# Lösungen

## 1

### a

#### Daten

```
a = 78.42; Δa = 0.01;
b = 85.84; Δb = 0.01;
γ = 0.6285; Δγ = 0.0015;
c[a_, b_, γ_] := Sqrt[a^2 + b^2 - 2 a b Cos[γ]]
```

```
cc = c[a, b, γ]
```

51.2613

```
Δc = (Abs[D[c[a1, b1, γ1], a1]] Δa + Abs[D[c[a1, b1, γ1], b1]] Δb +
    Abs[D[c[a1, b1, γ1], γ1]] Δγ) /. {a1 → a, b1 → b, γ1 → γ}
```

0.121933

#### Einzelteile

```
(Abs[D[c[a1, b1, γ1], a1]] Δa +
  Abs[D[c[a1, b1, γ1], b1]] Δb + Abs[D[c[a1, b1, γ1], γ1]] Δγ)
```

$$0.005\,\mathrm{Abs}\left[\frac{2\,b1-2\,a1\,\mathrm{Cos}[\gamma 1]}{\sqrt{a1^2+b1^2-2\,a1\,b1\,\mathrm{Cos}[\gamma 1]}}\right]+0.005\,\mathrm{Abs}\left[\frac{2\,a1-2\,b1\,\mathrm{Cos}[\gamma 1]}{\sqrt{a1^2+b1^2-2\,a1\,b1\,\mathrm{Cos}[\gamma 1]}}\right]+0.0015\,\mathrm{Abs}\left[\frac{a1\,b1\,\mathrm{Sin}[\gamma 1]}{\sqrt{a1^2+b1^2-2\,a1\,b1\,\mathrm{Cos}[\gamma 1]}}\right]$$

```
(Abs[D[c[a1, b1, γ1], a1]] Δa ) /. {a1 → a, b1 → b, γ1 → γ}
```

0.00175242

```
(Abs[D[c[a1, b1, γ1], b1]] Δb ) /. {a1 → a, b1 → b, γ1 → γ}
```

0.00437079

```
( Abs[D[c[a1, b1, γ1], γ1]] Δγ) /. {a1 → a, b1 → b, γ1 → γ}
```

0.11581

Der Hauptteil des Fehlers kommt von !

### b

```
α[a_, b_, c_] := ArcCos[(b^2 + c^2 - a^2) / (2 b c)]
```

```
α[a, b, cc]
```

```
1.11845
```

```
α[a, b, cc] / Degree
```

```
64.0823
```

```
Δα = (Abs[D[α[a1, b, cc], a1]] Δa + Abs[D[α[a, b1, cc], b1]] Δb +
    Abs[D[α[a, b, c1], c1]] Δc) /. {a1 → a, b1 → b, c1 → cc}
```

```
0.000781818
```

```
Δα / Degree
```

```
0.0447949
```

```
{Abs[D[α[a1, b, cc], a1]] Δa, Abs[D[α[a, b1, cc], b1]] Δb , Abs[D[α[a, b, c1], c1]] Δc} /.
 {a1 → a, b1 → b, c1 → cc}
```

```
{0.000198145, 0.000160282, 0.000423392}
```

---

## 2.

```
Remove["Global`*"]
```

```
points = {{0, 0.497}, {1, 0.580}, {2, 0.839}, {3, 0.933}, {4, 1.044},
   {5, 1.141}, {6, 1.151}, {7, 1.313}, {8, 1.404}, {9, 1.409}, {10, 1.422}};
```

### a

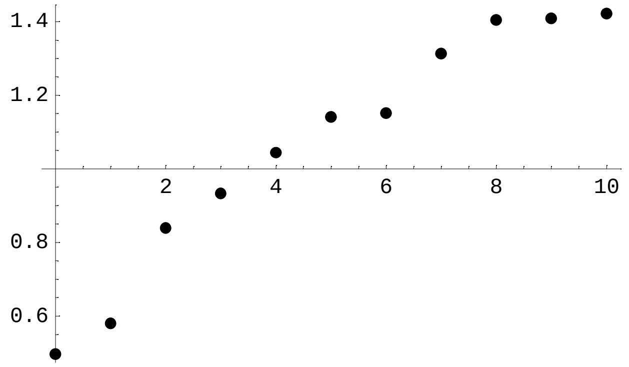
```
f1 = FindFit[points, a + b x, {a, b}, x]
```

```
{a → 0.589227, b → 0.0954818}
```

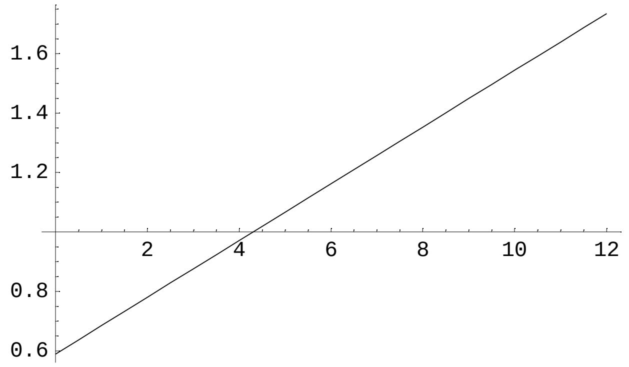
```
fg1[x_] := a + b x /. f1;  fg1[x]
```

```
0.589227 + 0.0954818 x
```

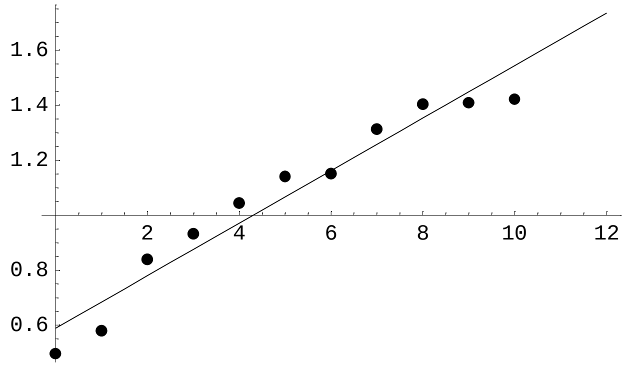
```
plotpoints = ListPlot[points, PlotStyle → PointSize[0.02]];
```




```
pl1 = Plot[fg1[x], {x, 0, 12}];
```




```
s1 = Show[pl1, plotpoints];
```




## b

```
trp1 = Transpose[points]; trp1 // MatrixForm
```

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 0.497 | 0.58 | 0.839 | 0.933 | 1.044 | 1.141 | 1.151 | 1.313 | 1.404 | 1.409 | 1.422 |

```
xListe1 = trp1[[1]]
```

{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

```
yListe1 = trp1[[2]]
```

{0.497, 0.58, 0.839, 0.933, 1.044, 1.141, 1.151, 1.313, 1.404, 1.409, 1.422}

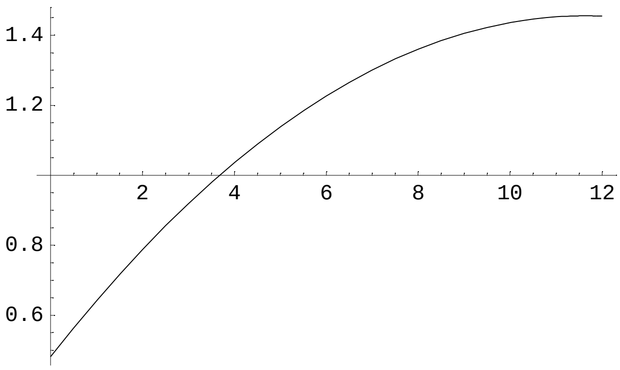
```
<< Statistics`MultiDescriptiveStatistics`

Correlation[xListe1, yListe1]

0.971685

Covariance[xListe1, yListe1]

1.0503
```

## c

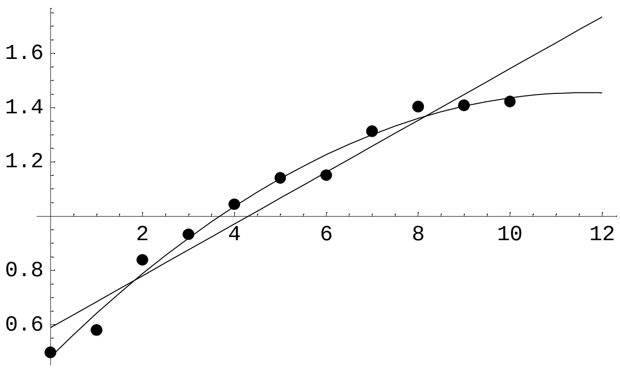
```
f2 = FindFit[points, a + b x + c x^2, {a, b, c}, x]

{a → 0.481448, b → 0.167335, c → -0.00718531}

fg2[x_] := a + b x + c x^2 /. f2;  fg2[x]
```

$0.481448 + 0.167335\, x - 0.00718531\, x^2$

```
pl2 = Plot[fg2[x], {x, 0, 12}];
```

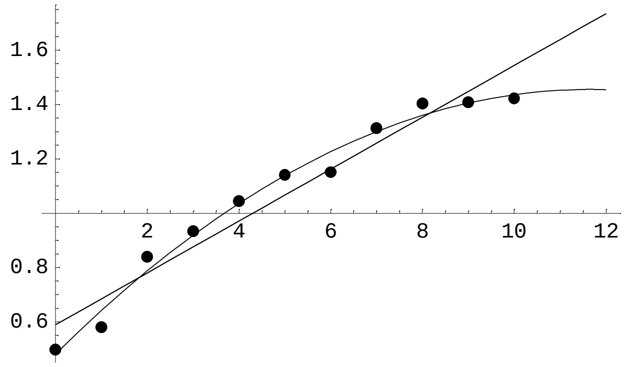



```
s2 = Show[pl1, pl2, plotpoints];
```



## d

```
Show[s1, s2];
```




# 3

## a

Monte Carlo: Experimentelle Bestimmung der relativen Häufigkeit als Wahrscheinlichkeit. Verwendung der Wahrscheinlichkeit als reale Messgrösse.

## b

Hier Monte Carlo. 1 m entspricht 1000 m = 1 km.

```
Flaeche = 88427 / 100000 km^2 // N
```

0.88427 $km^2$

# 4

```
Remove["Global`*"]

<< Statistics`DescriptiveStatistics`

DatenSatz = {5.6, 4.5, 5.3, 4.7, 4.5, 5.6, 4.7, 5.5, 4.8}
```

{5.6, 4.5, 5.3, 4.7, 4.5, 5.6, 4.7, 5.5, 4.8}

## a

```
Mean[DatenSatz]
```

5.02222

```
StandardDeviation[DatenSatz]
```

```
0.47111
```

```
locRep = LocationReport[DatenSatz]
```

```
{Mean → 5.02222, HarmonicMean → 4.98353, Median → 4.8}
```

```
dispRep = DispersionReport[DatenSatz]
```

```
{Variance → 0.221944, StandardDeviation → 0.47111, SampleRange → 1.1,
 MeanDeviation → 0.424691, MedianDeviation → 0.3, QuartileDeviation → 0.4375}
```

## b

Der Mittelwert der Grundgesamtheit müsste also mit    = 5.02222... und die Standardabweichung der Grundgesamtheit mit    = 0.47111... eingesetzt werden.

```
μ = Mean /. locRep
```

```
5.02222
```

```
σ = StandardDeviation /. dispRep
```

```
0.47111
```

## c

```
f[X_] := Cos[X - X^2] / (1 - X^2) + 1 / X;
```

```
f[X]
```

$$\frac{1}{X} + \frac{\text{Cos}[X - X^2]}{1 - X^2}$$

```
f[μ]
```

```
0.190111
```

```
D[f[X], X]
```

$$-\frac{1}{X^2} + \frac{2\,X\,\text{Cos}[X - X^2]}{(1 - X^2)^2} - \frac{(1 - 2\,X)\,\text{Sin}[X - X^2]}{1 - X^2}$$

```
Abs[D[f[X], X]] Abs[ΔX] /. {X → μ, ΔX → σ}
```

```
0.154753
```

Y +/-   Y = 0.190111 +/- 0.154753

Fehler fast so gross wie der Wert!

# 5

```
Remove["Global`*"]

μ = 12.54; σ = 1.96; Δx = 20.4 - μ

7.86

P (X ausserhalb von μ - Δx) ≤ σ^2 / (Δx^2)

P (-7.86 + 12.54 ausserhalb von X) ≤ 0.0621823
```

Die Wahrscheinlichkeit dieser Situation ist kleiner etwa 6.3 %. Vielleicht hat jemand 20.4 statt 10.4 geschrieben. Die Unmöglichkeit des Werts müsste man aber praktisch beurteilen.

# 6

Ablesen :

| | |
|---|---|
| | == > ca. 490 |
| 60 % | == > ca. [430, 540] |
| 80 % | == > ca. [400, 580] |

# 7

```
Remove["Global`*"]

M = {189, 196, 156, 173, 155, 179, 195, 186, 181, 168, 193, 158, 172,
  174, 157, 209, 165, 203, 143, 153, 203, 183, 153, 186, 154, 192, 214,
  157, 217, 156, 158, 182, 179, 206, 178, 173, 151, 177, 169, 177}; Sort[M]

{143, 151, 153, 153, 154, 155, 156, 156, 157, 157, 158, 158,
 165, 168, 169, 172, 173, 173, 174, 177, 177, 178, 179, 179, 181, 182,
 183, 186, 186, 189, 192, 193, 195, 196, 203, 203, 206, 209, 214, 217}
```

## a

```
μ = Mean[M]
```

$\frac{707}{4}$

```
N[%]

176.75

σ = StandardDeviation[M]
```

$\sqrt{\frac{9665}{26}}$

```
N[%]
```

19.2803

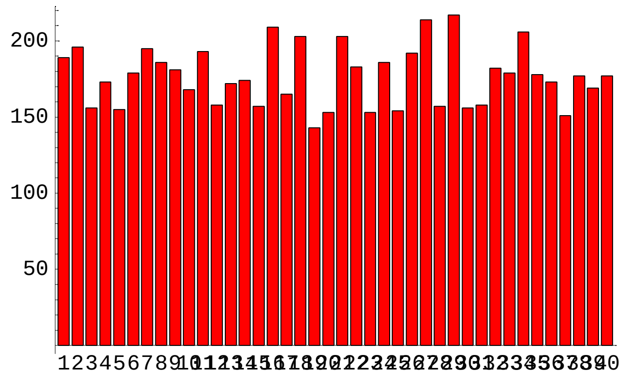
## b

```
<< Statistics`DescriptiveStatistics`;
<< Statistics`DataManipulation`;
<< Graphics`Graphics`; (*Version 5 *)
```

```
{Min[M], Max[M]}
```

{143, 217}

```
BarChart[M];
```




```
r = Table[k, {k, Min[M] - 2, Max[M] + 4, 8}]
```

{141, 149, 157, 165, 173, 181, 189, 197, 205, 213, 221}

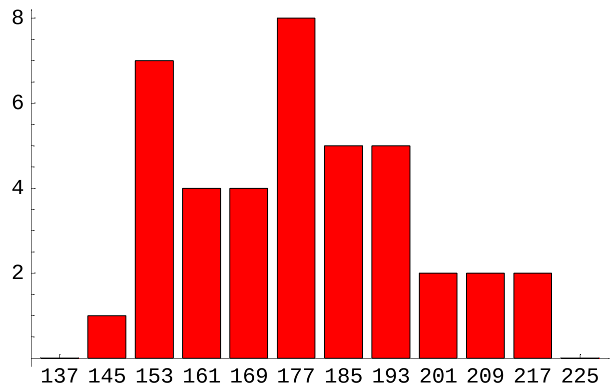
```
r1 = Table[k - 4, {k, Min[M] - 2, Max[M] + 12, 8}]
```

{137, 145, 153, 161, 169, 177, 185, 193, 201, 209, 217, 225}

```
Mr = RangeCounts[M, r]
```

{0, 1, 7, 4, 4, 8, 5, 5, 2, 2, 2, 0}

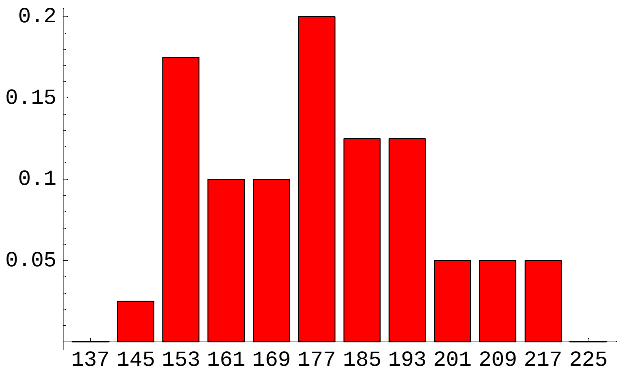
```
BarChart[Mr, BarLabels → r1];
```



```
Mr0Rel = Mr / Apply[Plus, Mr]
```

$\left\{0, \frac{1}{40}, \frac{7}{40}, \frac{1}{10}, \frac{1}{10}, \frac{1}{5}, \frac{1}{8}, \frac{1}{8}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, 0\right\}$

```
pl1 = BarChart[Mr0Rel, BarLabels → r1];
```




**c**

```
<< Statistics`NormalDistribution`
```

```
? NormalDistribution
```

NormalDistribution[mu, sigma] represents the normal (
Gaussian) distribution with mean mu and standard deviation sigma. Mehr…

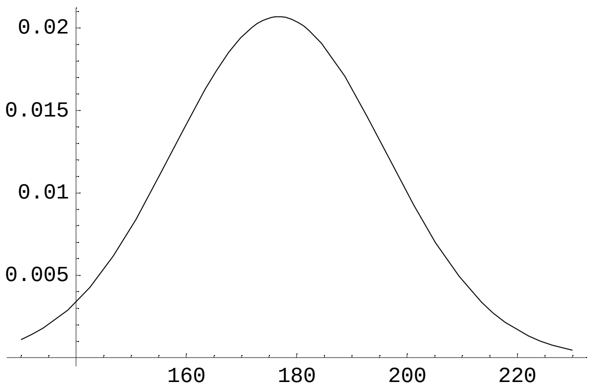
```
PDF[NormalDistribution[μ, σ], x]
```

$$e^{-\frac{13\left(-\frac{707}{4}+x\right)^2}{9665}} \sqrt{\frac{13}{9665\,\pi}}$$

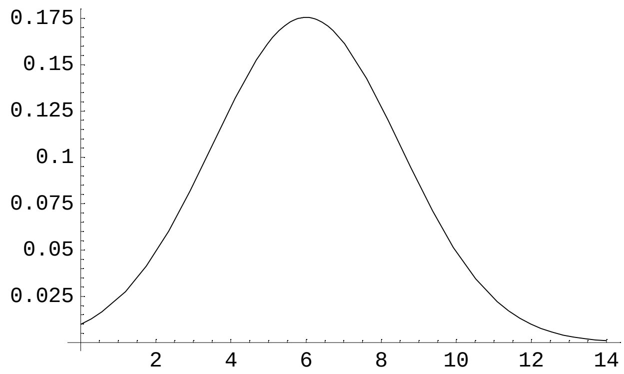
```
f[x_] := PDF[NormalDistribution[μ, σ], x]; f[x]
```

$$e^{-\frac{13\left(-\frac{707}{4}+x\right)^2}{9665}} \sqrt{\frac{13}{9665\,\pi}}$$

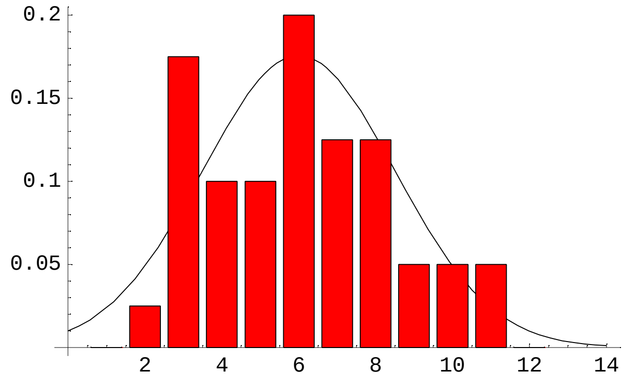
```
pl2 = Plot[PDF[NormalDistribution[μ, σ], x], {x, 130, 230}];
```



```
pl2 = Plot[ 1.1 PDF[NormalDistribution[6, 2.5], x], {x, 0, 14}];
```




```
Show[pl2, pl1];
```




Aus subjektiver Sicht scheint es ein Grenzfall zu sein. Nun sind mathematische Methoden gefragt, um die Wahrscheinlichkeit eines solchen Grenzfalls unter der Voraussetzung der Normalverteilung zu untersuchen.