Lösungen zu Uebungen in AlgGeo ◇ Solutions pour les excercices en AlgGéo ◇ T. B1 ◇ II / 12

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5 Testserien zur Vektorrechnung: • *5 séries de test concernant le calcul vectoriel:*

s 1 Serie 1 •: *Série 1*

Lösungen / solutions

1 a) $\vec{a}\cdot\vec{b} = \begin{pmatrix}5\\2\\0\end{pmatrix}\cdot\begin{pmatrix}2\\3\\0\end{pmatrix} = 10+6 = \underline{\underline{16}}$

b) $\vec{a}\,(\vec{a}+\vec{b}) = \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b} = \begin{pmatrix}5\\2\\0\end{pmatrix}\cdot\begin{pmatrix}5\\2\\0\end{pmatrix} + 16 = 25+4+16 = \underline{\underline{45}}$

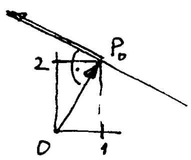
c) $\vec{a}\times\vec{b} = \begin{pmatrix}5\\2\\0\end{pmatrix}\times\begin{pmatrix}2\\3\\0\end{pmatrix} = \begin{pmatrix}0\\0\\15-4\end{pmatrix} = \underline{\underline{\begin{pmatrix}0\\0\\11\end{pmatrix}}}$

d) $F = \frac{1}{2}\,|\vec{a}\times\vec{b}| = \frac{1}{2}\cdot 11 = \underline{\underline{5.5}}$

e) $h = \frac{2F}{|\vec{a}|} = \frac{11}{\sqrt{5^2+2^2}} = \underline{\underline{\frac{11}{\sqrt{29}}}} \approx \underline{\underline{2.0426\ldots}}$

f) $\varphi = \arccos\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \arccos\frac{16}{\underbrace{\sqrt{5^2+2^2}}_{29}\cdot\underbrace{\sqrt{2^2+3^2}}_{13}} = \underline{\underline{\arccos\frac{16}{\sqrt{377}}}}$

$\approx \underline{\underline{0{,}602287\ldots \;\hat{\approx}\; 34.508\ldots^\circ}}$

2 

$\vec{r}_g = \underbrace{\overrightarrow{OP_0}}_{\vec{r}_0} + t\cdot\underbrace{\overrightarrow{OP_0}_{\perp}}_{\vec{a}} \Rightarrow \underline{\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix} + t\begin{pmatrix}-2\\1\end{pmatrix}}$

$A:\; y=0 \Rightarrow 0 = 2+t,\; t=-2,\; x = 1+(-2)(-2) = 5$

$\underline{\underline{A\,(5/0)}}$

$B:\; x=0 \Rightarrow 0 = 1-2t,\; t=\frac{1}{2},\; y = 2+\frac{1}{2}\cdot 1 = \frac{5}{2} = 2.5$

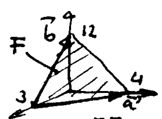
$\underline{\underline{B\,(0/2.5)}}$

$F = \frac{|\overline{OA}|\cdot|\overline{OB}|}{2} = \frac{5\cdot 2.5}{2} = \underline{\underline{\left(\frac{5}{2}\right)^2}} = \underline{\underline{6.25}}$

3 a) $Ax+By+Cz-144=0$

$A\cdot 3+0+0 = 144$ $\underline{\underline{A = 48}}$
$0+B\cdot 4+0 = 144$ $\underline{\underline{B = 36}}$
$0+0+C\cdot 12 = 144$ $\underline{\underline{C = 12}}$

$\left\{ A'=4,\; B'=3,\; C'=1,\; D'=-12 \right.$



b) $\overbrace{\phi-O}^{d_1}$?

$V = \frac{1}{3}F\cdot d_1$, $F=\frac{1}{2}|\vec{a}\times\vec{b}|$, $V=\frac{1}{3\cdot 2}x_0y_0z_0 = \frac{3\cdot 4\cdot 12}{3\cdot 2}$

$\Longrightarrow \frac{1}{2}\cdot\frac{1}{3}\cdot|\vec{a}\times\vec{b}|\cdot d_1 = \frac{1}{3\cdot 2}x_0\cdot y_0\cdot z_0$, $d_1 = \frac{x_0y_0z_0}{|\vec{a}\times\vec{b}|} = \frac{3\cdot 4\cdot 12}{\left|\begin{pmatrix}-3\\4\\0\end{pmatrix}\times\begin{pmatrix}-3\\0\\12\end{pmatrix}\right|}$

$d_1 = \frac{144}{\left|\begin{pmatrix}48\\36\\12\end{pmatrix}\right|} = \frac{144}{\sqrt{48^2+36^2+12^2}} = \frac{144}{\sqrt{3744}} \approx \underline{\underline{2{,}35339\ldots}}$

c) $P-\phi$? $\rightsquigarrow$.

$d_2 = 2\cdot d_1 \approx 4.70678\ldots$ (Geometrie)

4 Sei / soit $\vec{r}_0 = \vec{0} \Rightarrow \vec{r}_\phi = \lambda \begin{pmatrix}1\\1\\-1\end{pmatrix} + \mu \begin{pmatrix}-2\\1\\1\end{pmatrix}, \quad \vec{r}_g = t\begin{pmatrix}3\\0\\4\end{pmatrix}$

a) $V = (\vec{a} \times \vec{b}) \cdot \vec{c} = \left(\begin{pmatrix}1\\1\\-1\end{pmatrix} \times \begin{pmatrix}-2\\1\\1\end{pmatrix}\right) \cdot \begin{pmatrix}3\\0\\4\end{pmatrix} = \begin{pmatrix}2\\1\\3\end{pmatrix} \cdot \begin{pmatrix}3\\0\\4\end{pmatrix} = 2 \cdot 3 + 3 \cdot 4 = \underline{\underline{18}}$

b) $\varphi = \arccos \dfrac{\vec{a} \cdot \vec{c}}{|\vec{a}||\vec{c}|} = \arccos \dfrac{\begin{pmatrix}1\\1\\-1\end{pmatrix} \cdot \begin{pmatrix}3\\0\\4\end{pmatrix}}{\sqrt{1^2+1^2+1^2} \cdot \sqrt{3^2+4^2}} = \arccos \dfrac{-1}{\sqrt{3} \cdot \sqrt{25}} = \underline{\underline{\arccos \dfrac{-1}{\sqrt{75}}}}$

$\approx \underline{\underline{1.6865..}} \quad \hat{\approx} \quad \underline{\underline{96.63...^\circ}}$

c) $\vec{n}_1 = \vec{a} \times \vec{b}, \quad \vec{n}_2 = \vec{b} \times \vec{c}, \quad \varphi_1 = \arccos \dfrac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$

$\vec{n}_1 = \begin{pmatrix}1\\1\\-1\end{pmatrix} \times \begin{pmatrix}-2\\1\\1\end{pmatrix} = \begin{pmatrix}2\\1\\3\end{pmatrix}, \quad \vec{n}_2 = \begin{pmatrix}-2\\1\\1\end{pmatrix} \times \begin{pmatrix}3\\0\\4\end{pmatrix} = \begin{pmatrix}4\\11\\-3\end{pmatrix}$

$\rightarrow \varphi_1 = \arccos \dfrac{\begin{pmatrix}2\\1\\3\end{pmatrix} \cdot \begin{pmatrix}4\\11\\-3\end{pmatrix}}{\underbrace{\sqrt{2^2+1^2+3^2}}_{14} \cdot \underbrace{\sqrt{4^2+11^2+3^2}}_{146}} = \underline{\underline{\arccos \dfrac{10}{\sqrt{2044}}}} \approx \underline{\underline{1.34776...}} \approx \underline{\underline{77.22..^\circ}}$ 10P

5
$$\begin{aligned} 2x + y + z + w &= 5 \\ x + 2y + z + w &= 5k \\ x + y + 2z + w &= 5u \\ x + y + z + 2w &= 5 \end{aligned}$$

$\rightarrow \left\{\begin{array}{l|rrrr|l} I & 2 & 1 & 1 & 1 & 5 \\ II & 1 & 2 & 1 & 1 & 5k \\ III & 1 & 1 & 2 & 1 & 5u \\ IV & 1 & 1 & 1 & 2 & 5 \end{array}\right. \rightarrow \left\{\begin{array}{l|rrrr|ll} I & 2 & 1 & 1 & 1 & 5 & I' \\ II-I & -1 & 1 & 0 & 0 & 5k-5 & II' \\ III-I & -1 & 0 & 1 & 0 & 5u-5 & III' \\ IV-I & -1 & 0 & 0 & 1 & 0 & IV' \end{array}\right.$

$IV': \; -x + w = 0 \Rightarrow \underline{\underline{x = w}}$

$\rightarrow \begin{array}{ll} I' & 2x + y + z + x = 3x + y + z = 5 \\ II' & -x + y = 5k - 5 \\ III' & -x + z = 5u - 5 \end{array} \Bigg\} \rightarrow \left\{\begin{array}{rrr|ll} 3 & 1 & 1 & 5 & I' \\ -1 & 1 & 0 & 5k-5 & II' \\ -1 & 0 & 1 & 5u-5 & III' \end{array}\right.$

$\begin{array}{l|rrr|l} I'' = I' - III' & 4 & 1 & 0 & 10-5u \\ II' & -1 & 1 & 0 & -5+5k \\ III' & -1 & 0 & 1 & -5+5u \end{array} \qquad \begin{array}{l|rrr|l} I''' = I'' - II' & 5 & 0 & 0 & 15-5k-5u \\ II' & -1 & 1 & 0 & -5+5k \\ III' & -1 & 0 & 1 & -5+5u \end{array}$

$I''': \quad 5x = 15 - 5k - 5u \Rightarrow x = 3 - k - u$

$II': \quad y = -5 + 5k + x \Rightarrow y = -2 + 4k - u$

$III': \quad z = -5 + 5u + x \Rightarrow z = -2 + 4u - k$

$$\Rightarrow \quad \begin{aligned} x &= 3 - k - u \\ y &= -2 + 4k - u \\ z &= -2 - k + 4u \\ w &= 3 - k - u \end{aligned}$$

6P

s 2 Serie 2 • : *Série 2*

Lösungen

1) $\vec{n}_\phi = \begin{pmatrix}3\\2\\-5\end{pmatrix}$, $\vec{n}_\psi = \begin{pmatrix}4\\1\\2\end{pmatrix} \times \begin{pmatrix}-1\\2\\2\end{pmatrix} = \begin{pmatrix}-2\\-10\\9\end{pmatrix}$, $\vec{n}_\phi \cdot \vec{n}_\psi = |\vec{n}_\phi| \cdot |\vec{n}_\psi| \cdot \cos\alpha$

$(\alpha = \sphericalangle(\phi, \psi) = \sphericalangle(\vec{n}_\phi, \vec{n}_\psi) \Rightarrow \alpha = \arccos \dfrac{\vec{n}_\phi \cdot \vec{n}_\psi}{|\vec{n}_\phi| \cdot |\vec{n}_\psi|}$

$\alpha \approx \underline{\underline{147.865^\circ}}$

2) a) $\vec{a} \perp \vec{b}$: $\vec{a} \cdot \vec{b} = 0$, $\begin{pmatrix}2\\3\\4\end{pmatrix} \cdot \begin{pmatrix}x\\1\\2\end{pmatrix} = 0$, $2x + 3 \cdot 1 + 4 \cdot 2 = 0$, $\underline{\underline{x = -\frac{11}{2}}}$

b) $\vec{a} \cdot \vec{v} = \vec{a} \cdot \lambda \vec{b} = \lambda \cdot (\vec{a} \cdot \vec{b}) = 7$, $\lambda\,(2x + 3 \cdot 1 + 4 \cdot 2) = 7$

d.h. $\underline{\lambda \cdot (2x + 11) = 7}$ $\quad x = -\frac{11}{2}$: $\underline{\underline{\mathbb{L} = \{\}}}$

$x \neq -\frac{11}{2}$: $\underline{\underline{\lambda = \frac{7}{2x + 11}}}$

3) $\vec{a}_\perp = \begin{pmatrix}-1\\3\end{pmatrix}$, $\vec{a}_\perp \cdot \vec{r}_0 = |\vec{r}_0| \cdot |\vec{a}_\perp| \cdot \cos\varphi$, $|\vec{r}_0| \cdot \cos\varphi = d$

$\Rightarrow \quad d = \dfrac{\vec{a}_\perp \cdot \vec{r}_0}{|\vec{a}_\perp|} = \sqrt{\frac{5}{2}} \approx \underline{\underline{1.58114}}$

Entspr. $d_1 + d = \dfrac{\vec{a}_\perp \cdot \vec{OP}}{|\vec{a}_\perp|} \approx 5.05764$

$\Rightarrow \underline{\underline{d_1 \approx 3{,}47851}}$

4) a) $\vec{n} = \vec{b} \times \vec{c} \perp \phi$. Lös. wie 3), $\left\{ \begin{array}{l} \vec{n} = \begin{pmatrix}-1\\4\\-7\end{pmatrix} \\ \vec{n} \text{ hat die Rolle von } \vec{a}_\perp. \end{array} \right.$

$d = \dfrac{\vec{n} \cdot \vec{a}}{|\vec{n}|} \approx -1.51523$

$d_1 + d = \dfrac{\vec{n} \cdot \vec{OP}_0}{|\vec{n}|} \approx -3.03046$

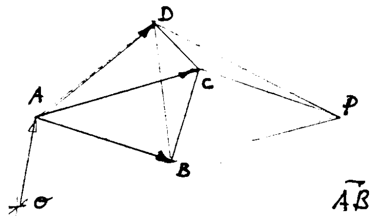
$\Rightarrow \underline{\underline{d_1 \approx 1.51523}}$

b) $V = \left| \frac{1}{6} (\vec{a} \times \vec{b}) \cdot \vec{c} \right| = \underline{\underline{\frac{5}{2}}}$

S 3 Serie 3 •: *Série 3*

Lösungen / solutions

①



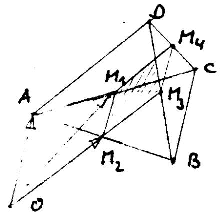
$$V_{ABPCD} = \frac{1}{3}\left|[\vec{AB}, \vec{AC}, \vec{AD}]\right|$$

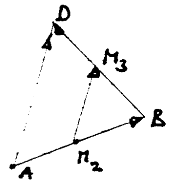
$$V_{ABCD} = \frac{1}{2} V_{ABPCD} = \frac{1}{6}\left|[\vec{AB}, \vec{AC}, \vec{AD}]\right| = \frac{1}{6}\left|(\vec{AB} \times \vec{AC}) \circ \vec{AD}\right|$$

$$\vec{AB} = \begin{pmatrix}6\\2\\0\end{pmatrix}, \quad \vec{AC} = \begin{pmatrix}8\\-1\\1\end{pmatrix}, \quad \vec{AD} = \begin{pmatrix}5\\1\\6\end{pmatrix}$$

$$\Rightarrow V_{ABCD} = \frac{1}{6}\left|\left(\begin{pmatrix}6\\2\\0\end{pmatrix} \times \begin{pmatrix}8\\-1\\1\end{pmatrix}\right) \circ \begin{pmatrix}5\\1\\6\end{pmatrix}\right| = \frac{1}{6}\left|\begin{pmatrix}2\\-6\\-22\end{pmatrix} \circ \begin{pmatrix}5\\1\\6\end{pmatrix}\right| = \frac{1}{6}\left|10 - 6 - 132\right| = \left|-\frac{128}{6}\right|$$

$$\Rightarrow \underline{\underline{V_{ABCD} = 21\tfrac{1}{3} = 21.33\ldots}}$$





$$F_{M_1M_2M_3M_4} = \left|\vec{M_2M_3} \times \vec{M_2M_1}\right|$$

$$\vec{OM_2} = \vec{OA} + \frac{1}{2}\vec{AB}, \quad \vec{OM_3} = \vec{OA} + \vec{AB} + \frac{1}{2}\vec{BD}$$

$$\vec{M_2M_3} = \vec{OM_3} - \vec{OM_2} = \vec{AB} + \frac{1}{2}\vec{BD} - \frac{1}{2}\vec{AB} = \frac{1}{2}(\vec{AB} + \vec{BD})$$

$$\Longrightarrow \quad \vec{M_2M_3} = \frac{1}{2}(\vec{AB} + \vec{BD}) = \frac{1}{2}\left(\begin{pmatrix}6\\2\\0\end{pmatrix} + \begin{pmatrix}-1\\-1\\6\end{pmatrix}\right) = \frac{1}{2}\cdot\begin{pmatrix}5\\1\\6\end{pmatrix}$$

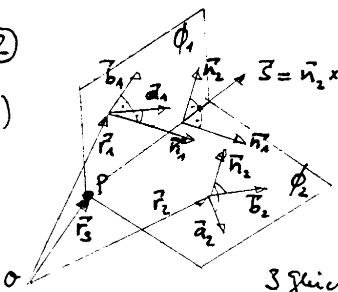
entspr. / correspond. $\vec{M_2M_1} = \frac{1}{2}(\vec{BA} + \vec{AC}) = \frac{1}{2}\left(\begin{pmatrix}-6\\-2\\0\end{pmatrix} + \begin{pmatrix}8\\-1\\1\end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix}2\\-3\\1\end{pmatrix}$

$$F_{M_1M_2M_3M_4} = \frac{1}{2}\cdot\frac{1}{2}\cdot\left|\begin{pmatrix}5\\1\\6\end{pmatrix} \times \begin{pmatrix}2\\-3\\1\end{pmatrix}\right| = \frac{1}{4}\left|\begin{pmatrix}19\\7\\-17\end{pmatrix}\right| = \frac{1}{4}\sqrt{19^2+7^2+17^2} = \frac{1}{4}\sqrt{699}$$

$$\underline{\underline{F_{M_1M_2M_3M_4} = \frac{1}{4}\sqrt{699} = 6.60965\ldots}}$$

②

(a)



$$\vec{n_1} = \vec{a_1} \times \vec{b_1} \text{ resp. } \vec{b_1} \times \vec{a_1} \qquad \begin{pmatrix}1\\0\\1\end{pmatrix} \times \begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}-1\\1\\1\end{pmatrix}$$

$$\vec{n_2} = \vec{a_2} \times \vec{b_2} \text{ resp. } \vec{b_2} \times \vec{a_2} \qquad \begin{pmatrix}-1\\2\\0\end{pmatrix} \times \begin{pmatrix}0\\0\\2\end{pmatrix} = \begin{pmatrix}4\\2\\0\end{pmatrix}$$

$$\vec{s} = \vec{n_1} \times \vec{n_2} \text{ resp. } \vec{n_2} \times \vec{n_1} \qquad \begin{pmatrix}-1\\1\\1\end{pmatrix} \times \begin{pmatrix}4\\2\\0\end{pmatrix} = \begin{pmatrix}-2\\4\\-6\end{pmatrix}, \begin{pmatrix}1\\-2\\3\end{pmatrix}$$

$$P: \quad \vec{x}_{\Phi_1} = \vec{x}_{\Phi_2}: \quad \begin{pmatrix}2\\1\\3\end{pmatrix} + \lambda_1\begin{pmatrix}1\\0\\1\end{pmatrix} + \mu_1\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}-2\\2\\2\end{pmatrix} + \lambda_2\begin{pmatrix}-1\\2\\0\end{pmatrix} + \mu_2\begin{pmatrix}0\\0\\2\end{pmatrix}$$

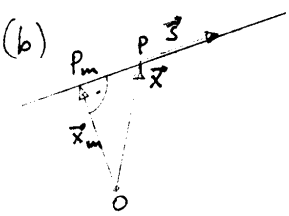
3 Gleich. / 4 Unbekannte ⇒ Wähle $\lambda_2 = 0$
3 Equat. / 4 inconnues ⇒ choisir $\lambda_2 = 0$

$$\Rightarrow \begin{array}{ll} \text{I} & \lambda_1\cdot 1 + \mu_1\cdot 1 - \mu_2\cdot 0 = -4 \\ \text{II} & \lambda_1\cdot 0 + \mu_1\cdot 1 - \mu_2\cdot 0 = 1 \\ \text{III} & \lambda_1\cdot 1 + \mu_1\cdot 0 - \mu_2\cdot 2 = -1 \end{array}$$

II: $\mu_1 = 1$ I: $\lambda_1 + 1 = -4 \Rightarrow \lambda_1 = -5$
III: $-5 - 2\mu_2 = -1 \Rightarrow \mu_2 = -2$ $\Rightarrow \vec{OP} = \begin{pmatrix}-2\\2\\2\end{pmatrix} + 0\cdot\begin{pmatrix}-1\\2\\0\end{pmatrix} - 2\begin{pmatrix}0\\0\\2\end{pmatrix} = \begin{pmatrix}-2\\2\\-2\end{pmatrix}$

$$\underline{\underline{\vec{x}_s = \vec{OP} + \vec{s} = \begin{pmatrix}-2\\2\\-2\end{pmatrix} + t\cdot\begin{pmatrix}1\\-2\\3\end{pmatrix} = \begin{pmatrix}0\\-2\\4\end{pmatrix} + t\begin{pmatrix}1\\-2\\3\end{pmatrix}}}$$

(b)



$$|\vec{OP}| \text{ min.} \Leftrightarrow \vec{x} \perp \vec{s} \Leftrightarrow \vec{x} \circ \vec{s} = 0 \Leftrightarrow \begin{pmatrix}t\\-2-2t\\4+3t\end{pmatrix} \circ \begin{pmatrix}1\\-2\\3\end{pmatrix} = 0$$

$$\Leftrightarrow t + 4 + 4t + 12 + 9t = 14t + 16 = 0 \Rightarrow t = -\frac{8}{7}$$

$$\Rightarrow \vec{x}_m = \vec{OP_m} = \begin{pmatrix}0\\-2\\4\end{pmatrix} + \left(-\frac{8}{7}\right)\cdot\begin{pmatrix}1\\-2\\3\end{pmatrix} = \begin{pmatrix}-8/7\\2/7\\4/7\end{pmatrix}$$

$$\underline{\underline{P_{min} = \left(-\frac{8}{7} \,\middle|\, \frac{2}{7} \,\middle|\, \frac{4}{7}\right) \approx (-1.14\ldots \,|\, 0.28\ldots \,|\, 0.57\ldots)}} \qquad d = \frac{\sqrt{84}}{7} \approx 1.309\ldots$$

③

$$\begin{array}{lrl} \text{I.} & 3x + y - 20w &= 11 \\ \text{II.} & 2x - 2y + z - w &= 2 \\ \text{III.} & x + y + 11z &= 7 \\ \text{IV.} & -x - 8z + 16w &= -1 \\ \text{V.} & 4x + 2y + 11z - 20w &= 18 \end{array}$$

$\text{I} + \text{III} = \text{V} \Rightarrow \cancel{\text{V}}$

Cramer: $D_0 = \begin{vmatrix} 3 & 1 & 0 & -20 \\ 2 & -2 & 1 & -1 \\ 1 & 1 & 11 & 0 \\ -1 & 0 & -8 & 16 \end{vmatrix} = -1593$, $D_x = \begin{vmatrix} 11 & 1 & 0 & -20 \\ 2 & -2 & 1 & -1 \\ 7 & 1 & 11 & 0 \\ -1 & 0 & -8 & 16 \end{vmatrix} = -6345$

$D_y = \begin{vmatrix} 3 & 11 & 0 & -20 \\ 2 & 2 & 1 & -1 \\ 1 & 7 & 11 & 0 \\ -1 & -1 & -8 & 16 \end{vmatrix} = -4608$, $D_z = \begin{vmatrix} 3 & 1 & 11 & -20 \\ 2 & -2 & 2 & -1 \\ 1 & 1 & 7 & 0 \\ -1 & 0 & -1 & 16 \end{vmatrix} = -18$, $D_w = \begin{vmatrix} 3 & 1 & 0 & 11 \\ 2 & -2 & 1 & 2 \\ 1 & 1 & 11 & 7 \\ -1 & 0 & -8 & -1 \end{vmatrix} = -306$

$\Rightarrow x = \frac{-6345}{-1593} = \frac{235}{59} \approx \underline{\underline{3.98305...}}$ $\quad y = \frac{-4608}{-1593} = \frac{512}{177} \approx \underline{\underline{2.89266...}}$

$z = \frac{-18}{-1593} = \frac{2}{177} \approx \underline{\underline{0{,}0112994...}}$ $\quad w = \frac{-306}{-1593} = \frac{34}{177} \approx \underline{\underline{0{,}19209...}}$

④ $K: (x-3)^2 + (y-4)^2 = 100 \Rightarrow \vec{OM} = \binom{3}{4}$

$g_1: \binom{x}{y} = \binom{2}{1} + t\binom{-2}{1}$ z.B/par.a.: $\begin{matrix} y = 1+t \\ t = y-1 \end{matrix}$ $\quad x = 2-2t = 2-2(y-1) = -2y+4$
$\Rightarrow x = -2y+4$

$T \in (g_1 \cap K): (\overbrace{-2y+4}^{x} - 3)^2 + (y-4)^2 = (-2y+1)^2 + (y-4)^2 = 4y^2 - 4y + 1 + y^2 - 8y + 16 = 100$

$\Rightarrow 5y^2 - 12y - 83 = 0 \Rightarrow y_{1,2} = \frac{12 \pm \sqrt{12^2 + 4 \cdot 5 \cdot 83}}{2 \cdot 5} = \frac{12 \pm \sqrt{1804}}{10} = \begin{cases} 5.44735... \\ -3.04735... \end{cases}$

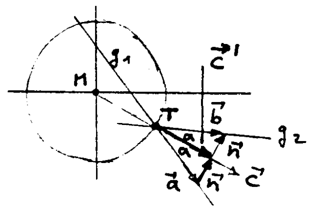
$\Rightarrow x_{1,2} = \begin{cases} -6.89470... \\ 10.0947... \end{cases} \Rightarrow T_1 \approx (-6.89470.. / 5.44735..),\ T_2 \approx (10.0947.. / -3.04735...)$

$T = (u/v),\ u > 0 \Rightarrow T = T_2 \approx \underline{(10.0947.. / -3.04735...)}$

$g_1: \vec{x} = \vec{r_0} + t \cdot \vec{a}$

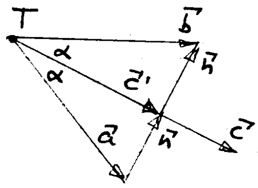
$g_2: \vec{x} = \vec{OT} + q \cdot \vec{b}$

$\vec{a} = \binom{-2}{1}$



$\vec{c} = \vec{MT} = \vec{OT} - \vec{OM} = \binom{7.0947..}{-7.04735...}$

$\vec{c}\,' = \frac{\vec{c}}{c} \cdot a \cdot \cos\alpha = \vec{c} \cdot \frac{(\vec{a} \circ \vec{c})}{c^2}$



$\vec{n} = \vec{c}\,' - \vec{a}$

$\vec{b} = \vec{c}\,' + \vec{n}$
$= 2\vec{c}\,' - \vec{a}$

$\Rightarrow \vec{b} = 2\vec{c}\,' - \vec{a} = 2 \cdot \vec{c} \cdot \frac{(\vec{a} \circ \vec{c})}{c^2} - \vec{a}$

$g_2: \vec{x} = \vec{OT} + q \cdot \left(2 \cdot \vec{c}\, \frac{(\vec{a} \circ \vec{c})}{c^2} - \vec{a}\right)$

$\approx \binom{10.0947..}{-3.04735..} + q \cdot \left(\underbrace{\frac{2 \cdot}{7.09..^2 + 7.04..^2} \underbrace{(-2 \cdot 7{,}094... - 7.047..)}_{-21.23675}}_{-0.4247354} \cdot \binom{7.0947..}{-7.047...} - \binom{-2}{1} \right)$

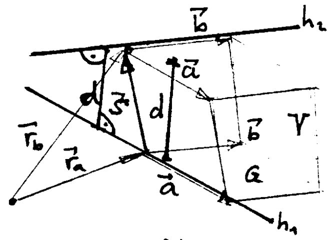
$= \binom{10.0947..}{-3.04735...} + q \binom{-1{,}0133701...}{+1{,}9932589..}$

$\Rightarrow \underline{\underline{\binom{x}{y} \approx \binom{10.09}{-3.05} + q \binom{-1.01}{+1.99}}}$ $\quad (\alpha \approx 18{,}4^\circ)$

$\Rightarrow$ $1.99..x + 1.01..y \approx 17{,}033...$

$\underline{\underline{x + 0{,}508..y \approx 8{,}545...}}$

⑤



$$V = G\cdot d = |\vec a\times\vec b|\cdot d$$

$$V = |[\vec a,\vec b,\vec s]| \ , \quad \vec s = \vec r_b - \vec r_a$$

$$\Longrightarrow \quad |\vec a\times\vec b|\cdot d = |[\vec a,\vec b, \vec r_b-\vec r_a]|$$

$$d = \frac{|[\vec a,\vec b,\vec r_b-\vec r_a]|}{|\vec a\times\vec b|}$$

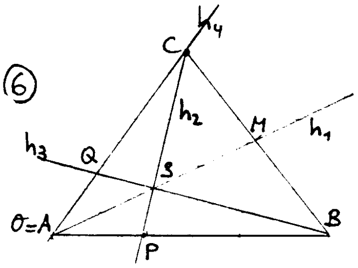
$$\vec s = \vec r_b - \vec r_a = \begin{pmatrix}2\\2\\2\end{pmatrix} - \begin{pmatrix}2\\1\\0\end{pmatrix} = \begin{pmatrix}0\\1\\2\end{pmatrix} \ , \quad \vec a\times\vec b = \begin{pmatrix}1\\1\\1\end{pmatrix}\times\begin{pmatrix}-5\\-4\\6\end{pmatrix} = \begin{pmatrix}10\\-11\\1\end{pmatrix}$$

$$[\vec a,\vec b,\vec s] = (\vec a\times\vec b)\circ\vec s = \begin{pmatrix}10\\-11\\1\end{pmatrix}\circ\begin{pmatrix}0\\1\\2\end{pmatrix} = -9$$

$$\Longrightarrow \quad d = \frac{9}{\sqrt{10^2+11^2+1}} = \frac{9}{\sqrt{222}} = 0,604045\ldots$$

$$\underline{\underline{d \approx 0,604}}$$

⑥



$$\vec{AM} = \vec{AB} + \tfrac12\vec{BC} = \begin{pmatrix}b\\0\end{pmatrix} + \tfrac12\begin{pmatrix}r-b\\s\end{pmatrix} = \begin{pmatrix}r+b\\s\end{pmatrix}\cdot\tfrac12$$

$$h_1: \ \vec x = \lambda\vec{AM} = \lambda\cdot\tfrac12\begin{pmatrix}r+b\\s\end{pmatrix} = \lambda'\begin{pmatrix}r+b\\s\end{pmatrix}$$

$$|\vec{AP}|:|\vec{PB}| = 1:2 \Rightarrow \vec{OP} = \vec{AP} = \tfrac13\vec{AB} = \begin{pmatrix}b\\0\end{pmatrix}\cdot\tfrac13$$

$$h_2: \ \vec x = \vec{AP} + t\cdot\vec{PC} = \begin{pmatrix}b/3\\0\end{pmatrix} + t\cdot\begin{pmatrix}r-b/3\\s\end{pmatrix}$$

$$S = h_1\cap h_2: \ \lambda'\begin{pmatrix}r+b\\s\end{pmatrix} = \begin{pmatrix}b/3\\0\end{pmatrix} + t\begin{pmatrix}r-b/3\\s\end{pmatrix} \ \Big|\ \begin{matrix}I\\II\end{matrix}$$

$II \Rightarrow \lambda' = t$

$I \Rightarrow t(\cancel{r}+b) - t(\cancel{r}-\tfrac b3) = \tfrac b3$

$t\cdot\tfrac{4b}{3} = \tfrac b3 \Rightarrow t = \tfrac14$

$$\Rightarrow \vec{AS} = \tfrac14\begin{pmatrix}r+b\\s\end{pmatrix} \Rightarrow h_3: \ \vec x = \vec{AB} + \lambda\vec{BS} = \begin{pmatrix}b\\0\end{pmatrix} + \lambda\begin{pmatrix}1/4(r+b)-b\\s/4\end{pmatrix} = \begin{pmatrix}b\\0\end{pmatrix} + \lambda\begin{pmatrix}r/4 - 3b/4\\s/4\end{pmatrix}$$

$$h_4: \ \vec x = t\cdot\vec{AC} = t\cdot\begin{pmatrix}r\\s\end{pmatrix}$$

$$Q = h_3\cap h_4: \ t\cdot\begin{pmatrix}r\\s\end{pmatrix} = \begin{pmatrix}b\\0\end{pmatrix} + \lambda\begin{pmatrix}r/4 - 3b/4\\s/4\end{pmatrix} \Longrightarrow \begin{matrix} t\cdot r = b + \lambda\frac r4 - \lambda\cdot\frac{3b}{4} & I \\ t\cdot s = \lambda\cdot\frac s4 \Rightarrow t = \frac\lambda4 & II\end{matrix}$$

$$I: \ \cancel{tr} = b + \cancel{tr} - 3bt \Longrightarrow t = \tfrac13$$

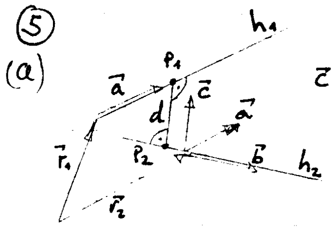
$$\Longrightarrow \vec{AQ} = \tfrac13\vec{AC} \ : \ |\vec{AQ}| : |\vec{QC}| = \tfrac13 : \tfrac23 = 1:2$$

<u>Resultat / résultat:</u> $|\vec{AQ}| : |\vec{QC}| = 1:2$, $\underline{\underline{x = 2}}$

-3-

(5)

(a)



$\vec{c} = \vec{a} \times \vec{b}$

$$\left.\begin{array}{l}\vec{c} \perp \vec{a} \Rightarrow \perp h_1 \\ \vec{c} \perp \vec{b} \Rightarrow \perp h_2\end{array}\right\} \Rightarrow \vec{c} \parallel d$$

$$\vec{c} = \begin{pmatrix}1\\1\\1\end{pmatrix} \times \begin{pmatrix}-5\\-4\\6\end{pmatrix} = \begin{pmatrix}10\\-11\\1\end{pmatrix}$$

$$\phi_1 = (h_1, d) \quad , \quad \phi_2 = (h_2, d) \quad \left\{\begin{array}{l} P_1 = \phi_2 \cap h_1 \\ P_2 = \phi_1 \cap h_2\end{array}\right.$$

$$\left.\begin{array}{l}\phi_1: \quad \vec{r}_{\phi_1} = \begin{pmatrix}2\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\1\end{pmatrix} + \mu\begin{pmatrix}10\\-11\\1\end{pmatrix} \quad (= \vec{r}_1 + \lambda\vec{a} + \mu\vec{c}) \\ h_2: \quad \vec{r}_{h_2} = \begin{pmatrix}2\\2\\2\end{pmatrix} + t\begin{pmatrix}-5\\-4\\+6\end{pmatrix}\end{array}\right\} P_2: \quad \vec{r}_{\phi_1} = \vec{r}_{h_2}$$

$$\Rightarrow \left.\begin{array}{l} 2 + \lambda + 10\mu = 2 - 5t \\ 1 + \lambda - 11\mu = 2 - 4t \\ 0 + \lambda + \mu = 2 + 6t \end{array}\right] \Rightarrow \left.\begin{array}{l} 1 + 21\mu = -t \\ 1 - 12\mu = -10t\end{array}\right\} \Rightarrow \begin{array}{l} 12 + 21 = -12t - 210t \\ 33 = -222t \end{array}$$

$$\Rightarrow t = -\frac{33}{222} \approx -0.1486487\ldots$$

$$\Rightarrow \vec{OP}_2 = \begin{pmatrix}2\\2\\2\end{pmatrix} - 0.1486487\ldots\begin{pmatrix}-5\\-4\\+6\end{pmatrix} = \begin{pmatrix}2.7432432\ldots\\2.5945946\ldots\\1.1081081\ldots\end{pmatrix}$$

$$\vec{OP}_2 + \lambda \cdot \vec{c} = \vec{OP}_1 = \vec{r}_1 + t \cdot \vec{a} \Rightarrow \left.\begin{array}{l} 2.743\ldots + \lambda \cdot 10 = 2 + t \cdot 1 \\ 2.594\ldots + \lambda(-11) = 1 + t \cdot 1 \\ 1.1081\ldots + \lambda \cdot 1 = 0 + t \end{array}\right\} \begin{array}{l}\text{Syst. l. a.} \\ \text{Syst. lin. dép.}\end{array}$$

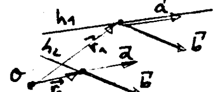
$$\Rightarrow \text{II} - \text{III}: \quad 1.486486\ldots - 12\lambda = 1 \Rightarrow \lambda = 0.04040\ldots, \quad t = 1.1485135\ldots$$

$$\Rightarrow \vec{OP}_1 = \vec{r}_1 + t\vec{a} = \begin{pmatrix}2\\1\\0\end{pmatrix} + 1.1485135 \cdot \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}3.1485135\\2.1485135\\1.1485135\end{pmatrix}$$

$$d = |\vec{OP}_2 - \vec{OP}_1| = \sqrt{(2.7432432 - 3.1485135)^2 + (2.5945946 - 2.1485135)^2 + (1.108108\ldots - 1.148513\ldots)^2}$$

$$= \underline{\underline{0.60404\ldots}}$$

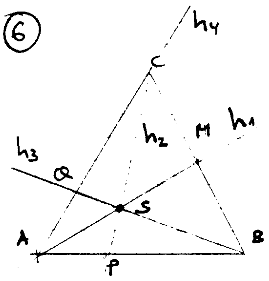
(b) Einfacher / plus simple:



$$d = \frac{V}{F} = \frac{V(\vec{a}, \vec{b}, \vec{r}_2 - \vec{r}_1)}{F(\vec{a}, \vec{b})} = \frac{|[\vec{a}, \vec{b}, \vec{r}_2 - \vec{r}_1)|}{|\vec{a} \times \vec{b}|}$$

$$= \underline{\underline{0.60404\ldots}}$$

(6)



$$A = O \qquad \vec{OM} = \vec{AB} + \frac{1}{2}\vec{BC} = \begin{pmatrix}b\\0\end{pmatrix} + \frac{1}{2}\begin{pmatrix}r-b\\s\end{pmatrix} \qquad h_1: \ \vec{x} = \lambda\,\vec{OM} = \lambda\begin{pmatrix}(r+b)/2\\s/2\end{pmatrix} = \lambda'\begin{pmatrix}r+b\\s\end{pmatrix}$$

$$|\vec{AP}| : |\vec{PB}| = 1:2 \Rightarrow \vec{OP} = \frac{1}{3}\vec{AB} = \begin{pmatrix}b/3\\0\end{pmatrix}$$

$$h_2: \quad \vec{x} = \vec{OP} + t \cdot \vec{PC} = \begin{pmatrix}b/3\\0\end{pmatrix} + t \cdot \begin{pmatrix}r - b/3\\s\end{pmatrix}$$

$$S = h_1 \cap h_2: \quad \lambda'\begin{pmatrix}r+b\\s\end{pmatrix} = \begin{pmatrix}b/3\\0\end{pmatrix} + t\begin{pmatrix}r - b/3\\s\end{pmatrix} \Bigg\} \Rightarrow \lambda' = t \quad (\text{II})$$

$$\Rightarrow \text{I}: \ t \cdot (r+b) - t\left(r - \frac{b}{3}\right) = t\left(\frac{4b}{3}\right) = \frac{b}{3} \Rightarrow t = 1/4$$

$$\Rightarrow \vec{OS} = \frac{1}{4}\begin{pmatrix}r+b\\s\end{pmatrix}$$

$$\Rightarrow h_3: \quad \vec{r} = \vec{OB} + \lambda\,\vec{BS} = \begin{pmatrix}b\\0\end{pmatrix} + \lambda\begin{pmatrix}\frac{1}{4}(r+b) - b\\ \frac{s}{4}\end{pmatrix} = \begin{pmatrix}b\\0\end{pmatrix} + \lambda\begin{pmatrix}\frac{r}{4} - \frac{b \cdot 3}{4}\\ \frac{s}{4}\end{pmatrix}$$

$$h_4: \quad \vec{r} = t \cdot \vec{OC} = t\begin{pmatrix}r\\s\end{pmatrix}$$

$$Q = h_3 \cap h_4: \quad t\begin{pmatrix}r\\s\end{pmatrix} = \begin{pmatrix}b\\0\end{pmatrix} + \lambda\begin{pmatrix}\frac{r}{4} - \frac{b \cdot 3}{4}\\ \frac{s}{4}\end{pmatrix} \Rightarrow \begin{array}{|l} \text{I}: \quad t \cdot r = b + \lambda\frac{r}{4} - \lambda\frac{b \cdot 3}{4} \\ \text{II}: \quad t \cdot s = \lambda \cdot \frac{s}{4} \Rightarrow t = \frac{\lambda}{4}\end{array}$$

$$\text{I} \Rightarrow \quad t \cdot r = b + 4t\,\frac{r}{4} - 4t \cdot \frac{b \cdot 3}{4} \Rightarrow t(r - r + 3b) = b \Rightarrow t = \frac{1}{3}$$

$$\Rightarrow\Rightarrow \quad \vec{OQ} = \frac{1}{3} \cdot \vec{OC} \qquad |\vec{AQ}| : |\vec{QC}| = \frac{1}{3} : \frac{2}{3} = 1:2$$

Resultat / résultat: $\underline{\underline{|\vec{AQ}| : |\vec{QC}| = 1:2 \quad , \quad x = 2}}$

s 4 Serie 4 •: *Série 4*

Lösungen

① $\vec{w} = \begin{pmatrix}5\\-12\\6\end{pmatrix} = \lambda\vec{x} + \mu\vec{y} + \nu\vec{z} = \lambda\begin{pmatrix}0\\-2\\4\end{pmatrix} + \mu\begin{pmatrix}1\\-2\\2\end{pmatrix} + \nu\begin{pmatrix}1\\2\\4\end{pmatrix}$

$$\begin{aligned} 5 &= \lambda\cdot 0 + \mu\cdot 1 + \nu\cdot 1 \\ -12 &= \lambda(-2) + \mu(-2) + \nu\cdot 2 \\ 6 &= \lambda\cdot 4 + \mu\cdot 2 + \nu\cdot 4 \end{aligned}$$

Rechner oder von Hand: $\lambda = -\frac{3}{5} = -0.6$, $\mu = \frac{29}{5} = 5{,}8$, $\nu = -\frac{4}{5} = -0{,}8$

② $\vec{AB} = t\cdot\vec{AC} \leadsto \begin{pmatrix}-7-5\\-3+6\\+1-2\end{pmatrix} = t\begin{pmatrix}x-5\\5+6\\z-2\end{pmatrix}$ $3 = 11t$, $t = \frac{3}{11}$

$\Rightarrow -12 = \frac{3}{11}(x-5) \Rightarrow x = -39$

$-1 = \frac{3}{11}(z-2) \Rightarrow z = -\frac{5}{3}$

$\Rightarrow \vec{AC} = \begin{pmatrix}-44\\11\\-\frac{11}{3}\end{pmatrix}$

③ $|AQ| = |BQ| = |CQ| \Rightarrow \left|\begin{pmatrix}3-x\\2-y\end{pmatrix}\right|^2 = \left|\begin{pmatrix}-1-x\\4-y\end{pmatrix}\right|^2 = \left|\begin{pmatrix}3-x\\-1-y\end{pmatrix}\right|^2$

$\Rightarrow \sqrt{(3-x)^2+(2-y)^2} = \sqrt{(-1-x)^2+(4-y)^2} = \sqrt{(3-x)^2+(-1-y)^2}$

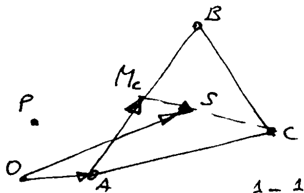
$9-6x+x^2+4-4y+y^2 = 1+2x+x^2+16-8y+y^2 = 9-6x+x^2+1+2y+y^2$ {2 Gleichung 2mal "="

$\Rightarrow$ z.B. $\begin{aligned} 4x-14y+7&=0 \\ -8x+4y-4&=0 \end{aligned}$ $x = -\frac{7}{24}$, $y = \frac{5}{12}$

④ $\vec{OA} = \vec{OP} - \vec{AP} = \begin{pmatrix}0\\1\end{pmatrix} - \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}-3\\-1\end{pmatrix}$

$\vec{OB} = \vec{OP} - \vec{BP} = \begin{pmatrix}0\\1\end{pmatrix} - \begin{pmatrix}-1\\4\end{pmatrix} = \begin{pmatrix}1\\-3\end{pmatrix}$

$\vec{OC} = \vec{OP} - \vec{CP} = \begin{pmatrix}0\\1\end{pmatrix} - \begin{pmatrix}1\\-3\end{pmatrix} = \begin{pmatrix}-1\\4\end{pmatrix}$



$\frac{1}{2} - \frac{1}{3}\cdot\frac{1}{2} = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$

$\vec{OS} = \underbrace{\vec{OA} + \underbrace{\tfrac{1}{2}\vec{AB}}_{\vec{AM_c}}}_{\vec{OM_c}} + \frac{1}{3}\Big(\vec{OC} - \underbrace{(\vec{OA} + \tfrac{1}{2}\vec{AB})}_{\vec{OM_c}}\Big) = \underbrace{\tfrac{1}{3}\vec{OC}}_{\text{r.s.}} + \underbrace{\tfrac{2}{3}\vec{OA}}_{\text{r.s.}} + \tfrac{1}{3}\underbrace{\vec{AB}}_{\vec{OB}-\vec{OA}}$

$= \frac{1}{3}\vec{OC} + \frac{1}{3}\vec{OB} + \frac{1}{3}\vec{OA}$

$\Rightarrow \vec{OS} = \frac{1}{3}(\vec{OA} + \vec{OB} + \vec{OC}) = \frac{1}{3}\begin{pmatrix}-3+1-1\\-1-3+4\end{pmatrix} = \frac{1}{3}\begin{pmatrix}-3\\0\end{pmatrix}$

$\Rightarrow \vec{OS} = \begin{pmatrix}-1\\0\end{pmatrix}$, $S(-1|0)$

s 5 Serie 5 •: *Série 5*

Test Vektoren, B

```
Remove["Global`@*"]
```

Aufgabe 1

Programme

```
(New Kernel) In[1]:=
lenVec[v_]:=Sqrt[Sum[v[[i]]^2,{i,Length[v]}]]

(New Kernel) In[2]:=
cross1[v1_,v2_]:=Det[{{e1,v1[[1]],v2[[1]]},
                      {e2,v1[[2]],v2[[2]]},
                      {e3,v1[[3]],v2[[3]]}}]

(New Kernel) In[3]:=
cross[v1_,v2_]:=Flatten[
               {{v1[[2]] v2[[3]]-v1[[3]] v2[[2]]},
                {v1[[3]] v2[[1]]-v1[[1]] v2[[3]]},
                {v1[[1]] v2[[2]]-v1[[2]] v2[[1]]}}]

(New Kernel) In[13]:=
gamma[v1_,v2_]:=ArcCos[v1.v2/lenVec[v1]/lenVec[v2]]
```

Lösungen

```
(New Kernel) In[5]:=
a={2,-1,3};b={4,5,-2};

(New Kernel) In[6]:=
a.b

(New Kernel) Out[6]=
-3

(New Kernel) In[7]:=
lenVec[a] lenVec[b]

(New Kernel) Out[7]=
3 Sqrt[70]

(New Kernel) In[8]:=
N[%]

(New Kernel) Out[8]=
25.0998
```



```
(New Kernel) In[9]:=
gamma[a,b]

(New Kernel) Out[9]=
               1
ArcCos[-(---------)]
          Sqrt[70]

(New Kernel) In[10]:=
N[%]

(New Kernel) Out[10]=
1.69061

(New Kernel) In[11]:=
cross[a,b]

(New Kernel) Out[11]=
{-13, 16, 14}

(New Kernel) In[12]:=
cross1[a,b]

(New Kernel) Out[12]=
-13 e1 + 16 e2 + 14 e3
```

Aufgabe 2

```
(New Kernel) In[14]:=
a={4,5,-2}

(New Kernel) Out[14]=
{4, 5, -2}

(New Kernel) In[15]:=
p={x,y,z}

(New Kernel) Out[15]=
{x, y, z}

(New Kernel) In[16]:=
ebene[a_,p_,d_]:=a.p+d; ebene[a,p,d]

(New Kernel) Out[16]=
d + 4 x + 5 y - 2 z

(New Kernel) In[17]:=
Solve[ebene[a,{2,-1,3},d]==0,d]

(New Kernel) Out[17]=
{{d -> 3}}

(New Kernel) In[18]:=
HNF[p_]:=ebene[a,p,3]/lenVec[a]; HNF[p]

(New Kernel) Out[18]=
3 + 4 x + 5 y - 2 z
-------------------
     3 Sqrt[5]
```

(New Kernel) In[19]:=
```
HNF[{0,0,0}]
```
(New Kernel) Out[19]=
$$\frac{1}{\text{Sqrt}[5]}$$
(New Kernel) In[20]:=
```
N[%]
```
(New Kernel) Out[20]=
```
0.447214
```
(New Kernel) In[21]:=
```
Solve[HNF[{x,0,0}]==0,x]
```
(New Kernel) Out[21]=
$$\{\{x \to -(\tfrac{3}{4})\}\}$$

Aufgabe 3

a)

(New Kernel) In[22]:=
```
Remove[k,g,sol]
```
(New Kernel) In[23]:=
```
mVec={1,1,1}
```
(New Kernel) Out[23]=
```
{1, 1, 1}
```
(New Kernel) In[24]:=
```
k[v_]:=(lenVec[v-mVec]^2==5^2); k[{x,y,z}]
```
(New Kernel) Out[24]=
$$(-1 + x)^2 + (-1 + y)^2 + (-1 + z)^2 == 25$$
(New Kernel) In[25]:=
```
g[t_]:={10,0,2}+ t {-2,0,0}
```
(New Kernel) In[26]:=
```
sol=Flatten[Solve[k[g[t]],t]]
```
(New Kernel) Out[26]=
$$\{t \to \frac{18 - 2\ \text{Sqrt}[23]}{4},\ t \to \frac{18 + 2\ \text{Sqrt}[23]}{4}\}$$
(New Kernel) In[27]:=
```
N[%]
```
(New Kernel) Out[27]=
```
{t -> 2.10208, t -> 6.89792}
```


(New Kernel) In[28]:=
```
{g[t]/.sol[[1]],g[t]/.sol[[2]]}
```
(New Kernel) Out[28]=
$$\{\{10 + \frac{-18 + 2\ \text{Sqrt}[23]}{2},\ 0,\ 2\},\ \{10 + \frac{-18 - 2\ \text{Sqrt}[23]}{2},\ 0,\ 2\}\}$$
(New Kernel) In[29]:=
```
N[%]
```
(New Kernel) Out[29]=
```
{{5.79583, 0, 2.}, {-3.79583, 0, 2.}}
```
(New Kernel) In[30]:=
```
punkt=g[t]/.sol[[1]]
```
(New Kernel) Out[30]=
$$\{10 + \frac{-18 + 2\ \text{Sqrt}[23]}{2},\ 0,\ 2\}$$
(New Kernel) In[31]:=
```
N[%]
```
(New Kernel) Out[31]=
```
{5.79583, 0, 2.}
```
(New Kernel) In[32]:=
```
k[punkt]//Simplify
```
(New Kernel) Out[32]=
```
True
```

b)

(New Kernel) In[43]:=
```
tgEbene[v_]:=((v-mVec).(punkt-mVec) - 5^2 ==0) ;
tgEbene[{x,y,z}]//Simplify
```
(New Kernel) Out[44]=
```
-25 - Sqrt[23] + Sqrt[23] x - y + z == 0
```
(New Kernel) In[45]:=
```
tgEbene[punkt]//Simplify
```
(New Kernel) Out[45]=
```
True
```
(New Kernel) In[46]:=
```
nVec={-Sqrt[23],-1,1}
```
(New Kernel) Out[46]=
```
{-Sqrt[23], -1, 1}
```

(New Kernel) In[47]:=

```
qVec={10,0,2}
```

(New Kernel) Out[47]=

```
{10, 0, 2}
```

(New Kernel) In[48]:=

```
PQ=punkt-qVec
```

(New Kernel) Out[48]=

```
 -18 + 2 Sqrt[23]
{----------------, 0, 0}
        2
```

(New Kernel) In[49]:=

```
fi= ArcCos[PQ.nVec/lenVec[PQ]/lenVec[nVec]]
```

(New Kernel) Out[49]=

```
        -(Sqrt[23] (-18 + 2 Sqrt[23]))
ArcCos[------------------------------]
                                   2
          5 Sqrt[(-18 + 2 Sqrt[23]) ]
```

(New Kernel) In[50]:=

```
winkel = 2 N[%]
```

(New Kernel) Out[50]=

```
0.573513
```

(New Kernel) In[51]:=

```
winkel 360 /2 /Pi //N
```

(New Kernel) Out[51]=

```
32.8599
```

(New Kernel) In[52]:=

```
%/2
```

(New Kernel) Out[52]=

```
16.4299
```