

Übungen in Analysis

◇ E+M I / 5 + 6 ◇

Probl. 1 Versuche, die Grenzwerte zu bestimmen für $n \rightarrow \infty$ (Rechner erlaubt - doch wie geht es ohne?):

(a) $a_n = \frac{1}{n}$

(b) $a_n = 2 + \frac{1}{\sqrt{n}}$

(c) $a_n = \frac{2n^2}{1 + 3n^2}$

(d) $a_n = \frac{4n^3 - 3n + 1}{n^4 - 2n^2}$

(e) $b_n = \sin(n) \cdot \frac{1}{n}$

(f) $b_n = e^{-\frac{1}{n}}$

Probl. 2 $f(x) = e^{(x^2 - \cos(2x))} \rightsquigarrow f$ gerade/ ungerade?

Probl. 3 $f(x) = y = e^{-x^2}$

(a) $x \geq 0 \rightsquigarrow f^{-1}(x) = ?$ Skizze!

(b) $f^{-1}(0.5) \approx ?$

Probl. 4 $\log(x^2) + \log\left(\frac{1}{x}\right) - \log(x) = ?$

Probl. 5 $\sin\left(x + \frac{\pi}{3}\right) = \sin(x) \cdot (?) + \cos(x) \cdot (?)$

(a) $\sin\left(x + \frac{\pi}{3}\right) = \sin(x) \cdot (?) + \cos(x) \cdot (?)$

(b) $\tan\left(x + \frac{\pi}{3}\right) = \dots ? \dots$

Probl. 6 $r(\varphi) = 1 + \cos\left(\frac{\varphi}{2}\right) \rightsquigarrow$ Polar... Skizze!

Probl. 7 $2 \cdot 3^x = 5^x \rightsquigarrow x = ?$

Probl. 8 $0.367367\dots = \frac{p}{q}$, $p, q \in \mathbb{N} \rightsquigarrow p, q = ?$

Probl. 9 $\langle a_n \rangle = \left\langle \frac{\sin\left(3\pi + \frac{4}{5}n^2\right)}{n^2} \right\rangle \rightsquigarrow a_n \rightarrow ?$

Probl. 10 $\langle a_n \rangle = \left\langle \frac{n^2 - 2n + 5}{n^3 + n^2 + 1} \right\rangle \rightsquigarrow a_n \rightarrow ?$

Probl. 11 $\langle a_n \rangle = \left\langle \frac{\ln(n)}{n^2} \right\rangle \rightsquigarrow a_n \rightarrow ?$

Hinweis: Skizze! $\rightsquigarrow \ln(n), n$

Probl. 12 $\langle a_n \rangle = \left\langle \left(1 + \frac{1}{n} + \frac{1}{n^2}\right) \cdot \left(5 + \frac{2+n}{n}\right) \right\rangle \rightsquigarrow a_n \rightarrow ?$

Probl. 13 $\langle a_n \rangle = \left\langle e^{\sin(\pi + \frac{1}{n})} \right\rangle \rightsquigarrow a_n \rightarrow ?$

Probl. 14 $\langle a_n \rangle = \left\langle \frac{\cos(2\pi + \frac{1}{5}n^3)}{n^2} \right\rangle \rightsquigarrow a_n \rightarrow ?$

Probl. 15 $\langle a_n \rangle = \left\langle \frac{n^2 - 2n + 4}{4n^3 + n^2 - 1} \right\rangle \rightsquigarrow a_n \rightarrow ?$

Probl. 16 $\langle a_n \rangle = \left\langle \frac{\ln(n)}{n^{1.5}} \right\rangle \rightsquigarrow a_n \rightarrow ?$

Hinweis: Skizze! $\rightsquigarrow \ln(n), n$

Probl. 17 $\langle a_n \rangle = \left\langle \left(2 + \frac{3}{n} + \frac{4}{n^2}\right) \cdot \left(5 + \frac{6+7n}{8n}\right) \right\rangle \rightsquigarrow a_n \rightarrow ?$

Probl. 18 $\langle a_n \rangle = \left\langle 5 e^{\tan(\pi + \frac{2}{n^3})} \right\rangle \rightsquigarrow a_n \rightarrow ?$

Probl. 19 $\langle a_n \rangle = \left\langle \frac{n+1}{\sqrt{n+1}} \right\rangle \rightsquigarrow a_n \rightarrow ?$

Probl. 20 $\langle a_n \rangle = \left\langle \frac{(3n+1)}{\left(\frac{n(2n-1)}{n+2}\right)} \right\rangle \rightsquigarrow a_n \rightarrow ?$

Probl. 21 Skizzieren?

(a) $f(x) = x^2 + \sin(x)$

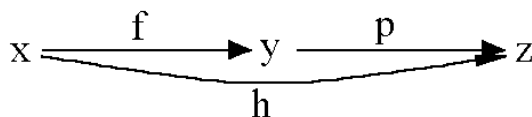
(b) $f(x) = [x^2] + 1$

(c) $f(x) = \sin([x^2 + 1]) - 1$

Probl. 22 $x = 4 + \frac{4}{4 + \frac{4}{4 + \frac{4}{\ddots}}} \stackrel{?}{=} \frac{p}{q}, p, q \in \mathbb{N}$

$x = 5 + \frac{3}{3 + \frac{3}{3 + \frac{3}{\ddots}}} \stackrel{?}{=} \frac{r}{s}, = ?$

Probl. 23



$f(x) = \arccos(x) = y$

$h(x) = p(f(x)) = p(y) = y^2 - 2y + 2 = z$

$h(x) = (p \circ f)(x)$

(a) $h(x) \rightsquigarrow$ Diagramm?
 $h(x) \rightsquigarrow$

(b) $D_h = ?, W_h = ?$

(c) $h(0) = ?, h(1) = ?$

Probl. 24 $f(x) = x^2 + x + \frac{1}{4}, g(x) = x^2 - x$

(a) Nullstellen von $h(x) = f(x) \cdot g(x)$?

(b) $u(x) = \frac{1}{f(x) - g(x)} \rightsquigarrow$ i. Diagramm?
ii. Pole?

Probl. 25 $h(x) = \frac{f(x)}{g(x)} + x$
 $f(x), g(x) \rightsquigarrow$ Probl. 4

(a) Diagramm?

(b) Verhalten für grosse $|x|$?

Probl. 26 $f(x) = e^{-x^2}$, $g(x) = x$

(a) Diagramm?

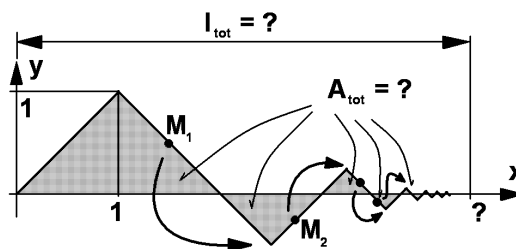
(b) $f(x) = g(x) \rightsquigarrow x \approx ?$

(c) $m \leq f(x) \leq M \rightsquigarrow m = ?$, $M = ?$

(d) $f(0) = ?$, $f(\ln(e)) = ?$, $f(1) = ?$

Probl. 27 $3^x = 2^{2x} \cdot e^3 \cdot 3^{-2x} \rightsquigarrow x = ?$

Probl. 28 $A_{tot} = ?$, $l_{tot} = ?$



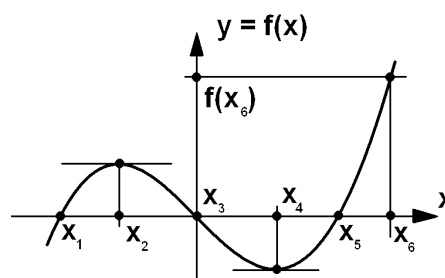
Probl. 29 $\lim_{n \rightarrow \infty} \frac{n \cdot \cos(2n) - \sin(n^2) + 8n^2 - 4n + 5}{2n^2 + 4n - 5 \sin(n)} = ?$

Probl. 30 $f(x) = a \cdot (x - x_1)(x - x_3)(x - x_5)$

$x_1 = -2$, $x_3 = 0$, $x_5 = 2$

$x_6 = 4$, $f(x_6) = 48$

$x_2 = ?$, $x_4 = ?$



Probl. 31 $\langle a_n \rangle = \langle (3 + \frac{3}{2^2} + \frac{3}{2^4} + \frac{3}{2^6} \dots) \cdot (\frac{5}{2} + \frac{5}{2^3} + \frac{5}{2^5} + \dots) \rangle \rightsquigarrow a_n \rightarrow ?$

Probl. 32 $\langle a_n \rangle = \sum_{k=1}^n (\frac{1}{2^k} - \frac{1}{3^k}) \rightsquigarrow a_n \rightarrow ?$

Probl. 33 Studiere die Funktion $Nest(f, x, n) := \underbrace{f(f(f(\dots(f(x))\dots)))}_n$

- (a) Skizziere $Nest(\cos, x, n)$, $n \in \{1, 2, \dots, 10\}$
- (b) Skizziere $Nest(\sin, x, n)$, $n \in \{1, 2, \dots, 10\}$
- (c) $\lim_{n \rightarrow \infty} Nest(\sin, x, n) = ?$ (Vermutung?)