Lösungen

1

a1

```
f[x_, y_] := (x - y) E^((x - y) (x + y))
```

```
f[x, y]
```

$e^{(x-y)(x+y)} (x - y)$

```
g[x_, y_, dx_, dy_] := Abs[D[f[x, y], x]] Abs[dx] + Abs[D[f[x, y], y]] Abs[dy];
x1 = 1; y1 = 1;
x2 = 2; y2 = 1;
x3 = 1; y3 = 2;
dx = 0.2; dy = dx;
```

```
g[x, y, dx, dy]
```

$0.2\,\text{Abs}[e^{(x-y)(x+y)} + 2\,e^{(x-y)(x+y)}\,x\,(x - y)] + 0.2\,\text{Abs}[-e^{(x-y)(x+y)} - 2\,e^{(x-y)(x+y)}\,(x - y)\,y]$

```
g[u, v, dv, dv] /. {x → u, y → v}
```

$\text{Abs}[dv]\,\text{Abs}[e^{(u-v)(u+v)} + 2\,e^{(u-v)(u+v)}\,u\,(u - v)] + \text{Abs}[dv]\,\text{Abs}[-e^{(u-v)(u+v)} - 2\,e^{(u-v)(u+v)}\,(u - v)\,v]$

```
g[x, y, dx, dy] /. {x → x1, y → y1}
```

0.4

```
g[x, y, dx, dy] /. {x → x2, y → y2}
```

32.1369

```
g[x, y, dx, dy] /. {x → x3, y → y3}
```

0.0398297

a2

```
q[x_, y_, dx_, dy_] :=
  Sqrt[(Abs[D[f[x, y], x]] Abs[dx])^2 + (Abs[D[f[x, y], y]] Abs[dy])^2];
```

```
q[x, y, dx, dy] /. {x → x1, y → y1}
```

0.282843

```
q[x, y, dx, dy] /. {x → x2, y → y2}
```

23.4236

```
q[x, y, dx, dy] /. {x → x3, y → y3}
```

0.0314881

b

```
h0[u_, v_, du_, dv_] :=
  Max[f[u, v], f[u + du, v + dv], f[u - du, v + dv], f[u + du, v - dv], f[u - du, v - dv]];
h1[u_, v_, du_, dv_] := Max[f[u + du, v + dv],
   f[u - du, v + dv], f[u + du, v - dv], f[u - du, v - dv]];
```

```
h2[u_, v_, du_, dv_] :=
  Max[Abs[f[u + du, v + dv] - f[u, v]], Abs[f[u - du, v + dv] - f[u, v]],
   Abs[f[u + du, v - dv] - f[u, v]], Abs[f[u - du, v - dv] - f[u, v]]];
```

```
{h0[x1, y1, dx, dy], h0[x2, y2, dx, dy], h0[x3, y3, dx, dy]}
```

{0.890216, 93.3609, -0.0209938}

```
{h1[x1, y1, dx, dy], h1[x2, y2, dx, dy], h1[x3, y3, dx, dy]}
```

{0.890216, 93.3609, -0.0209938}

```
{h2[x1, y1, dx, dy], h2[x2, y2, dx, dy], h2[x3, y3, dx, dy]}
```

{0.890216, 73.2753, 0.0493923}

```
{f[x1, y1], f[x2, y2], f[x3, y3]} // N
```

{0., 20.0855, -0.0497871}

```
{f[x1, y1], f[x2, y2], f[x3, y3]}
```

$\left\{0, e^3, -\frac{1}{e^3}\right\}$

c

e-Funktion statt linear

2

```
Binomial[10, 1]
```

10

```
Remove["Global`*"]
```

```
f[x_, n_, MM_, NN_] := Binomial[n, x] (MM / NN) ^x  (1 - MM / NN) ^ (n - x)
```

```
NN = 4000; MM = 0.2 / 100 NN;
```

a

```
f[0, 10, MM, NN]
0.980179
```

b

```
f[1, 10, MM, NN]
0.0196429
```

c

```
f[2, 10, MM, NN]
0.00017714
```

d

```
f[3, 10, MM, NN]
```

9.4664×10^{-7}

e

```
pMax6 = Sum[f[k, 10, MM, NN], {k, 0, 6}]
1.
```

f

```
pMin7 = 1 - f[0, 10, MM, NN] - Sum[f[k, 10, MM, NN], {k, 1, 6}]
```

5.20417×10^{-17}

3

```
Binomial[10, 1]
10
h[x_, n_, MM_, NN_] := Binomial[MM, x] Binomial[NN - MM, n - x] / Binomial[NN, n]
NN = 4000; MM = 0.2 / 100 NN;
```

a

```
h[0, 10, MM, NN]
```

0.980157

b

```
h[1, 10, MM, NN]
```

0.0196868

c

```
h[2, 10, MM, NN]
```

0.000155656

d

```
h[3, 10, MM, NN]
```

6.24969×10^{-7}

e

```
qMax6 = Sum[h[k, 10, MM, NN], {k, 0, 6}]
```

1.

f

```
qMin7 = 1 - Sum[h[k, 10, MM, NN], {k, 0, 6}]
```

-3.11307×10^{-13}

```
qMin7 = 1 - Sum[h[k, 10, MM, NN], {k, 0, 6}] // Chop
```

0

g

```
((pMax6 - qMax6) / pMax6) 100 // Abs
```

3.11307×10^{-11}

```
((pMax6 - qMax6) / pMax6) 100 // Abs // Chop
```

```
0
```

h

```
(pMin7 - qMin7) / pMin7 100 // Abs
```

```
100.
```

Fantasiezahl, infolge ungenauer Rundungsdifferenzen

j

```
{pMax6, qMax6}
```

```
{1., 1.}
```

Nicht wesentlich verschieden.

```
{pMin7, qMin7}
```

$\{5.20417 \times 10^{-17}, 0\}$

Belanglos, da sehr kleine Zahlen, vermutlich Rundungsdifferenzen.

Man könnte also auf den Schüttelprozess verzichten.

4

```
Remove["Global`*"]
```

```
<< Statistics`DescriptiveStatistics`
```

```
M = {20.10, 20.50, 21.08, 21.72, 21.14, 20.60, 20.31,
   21.54, 21.47, 20.70, 19.57, 20.33, 20.15, 19.31, 20.53, 20.59,
   21.80, 20.29, 21.13, 19.90, 20.81, 20.60, 20.16, 19.47, 20.00};
```

a

```
Mean[M]
```

```
20.552
```

```
LocationReport[M]
```

```
{Mean → 20.552, HarmonicMean → 20.5308, Median → 20.53}
```

```
DispersionReport[M]
```

```
{Variance → 0.454183, StandardDeviation → 0.673931, SampleRange → 2.49,
 MeanDeviation → 0.52448, MedianDeviation → 0.43, QuartileDeviation → 0.4775}
```

```
ShapeReport[M]
```

```
{Skewness → 0.139462, QuartileSkewness → 0.17801, KurtosisExcess → -0.61835}
```

```
std = StandardDeviation[M]
```

```
0.673931
```

b1 Linearer absoluter Fehler des Mittelwerts als standardisierter Fehler des Mittelwerts

```
Δx = 0.02;
MaxFehlerMittelwert = Length[M] Δx / Length[M]
```

```
0.02
```

b2 Standardfehler des Mittelwerts

```
Δx = 0.02;
StandardFehlerMittelwert = std / Sqrt[Length[M]]
```

```
0.134786
```

c1 Linearer absoluter Fehler der Standardabweichung als standardisierter Fehler

```
MaxFehlerStD = Sum[Abs[M[[k]] - Mean[M]] Abs[Δx], {k, 1, Length[M]}] /
  Sqrt[(Length[M] - 1) Sum[(M[[k]] - Mean[M])^2 , {k, 1, Length[M]}]]
```

```
0.0162133
```

c2 Standardfehler der Standardabweichung

Dieser Fehler berechnet sich als Wurzel aus der Varianz des Standardfehlers, wobei dann die die Formel für die Wurzel der Varianz einer Stichprobe in die Varianzformel eingeht. Das führt zu einem Versuch eines "Fests algebraischer Umformungen", auf das auf der Seite der Anwendungen des Aufwandes wegen nicht gerne eingegangen wird. Man kann aber vermuten, dass der genannte Fehler in der Grössenordnung des absoluten Fehlers der Standardabweichung liegt.

d

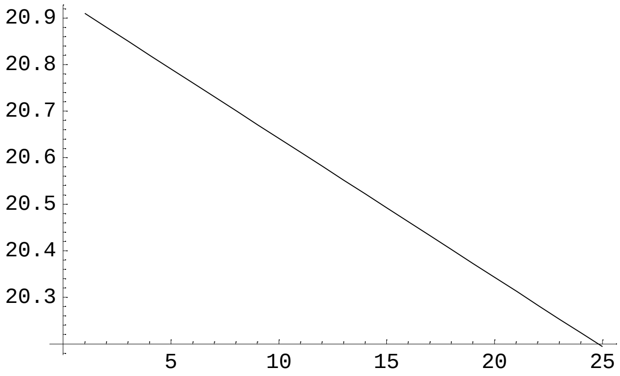
```
MPoints = Table[{k, M[[k]]}, {k, 1, Length[M]}]
```

```
{{1, 20.1}, {2, 20.5}, {3, 21.08}, {4, 21.72}, {5, 21.14}, {6, 20.6}, {7, 20.31},
 {8, 21.54}, {9, 21.47}, {10, 20.7}, {11, 19.57}, {12, 20.33}, {13, 20.15},
 {14, 19.31}, {15, 20.53}, {16, 20.59}, {17, 21.8}, {18, 20.29}, {19, 21.13},
 {20, 19.9}, {21, 20.81}, {22, 20.6}, {23, 20.16}, {24, 19.47}, {25, 20.}}
```

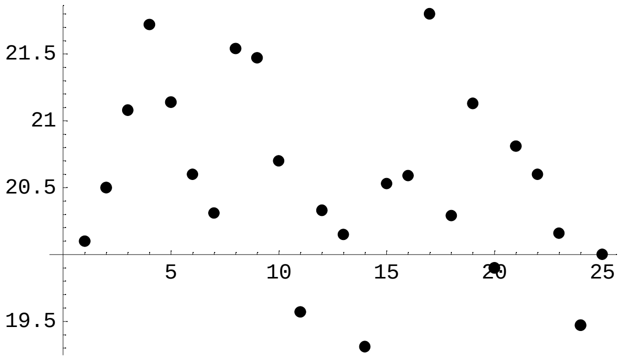
```
Fit[MPoints, {1, x}, x]
```

$20.9399 - 0.0298385\,x$

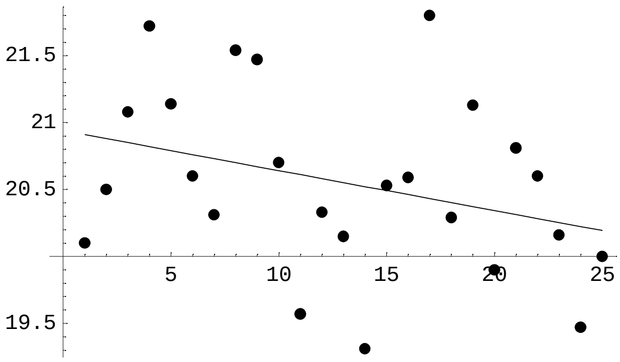
```
plotline = Plot[%, {x, 1, 25}];
```




```
plotpoints = ListPlot[MPoints, PlotStyle → PointSize[0.02]];
```



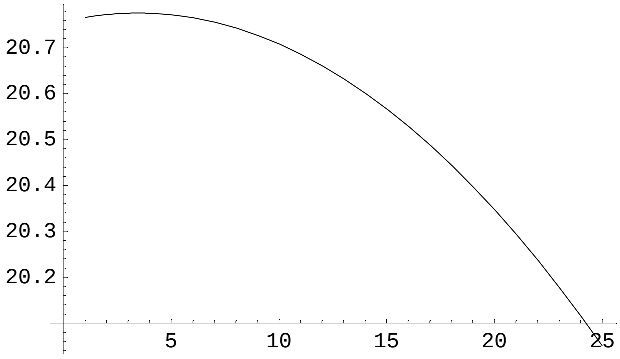

```
Show[plotpoints, plotline];
```



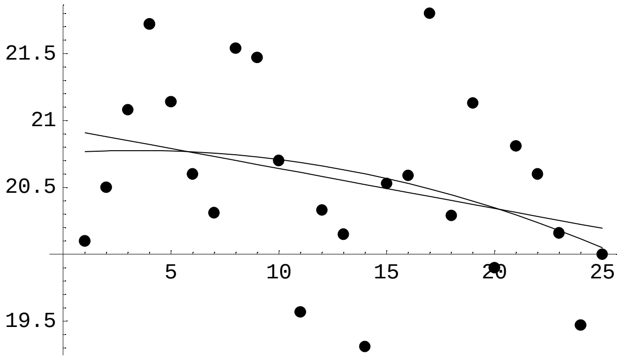

```
Fit[MPoints, {1, x, x^2}, x]
```

$20.7572 + 0.0107654\,x - 0.00156169\,x^2$

```
plotsqr = Plot[%, {x, 1, 25}];
```




```
Show[plotpoints, plotline, plotsqr];
```



e

Die Steigung -0.0298... ist ziemlich klein. Zufall?
Die Temperatur sind also tendenziell mit der Zeit

5

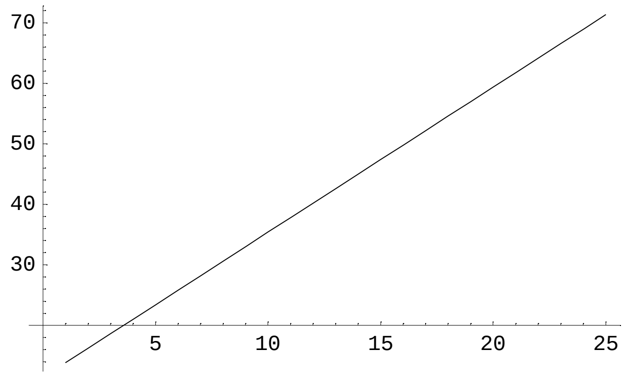
```
U = {{1, 97.6}, {2, 76.6}, {3, 46.5}, {4, 40.1}, {5, 30.1}, {6, 20.},
   {7, 17.3}, {8, 8.6}, {9, 4.1}, {10, 1.5}, {11, 0.1}, {12, 0.1}, {13, 1.9},
   {14, 4.5}, {15, 8.}, {16, 15.5}, {17, 26.5}, {18, 32.7}, {19, 46.8},
   {20, 50.6}, {21, 78.8}, {22, 90.7}, {23, 100.2}, {24, 105.9}, {25, 159.9}};
```

a

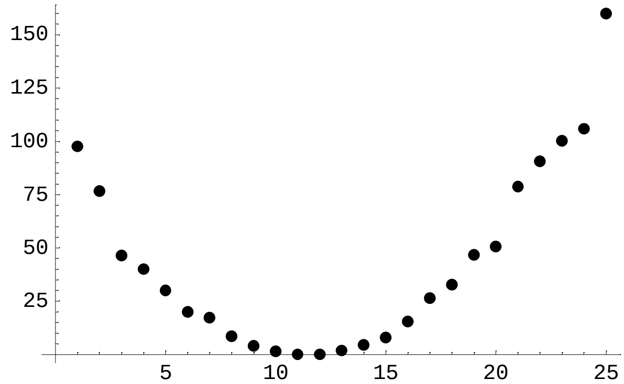
```
Fit[U, {1, x}, x]
```

```
11.43 + 2.39646 x
```

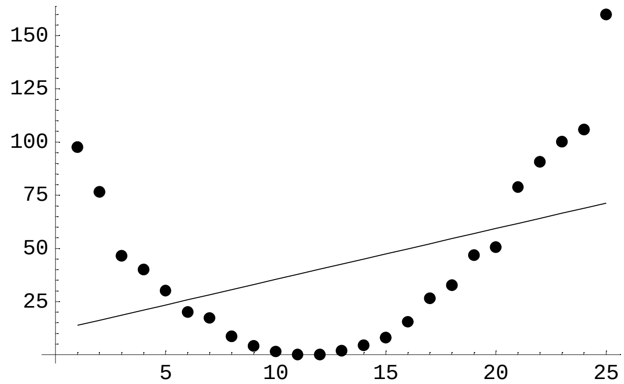
```
plotline = Plot[%, {x, 1, 25}];
```




```
plotpoints = ListPlot[U, PlotStyle → PointSize[0.02]];
```




```
Show[plotpoints, plotline];
```



b

```
tU = U // Transpose
```

```
{{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25},
 {97.6, 76.6, 46.5, 40.1, 30.1, 20., 17.3, 8.6, 4.1, 1.5, 0.1, 0.1, 1.9,
  4.5, 8., 15.5, 26.5, 32.7, 46.8, 50.6, 78.8, 90.7, 100.2, 105.9, 159.9}}
```

```
xList = tU[[1]]
```

```
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25}
```

```
yList = tU[[2]]
```

{97.6, 76.6, 46.5, 40.1, 30.1, 20., 17.3, 8.6, 4.1, 1.5, 0.1, 0.1, 1.9,
4.5, 8., 15.5, 26.5, 32.7, 46.8, 50.6, 78.8, 90.7, 100.2, 105.9, 159.9}

```
cVar = 1 / (Length[U] - 1)
  Sum[(xList[[k]] - Mean[xList]) (yList[[k]] - Mean[yList]), {k, 1, Length[U]}]
```

129.808

```
xVar = 1 / (Length[U] - 1) Sum[(xList[[k]] - Mean[xList]) ^2, {k, 1, Length[xList]}] // N
```

54.1667

```
yVar = 1 / (Length[U] - 1) Sum[(yList[[k]] - Mean[yList]) ^2, {k, 1, Length[yList]}] // N
```

1818.17

c

```
<< Statistics`MultiDescriptiveStatistics`
```

```
rxy = Correlation[xList, yList]
```

0.413637

```
ryx = Correlation[yList, xList]
```

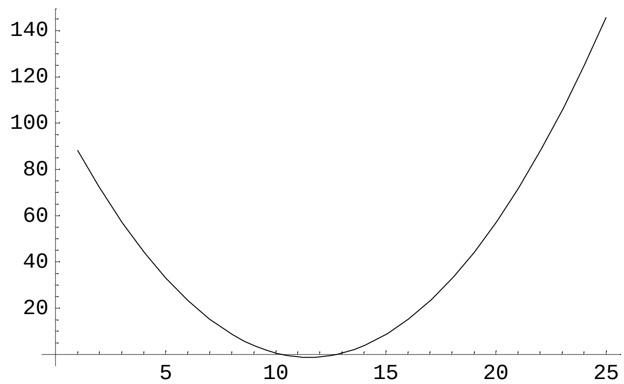
0.413637

d

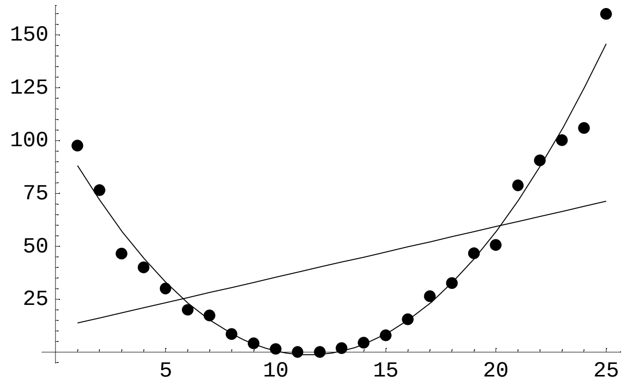
```
Fit[U, {1, x, x^2}, x]
```

$106.032 - 18.6261\,x + 0.808562\,x^2$

```
plotsqr = Plot[%, {x, 1, 25}];
```



```
Show[plotpoints, plotline, plotsqr];
```



6

a

Gesamthöhe linear auf 1 verkürzen

b

Ablesen: ca. 21.5

c

Ablesen: ca. 19 und 24

7

Hängt vom Text ab.
Siehe Wikipedia, T-Test

a
Zu was taugt das Verfahren mit dem t-Test? (Min nur einer Grundgesamtheit)

Testen von Hypothesen: Stimmt ein vorhandener Mittelwert einer Stichprobe mit dem unbekannten Erwartungswert der Grundgesamtheit bei unbekanntem   überein?

b
Was sind zu beachtende Voraussetzungen?

Die Annahme, dass eine normalverteilte Grundgesamtheit vorliegt.
Die Annahme, dass die Werte der (resp. deren Variablen) aus der Grundgesamtheit voneinander unabhängig sind.
Allfällige Eingschränkungen über die Grösse der Grundgesamtheit.