{1}{4} + \frac{i}{4}\right) e^{-\frac{1}{2} i \pi s} - \left(\frac{1}{4} - \frac{i}{4}\right) e^{\frac{i \pi s}{2}}$

```
% // N // Simplify
```

$2.5 - (0.25 + 0.25 i)\, e^{(0. - 1.5708 i) s} - (0.25 - 0.25 i)\, e^{(0. + 1.5708 i) s}$

```
fS[t]//ExpToTrig
```

$\frac{5}{2} - \frac{\text{Cos}[t]}{2} - \frac{\text{Sin}[t]}{2}$

```
% // ExpandAll
```

$\frac{5}{2} - \frac{\text{Cos}[t]}{2} - \frac{\text{Sin}[t]}{2}$

```
% // N
```

2.5 - 0.5 Cos[t] - 0.5 Sin[t]

```
fS1[s] // ExpToTrig
```

$$\frac{5}{2} - \frac{1}{2} \text{Cos}\left[\frac{\pi s}{2}\right] - \frac{1}{2} \text{Sin}\left[\frac{\pi s}{2}\right]$$

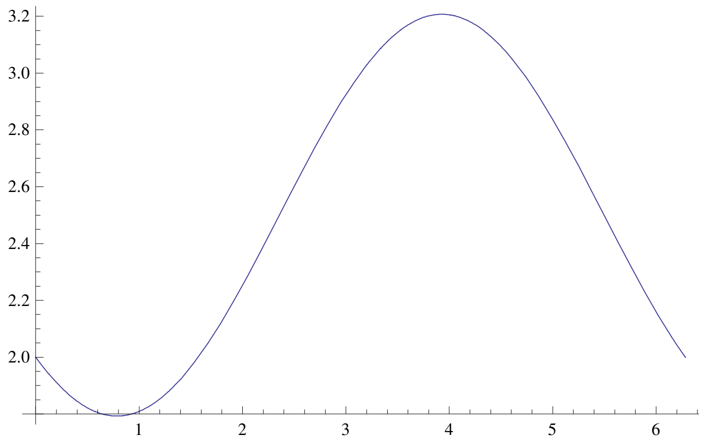
```
% // ExpandAll
```

$$\frac{5}{2} - \frac{1}{2} \text{Cos}\left[\frac{\pi s}{2}\right] - \frac{1}{2} \text{Sin}\left[\frac{\pi s}{2}\right]$$

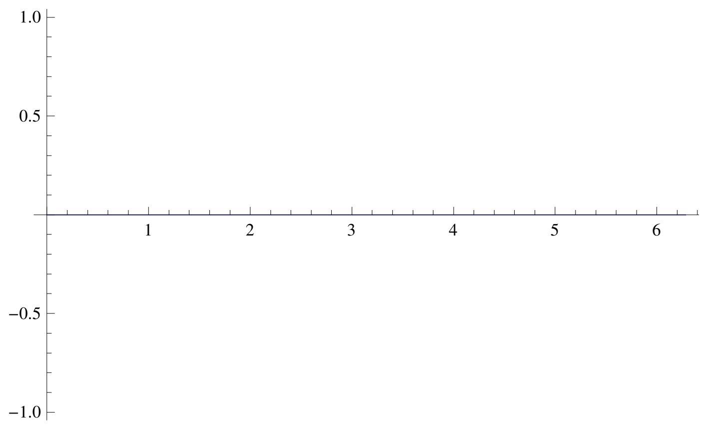
```
% // N
```

2.5 - 0.5 Cos[1.5708 s] - 0.5 Sin[1.5708 s]

```
Plot[Re[fS[t]],{t,0,2Pi}]
```

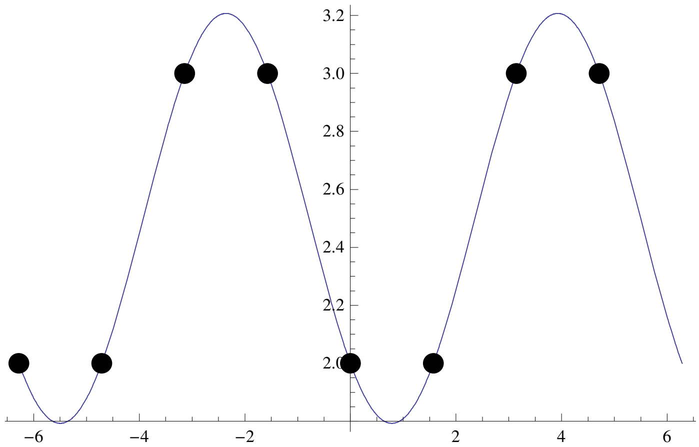



```
Plot[Im[fS[t]],{t,0,2Pi}]
```

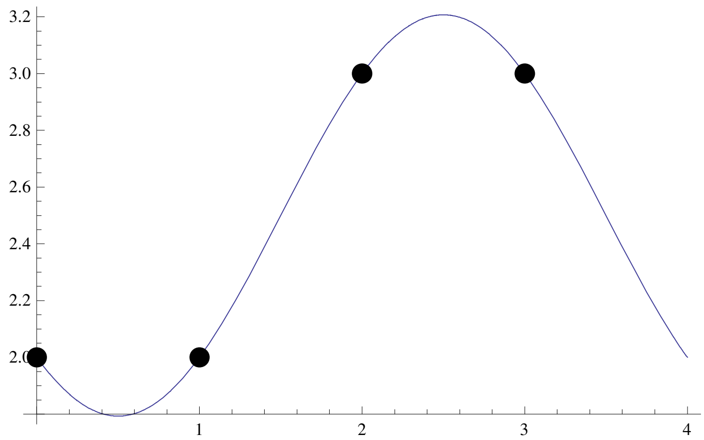


Man beachte im letzten Plot die Grösse der Amplitude.

```
Plot[{Re[fS[t]]},{t,-2Pi,2Pi},Epilog->epi]
```




```
Plot[{Re[fS1[s]]}, {s, 0, 4}, Epilog → epi1]
```



- **b**

  Um eine FFT machen zu können, braucht man eine 2-er Potenz als Anzahl der Intervalle.

- **c**

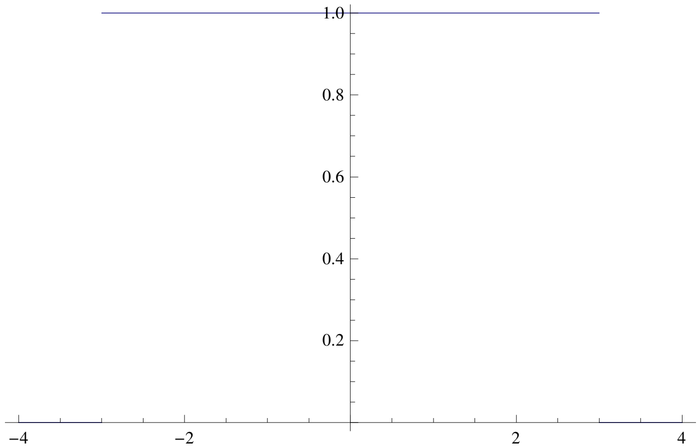
  Man muss z.B. 4 Messungen in einer Periode haben..

4

```
Remove["Global`*"];
```

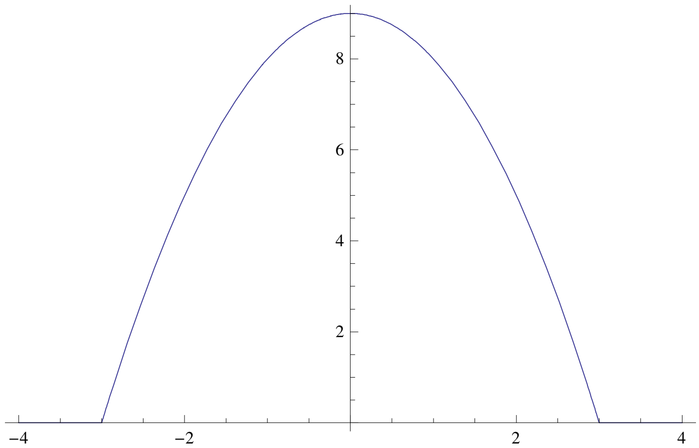
- **a**

```
f1[t_] := UnitStep[x + 3] - UnitStep[x - 3];
Plot[f1[x], {x, -4, 4}]
```



- **b**

```
f2[t_] := (9 - x^2) (UnitStep[x + 3] - UnitStep[x - 3]);
Plot[f2[x], {x, -4, 4}]
```



- **c (Achtung Faktor $\frac{1}{\sqrt{\pi}}$ bei anderer Definition der Fouriertransformation!!!)**

```
FourierTransform[f1[x], x, ω]
```

$$\frac{\sqrt{\frac{2}{\pi}}\ \mathrm{Sin}[3\,\omega]}{\omega}$$

- **d (Achtung Faktor $\frac{1}{\sqrt{\pi}}$ bei anderer Definition der Fouriertransformation!!!)**

```
FourierTransform[f2[x], x, ω]
```

$$\frac{i\, e^{-3 i \omega} \sqrt{\frac{2}{\pi}}}{\omega^3} - \frac{i\, e^{3 i \omega} \sqrt{\frac{2}{\pi}}}{\omega^3} - \frac{3\, e^{-3 i \omega} \sqrt{\frac{2}{\pi}}}{\omega^2} - \frac{3\, e^{3 i \omega} \sqrt{\frac{2}{\pi}}}{\omega^2}$$

```
FourierTransform[f2[x], x, ω] // Simplify
```

$$-\frac{e^{-3 i \omega} \sqrt{\frac{2}{\pi}} \left(-i + 3\,\omega + e^{6 i \omega} (i + 3\,\omega)\right)}{\omega^3}$$

```
% // ExpToTrig // Simplify
```

$$-\frac{2\sqrt{\frac{2}{\pi}}\,(3\,\omega\,\text{Cos}[3\,\omega]-\text{Sin}[3\,\omega])}{\omega^3}$$

- **e**

```
InverseFourierTransform[(Cos[Ω] - Sin[Ω]) / Ω, Ω, x]
```

$$\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{\frac{\pi}{2}}\ \text{Sign}[-1+x]-\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{\pi}{2}}\ \text{Sign}[1+x]$$

```
InverseFourierTransform[(Cos[Ω] - Sin[Ω]) / Ω, Ω, x] // Simplify
```

$$\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{\frac{\pi}{2}}\ (\text{Sign}[-1+x]-i\,\text{Sign}[1+x])$$

5

- **a**

```
Remove["Global`*"];
f[x_] := Cos[4 x] + I Sin[4 x]
FourierTransform[f[x], x, ω] // Simplify
```

$\sqrt{2\pi}$ DiracDelta[4 + ω]

```
1/√(2 π) FourierTransform[f[x], x, ω] // Simplify
```

DiracDelta[4 + ω]

```
√(2 π) FourierTransform[f[x], x, ω] // Simplify
```

2 π DiracDelta[4 + ω]

- **b**

```
Remove["Global`*"];
f[x_] := Sin[4 x] + I Cos[4 x]
FourierTransform[f[x], x, ω] // Simplify
```

$i\sqrt{2\pi}$ DiracDelta[-4 + ω]

```
1/√(2 π) FourierTransform[f[x], x, ω] // Simplify
```

i DiracDelta[-4 + ω]

```
√(2 π) FourierTransform[f[x], x, ω] // Simplify
```

2 i π DiracDelta[-4 + ω]

- **c**

```
Remove["Global`*"];
fHat[x_] := Cos[4 ω] + I Sin[4 ω]
InverseFourierTransform[fHat[x], ω, x] // Simplify
```

$\sqrt{2\pi}$ DiracDelta[-4 + x]

```
√(2 π) InverseFourierTransform[fHat[x], ω, x] // Simplify
```

2 π DiracDelta[-4 + x]

```
1/√(2π) InverseFourierTransform[fHat[x], ω, x] // Simplify
```

```
DiracDelta[-4 + x]
```

```
1/√(2π) FourierTransform[√(2π) InverseFourierTransform[fHat[x], ω, x], x, ω] // Simplify
```

$e^{4 i \omega}$

```
e^(2 i ω) // ExpToTrig
```

```
Cos[2 ω] + i Sin[2 ω]
```

6

- **Modul (in Zusatzfenster betreiben!)**

```
Remove["Global`*"];
```

```
four[fkt_, var_, perT_, start0Int_, n_, druck_] :=
  Module[{fktInt, tInt, nInt, znInt}, Print[" "]; Print["Output:"]; Print[" "];
   Print["Ausgabe:  ω, fktInt[var], a[0], a[k], b[k], c[k], Fourierreihen
      ff[var,n], ff[var], ffExp[var], ffKomplexTrig[var,n],
      ffKomplexExp[var,n], ffKomplex[var], Plot: z.B.
      Plot[Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]"];
   ω = 2 Pi / perT;  If[druck == 1, Print["ω = ", ω], " "];
   fktInt[tInt_] := Function[fkt[#]][tInt];
   If[druck == 1, Print["Funktion[", var, "]  = ", fktInt[var]], " "];
   a[0] = 2 / T Integrate[fktInt[var], {var, start0Int, start0Int + perT}];
   If[druck == 1, Print["a[0] = ", a[0]], " "];
   a[k_] := 2 / T Integrate[Cos[k ω var] fktInt[var], {var, start0Int,
       start0Int + perT}];  If[druck == 1, Print["a[k] = ", a[k]], " "];
   b[k_] := 2 / T Integrate[Sin[k ω var] fktInt[var], {var, start0Int,
       start0Int + perT}];  If[druck == 1, Print["b[k] = ", b[k]], " "];
   c[k_] := 1 / T Integrate[fktInt[var] E^(-I k ω var),
      {var, start0Int, start0Int + perT}]; If[druck == 1, Print["c[k] = ", c[k]], " "];
   ff[tInt_, znInt_] := a[0] / 2 + Sum[a[nInt] Cos[nInt ω tInt] +
       b[nInt] Sin[nInt ω tInt], {nInt, 1, znInt}];
    If[druck == 1, Print["Fourierreihe[", var, ", ", n, "]  = ", ff[var, n]], " "];
    If[druck == 1,
     Print["Num. Fourierreihe[", var, ", ", n, "]  = ", ff[var, n] // N], " "];
   ff[tInt_] := a[0] / 2 + Sum[a[nInt] Cos[nInt ω tInt] + b[nInt] Sin[nInt ω tInt],
      {nInt, 1, Infinity}];
    If[druck == 1, Print["Unendliche Fourierreihe[", var, "]  = ", ff[var]], " "];
   ffExp[tInt_] := ExpToTrig[a[0] / 2 +
      Sum[a[nInt] Cos[nInt ω tInt] + b[nInt] Sin[nInt ω tInt], {nInt, 1, Infinity}]];
    If[druck == 1, Print["Unendliche Fourierreihe komplex[",
     var, "]  = ", ffExp[var]], " "];
   ffKomplexTrig[tInt_, znInt_] := ExpToTrig[
     Sum[c[nInt] E^(I nInt ω tInt), {nInt, -znInt, znInt}]];
    If[druck == 1, Print["Komplexe Fourierreihe wieder trigonometrisch[",
     var, ", ", n, "]  = ", ffKomplexTrig[var, n]], " "];
   ffKomplexExp[tInt_, znInt_] := TrigToExp[
     Sum[c[nInt] E^(I nInt ω tInt), {nInt, -znInt, znInt}]];
    If[druck == 1, Print["Komplexe Fourierreihe[", var, ", ",
     n, "]  = ", ffKomplexExp[var, n]], " "];
   ffKomplex[tInt_] := Sum[c[nInt] E^(I nInt ω tInt), {nInt, -Infinity, Infinity}];
    If[druck == 1, Print["Komplexe Fourierreihe[", var, "]  = ", ffKomplex[var]], " "];
   If[druck == 1, Print["Plot"];
    Plot[Evaluate[{fktInt[var], ff[var, n]}], {var, start0Int, start0Int + perT}];, " "]
  ];
four[f, t, T, t0, 6, 0]
```

```
Output:


Ausgabe:  ω, fktInt[var], a[0], a[k], b[k], c[k], Fourierreihen ff[var,n], ff[var],
  ffExp[var], ffKomplexTrig[var,n], ffKomplexExp[var,n], ffKomplex[var],
  Plot: z.B. Plot[Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]
```

```
f[t_] := Abs[t - Pi] + Sin[t / 2];
T = 2 Pi;
t0 = -Pi;
(*  four[fkt_,var_,perT_,start0Int_,n_,druck_] *)
four[f, t, T, t0, 6, 1]
```

Output:

Ausgabe: ω, fktInt[var], a[0], a[k], b[k], c[k], Fourierreihen ff[var,n], ff[var], ffExp[var], ffKomplexTrig[var,n], ffKomplexExp[var,n], ffKomplex[var], Plot: z.B. Plot[Evaluate[ff[t,n]],{t,perT,perT+start0Int},PlotPoints→50]

$\omega = 1$

Funktion[t] $= \text{Abs}[-\pi + t] + \text{Sin}\left[\frac{t}{2}\right]$

$a[0] = 2\pi$

$$a[k] = \frac{2\,\text{Sin}[k\,\pi]}{k}$$

$$b[k] = \frac{2\left(-4\,k^3\,\text{Cos}[k\,\pi] - k\,\pi\,\text{Cos}[k\,\pi] + 4\,k^3\,\pi\,\text{Cos}[k\,\pi] + \text{Sin}[k\,\pi] - 4\,k^2\,\text{Sin}[k\,\pi]\right)}{k^2\left(-1 + 4\,k^2\right)\pi}$$

$$c[k] = \frac{e^{-i k \pi}\left(1 - e^{2 i k \pi} - 4\,k^2 + 4\,e^{2 i k \pi}\,k^2 + 4\,i\,k^3 + 4\,i\,e^{2 i k \pi}\,k^3 + 2\,i\,e^{2 i k \pi}\,k\,\pi - 8\,i\,e^{2 i k \pi}\,k^3\,\pi\right)}{2\,k^2\left(-1 + 4\,k^2\right)\pi}$$

$$\text{Fourierreihe}[t,\ 6] = \pi - \frac{2\,(-4 + 3\,\pi)\,\text{Sin}[t]}{3\,\pi} + \frac{(-16 + 15\,\pi)\,\text{Sin}[2\,t]}{15\,\pi} - \frac{2\,(-36 + 35\,\pi)\,\text{Sin}[3\,t]}{105\,\pi} + \frac{(-64 + 63\,\pi)\,\text{Sin}[4\,t]}{126\,\pi} - \frac{2\,(-100 + 99\,\pi)\,\text{Sin}[5\,t]}{495\,\pi} + \frac{(-144 + 143\,\pi)\,\text{Sin}[6\,t]}{429\,\pi}$$

Num. Fourierreihe[t, 6] $= 3.14159 - 1.15117\,\text{Sin}[t] + 0.660469\,\text{Sin}[2.\,t] - 0.448397\,\text{Sin}[3.\,t] + 0.338319\,\text{Sin}[4.\,t] - 0.27139\,\text{Sin}[5.\,t] + 0.226488\,\text{Sin}[6.\,t]$

Unendliche Fourierreihe[t] =

$$\pi + \frac{1}{2\pi}\,e^{-\frac{i t}{2}}\left(2\,i\,\text{ArcTan}\left[e^{-\frac{i t}{2}}\right] - 2\,i\,e^{i t}\,\text{ArcTan}\left[e^{-\frac{i t}{2}}\right] + 2\,i\,\text{ArcTan}\left[e^{\frac{i t}{2}}\right] - 2\,i\,e^{i t}\,\text{ArcTan}\left[e^{\frac{i t}{2}}\right] + 2\,i\,e^{\frac{i t}{2}}\,\pi\,\text{Log}\left[1 + e^{i t}\right] - 2\,i\,e^{\frac{i t}{2}}\,\pi\,\text{Log}\left[e^{-i t}\left(1 + e^{i t}\right)\right]\right)$$

Unendliche Fourierreihe komplex[t] =

$$\pi + \frac{1}{2\pi}\left(\text{Cos}\left[\frac{t}{2}\right] - i\,\text{Sin}\left[\frac{t}{2}\right]\right)\left(2\,i\,\text{ArcTan}\left[\text{Cos}\left[\frac{t}{2}\right] - i\,\text{Sin}\left[\frac{t}{2}\right]\right] + 2\,i\,\text{ArcTan}\left[\text{Cos}\left[\frac{t}{2}\right] + i\,\text{Sin}\left[\frac{t}{2}\right]\right] - 2\,i\,\text{ArcTan}\left[\text{Cos}\left[\frac{t}{2}\right] - i\,\text{Sin}\left[\frac{t}{2}\right]\right]\text{Cos}[t] - 2\,i\,\text{ArcTan}\left[\text{Cos}\left[\frac{t}{2}\right] + i\,\text{Sin}\left[\frac{t}{2}\right]\right]\text{Cos}[t] + 2\,i\,\pi\,\text{Cos}\left[\frac{t}{2}\right]\text{Log}[1 + \text{Cos}[t] + i\,\text{Sin}[t]] - 2\,i\,\pi\,\text{Cos}\left[\frac{t}{2}\right]\text{Log}\left[\text{Cos}[t] + \text{Cos}[t]^2 - i\,\text{Sin}[t] + \text{Sin}[t]^2\right] - 2\,\pi\,\text{Log}[1 + \text{Cos}[t] + i\,\text{Sin}[t]]\,\text{Sin}\left[\frac{t}{2}\right] + 2\,\pi\,\text{Log}\left[\text{Cos}[t] + \text{Cos}[t]^2 - i\,\text{Sin}[t] + \text{Sin}[t]^2\right]\text{Sin}\left[\frac{t}{2}\right] + 2\,\text{ArcTan}\left[\text{Cos}\left[\frac{t}{2}\right] - i\,\text{Sin}\left[\frac{t}{2}\right]\right]\text{Sin}[t] + 2\,\text{ArcTan}\left[\text{Cos}\left[\frac{t}{2}\right] + i\,\text{Sin}\left[\frac{t}{2}\right]\right]\text{Sin}[t]\right)$$

Komplexe Fourierreihe wieder trigonometrisch[t, 6] =

$$\pi - 2\,\text{Sin}[t] + \frac{8\,\text{Sin}[t]}{3\pi} + \text{Sin}[2\,t] - \frac{16\,\text{Sin}[2\,t]}{15\pi} - \frac{2}{3}\,\text{Sin}[3\,t] + \frac{24\,\text{Sin}[3\,t]}{35\pi} + \frac{1}{2}\,\text{Sin}[4\,t] - \frac{32\,\text{Sin}[4\,t]}{63\pi} - \frac{2}{5}\,\text{Sin}[5\,t] + \frac{40\,\text{Sin}[5\,t]}{99\pi} + \frac{1}{3}\,\text{Sin}[6\,t] - \frac{48\,\text{Sin}[6\,t]}{143\pi}$$

Komplexe Fourierreihe[t, 6] =

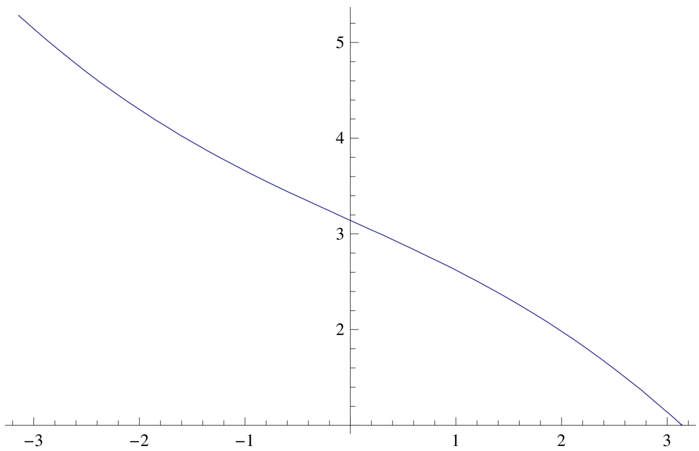
$$-i\,e^{-i\,t} + i\,e^{i\,t} + \frac{1}{2}\,i\,e^{-2\,i\,t} - \frac{1}{2}\,i\,e^{2\,i\,t} - \frac{1}{3}\,i\,e^{-3\,i\,t} + \frac{1}{3}\,i\,e^{3\,i\,t} + \frac{1}{4}\,i\,e^{-4\,i\,t} - \frac{1}{4}\,i\,e^{4\,i\,t} - \frac{1}{5}\,i\,e^{-5\,i\,t} + \frac{1}{5}\,i\,e^{5\,i\,t} + \frac{1}{6}\,i\,e^{-6\,i\,t} - \frac{1}{6}\,i\,e^{6\,i\,t} + \frac{4\,i\,e^{-i\,t}}{3\pi} - \frac{4\,i\,e^{i\,t}}{3\pi} - \frac{8\,i\,e^{-2\,i\,t}}{15\pi} + \frac{8\,i\,e^{2\,i\,t}}{15\pi} + \frac{12\,i\,e^{-3\,i\,t}}{35\pi} - \frac{12\,i\,e^{3\,i\,t}}{35\pi} - \frac{16\,i\,e^{-4\,i\,t}}{63\pi} + \frac{16\,i\,e^{4\,i\,t}}{63\pi} + \frac{20\,i\,e^{-5\,i\,t}}{99\pi} - \frac{20\,i\,e^{5\,i\,t}}{99\pi} - \frac{24\,i\,e^{-6\,i\,t}}{143\pi} + \frac{24\,i\,e^{6\,i\,t}}{143\pi} + \pi$$

Komplexe Fourierreihe[t] =

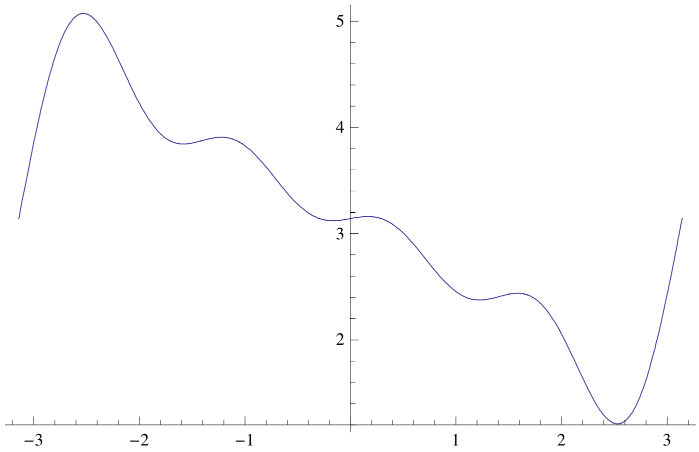
$$\pi + \frac{1}{3\pi}\,i\,e^{-\frac{i\,t}{2}}\left(-3\,e^{\frac{i\,t}{2}}\,\pi + 3\,\pi\,\text{ArcTan}\left[e^{\frac{i\,t}{2}}\right] - 3\,e^{i\,t}\,\pi\,\text{ArcTan}\left[e^{\frac{i\,t}{2}}\right] - 4\,e^{\frac{3\,i\,t}{2}}\,\text{Hypergeometric2F1}\left[\frac{1}{2}, 2, \frac{5}{2}, -e^{i\,t}\right] + 4\,e^{\frac{3\,i\,t}{2}}\,\pi\,\text{Hypergeometric2F1}\left[\frac{1}{2}, 2, \frac{5}{2}, -e^{i\,t}\right] + 3\,e^{\frac{i\,t}{2}}\,\pi\,\text{Log}\left[1 + e^{i\,t}\right]\right) - \frac{1}{3\pi}\,i\,e^{-i\,t}\left(-3\,e^{i\,t}\,\pi - 3\,e^{\frac{i\,t}{2}}\,\pi\,\text{ArcTan}\left[e^{-\frac{i\,t}{2}}\right] + 3\,e^{\frac{3\,i\,t}{2}}\,\pi\,\text{ArcTan}\left[e^{-\frac{i\,t}{2}}\right] - 4\,\text{Hypergeometric2F1}\left[\frac{1}{2}, 2, \frac{5}{2}, -e^{-i\,t}\right] + 4\,\pi\,\text{Hypergeometric2F1}\left[\frac{1}{2}, 2, \frac{5}{2}, -e^{-i\,t}\right] + 3\,e^{i\,t}\,\pi\,\text{Log}\left[e^{-i\,t}\left(1 + e^{i\,t}\right)\right]\right)$$

Plot

```
Plot[f[t], {t, -Pi, Pi}]
```



```
Plot[Evaluate[ff[t, 4]], {t, -Pi, Pi}, PlotPoints → 50]
```



7

```
Remove["Global`*"];
```

a

```
f1[x_] := 1 / 2 E^(-2 x^2);
fTransf1[Ω_] := 1 / (2 Pi) Integrate[f1[λ] E^(-I λ Ω), {λ, -Infinity, Infinity}];
fTransf1[Ω]
```

$$\frac{e^{-\frac{\Omega^2}{8}}}{4\sqrt{2\pi}}$$

```
f2[x_] := f1[x] / Sqrt[2 Pi];
fTransf2[Ω_] := 1 / Sqrt[2 Pi] Integrate[f2[λ] E^(-I λ Ω), {λ, -Infinity, Infinity}];
fTransf2[Ω]
```

$$\frac{e^{-\frac{\Omega^2}{8}}}{4\sqrt{2\pi}}$$

```
1 / Sqrt[2 Pi] Integrate[Evaluate[fTransf2[Ω] E^(I Ω x)], {Ω, -Infinity, Infinity}]
```

$$\frac{e^{-2 x^2}}{2\sqrt{2\pi}}$$

```
1 / Sqrt[2 Pi] Integrate[Evaluate[fTransf2[Ω] E^(I Ω x)], {Ω, -Infinity, Infinity}] == f2[x]
```

True

```
FourierTransform[f1[t], t, Ω]
```

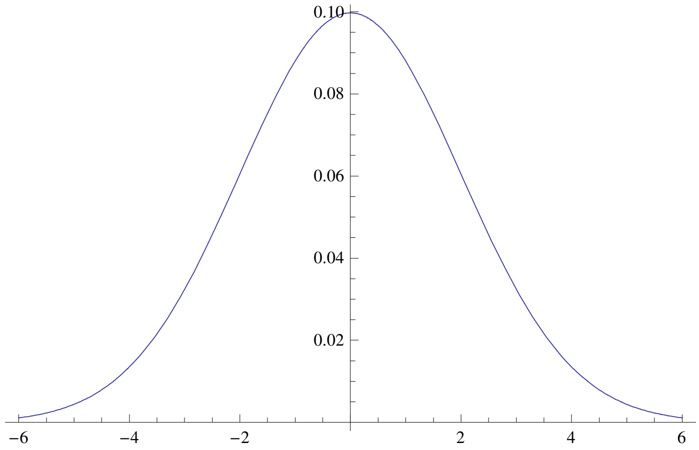
$$\frac{1}{4} e^{-\frac{\Omega^2}{8}}$$

```
FourierTransform[f2[t], t, Ω]
```

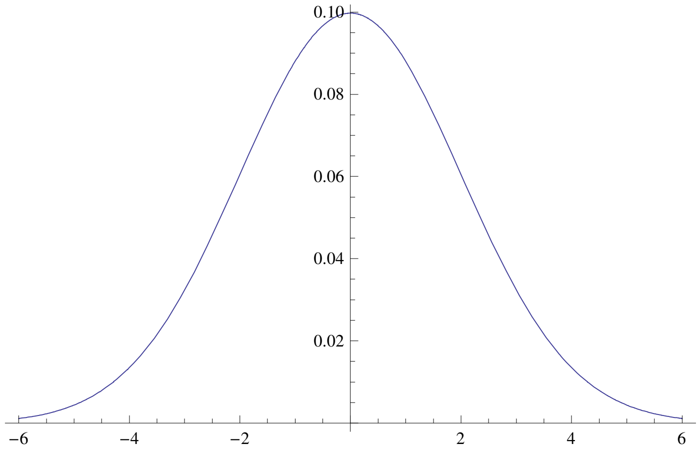
$$\frac{e^{-\frac{\Omega^2}{8}}}{4\sqrt{2\pi}}$$

b

```
Plot[Evaluate[fTransf1[Ω]], {Ω, -6, 6}]
```

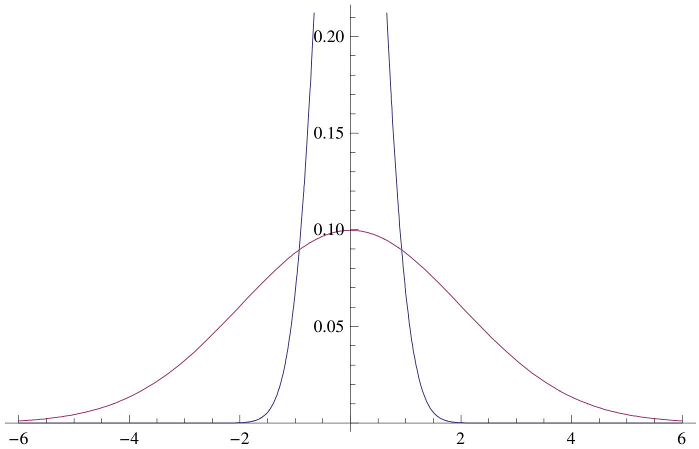



```
Plot[Evaluate[fTransf2[Ω]], {Ω, -6, 6}]
```

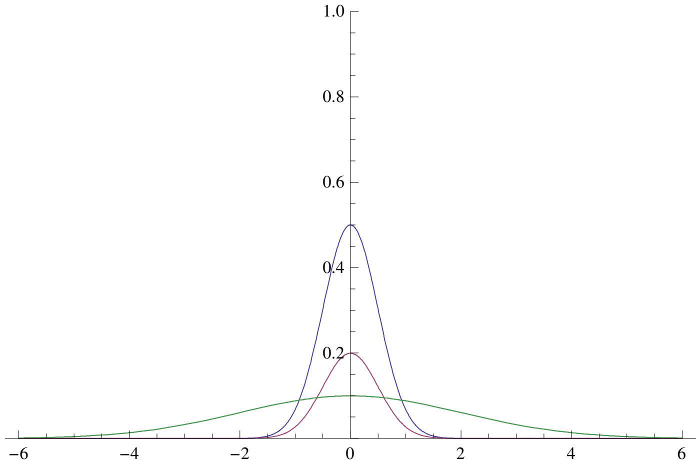


c

```
Plot[Evaluate[{f1[x], fTransf1[x]}], {x, -6, 6}]
```



```
Plot[Evaluate[{f1[x], f2[x], fTransf1[x], fTransf2[x]}], {x, -6, 6}, PlotRange → {0, 1}]
```




```
{f1[x], f2[x], fTransf1[x] , fTransf2[x]}
```

$$\left\{\frac{1}{2}\,e^{-2x^2},\ \frac{e^{-2x^2}}{2\sqrt{2\pi}},\ \frac{e^{-\frac{x^2}{8}}}{4\sqrt{2\pi}},\ \frac{e^{-\frac{x^2}{8}}}{4\sqrt{2\pi}}\right\}$$

```
fTransf1[x] == fTransf2[x]
```

True

Bemerkenswert: Alles Gauss-Glocken.