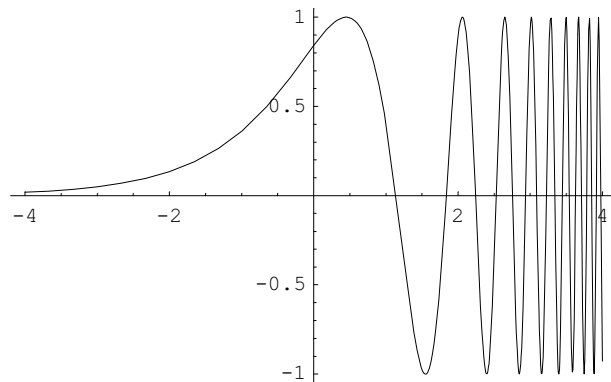


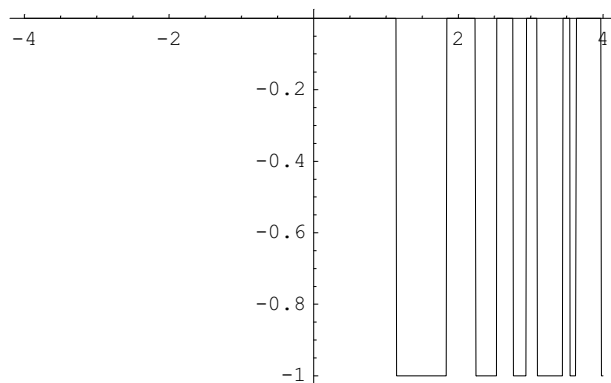
# Studiere die Beispiele!

## ■ 1. Plots ungewohnter Funktionen

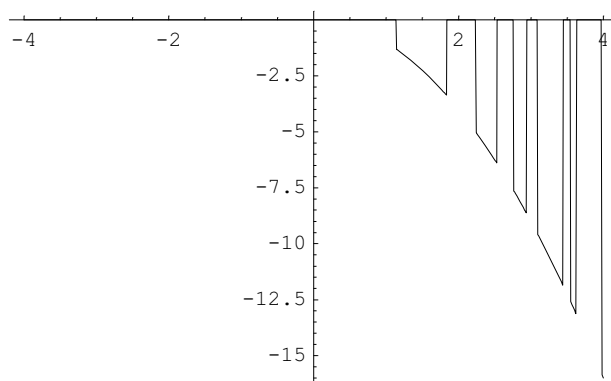
```
f0[x_] := Sin[E^x] ; Plot[f0[x], {x, -4, 4}] ;
```



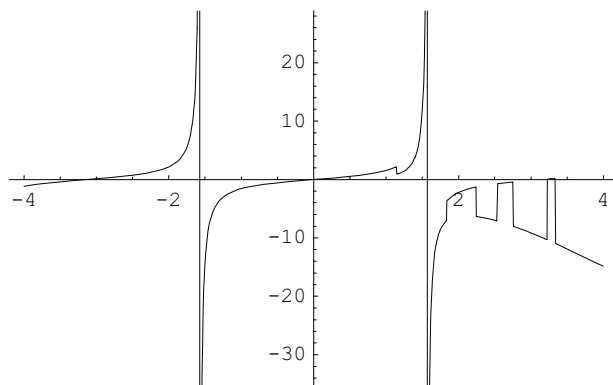
```
f1[x_] := Floor[Sin[E^x]] ; Plot[f1[x], {x, -4, 4}] ;
```



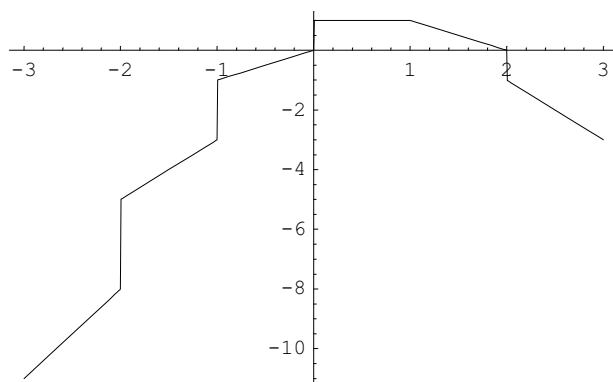
```
f2[x_] := Floor[Sin[E^x]] x^2 ; Plot[f2[x], {x, -4, 4}] ;
```



```
f3[x_] := Floor[Sin[E^x]] x^2 + Tan[x]; Plot[f3[x], {x, -4, 4}];
```

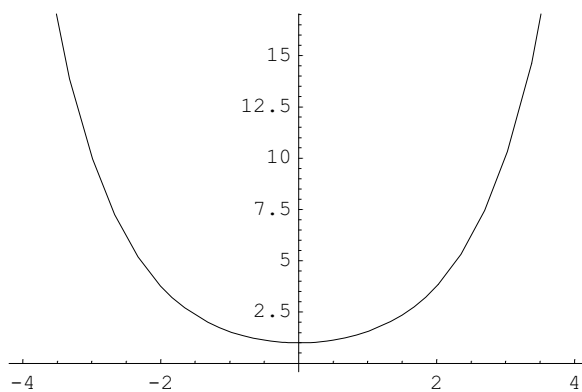


```
Plot[x Floor[1 - x] + Floor[1 + x], {x, -3, 3}];
```



## ■ 2. Umkehrfunktionen

```
g1[x_] := Cosh[x]; Plot[g1[x], {x, -4, 4}];
```

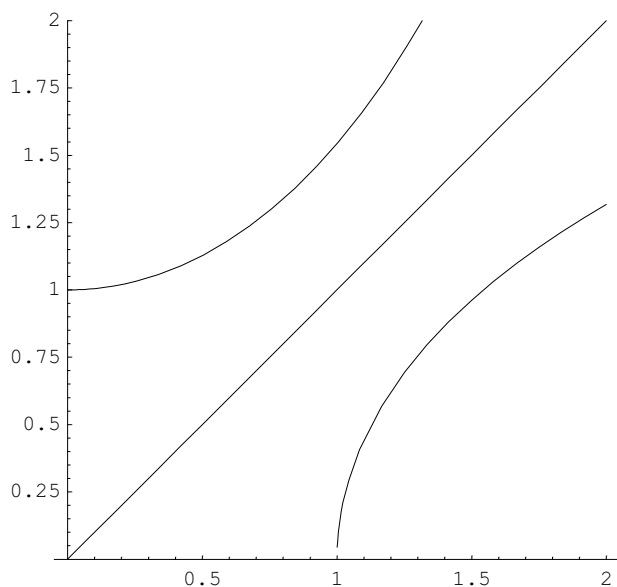


**Options[Plot]**

```
{AspectRatio →  $\frac{1}{\text{GoldenRatio}}$ , Axes → Automatic, AxesLabel → None,
  AxesOrigin → Automatic, AxesStyle → Automatic, Background → Automatic,
  ColorOutput → Automatic, Compiled → True, DefaultColor → Automatic,
  Epilog → {}, Frame → False, FrameLabel → None, FrameStyle → Automatic,
  FrameTicks → Automatic, GridLines → None, ImageSize → Automatic, MaxBend → 10.,
  PlotDivision → 30., PlotLabel → None, PlotPoints → 25, PlotRange → Automatic,
  PlotRegion → Automatic, PlotStyle → Automatic, Prolog → {}, RotateLabel → True,
  Ticks → Automatic, DefaultFont → $DefaultFont, DisplayFunction → $DisplayFunction,
  FormatType → $FormatType, TextStyle → $TextStyle}
```

Plot der Funktion und Umkehrfunktion:

```
h1[x_] := ArcCosh[x];
Plot[{g1[x], x, h1[x]}, {x, 0.001, 2}, AspectRatio → Automatic, PlotRange → {0, 2}];
```

**? Log**

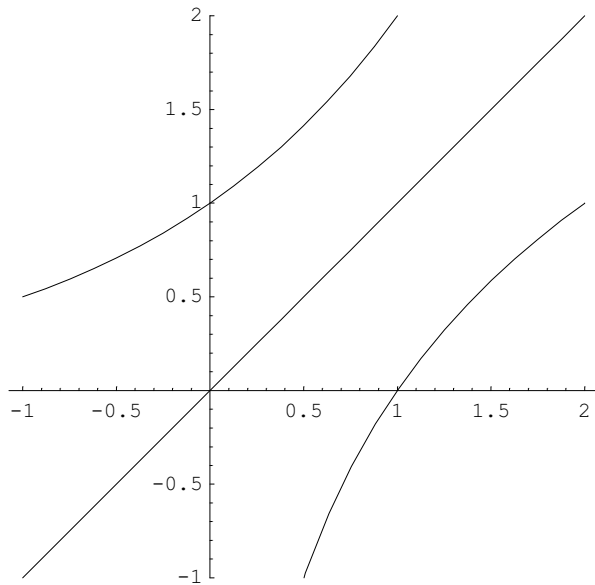
Log[z] gives the natural logarithm of z (logarithm to base e). Log[b, z] gives the logarithm to base b.

Plot der Funktion und Umkehrfunktion:

```

g2[x_] := 2^x;
h2[x_] := Log[2, x];
Plot[{g2[x], x, h2[x]}, {x, -1, 2}, AspectRatio -> Automatic, PlotRange -> {-1, 2}];

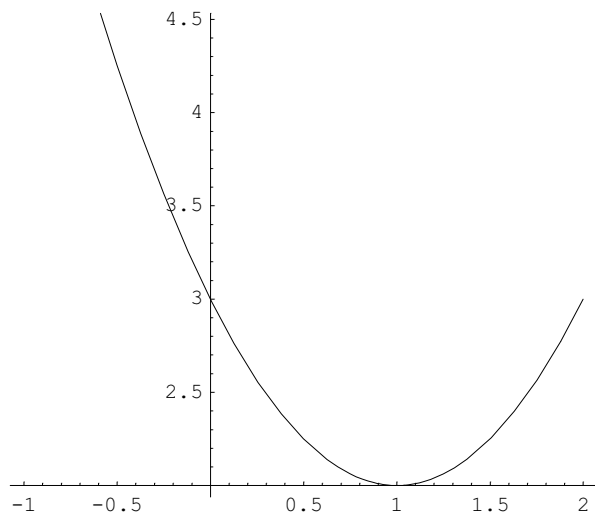
```



```

g3[x_] := x^2 - 2 x + 3;
Plot[{g3[x]}, {x, -1, 2}, AspectRatio -> Automatic];

```



```
Solve[g3[x] - 2 == 0, {x}]
```

```
{x -> 1}, {x -> 1}}
```

====> Minimum in 1

```
Remove[x, y, s]
```

```
Solve[g3[x] == y, {x}]
```

```
{{x -> 1 - Sqrt[-2 + y]}, {x -> 1 + Sqrt[-2 + y]}}
```

```
s = Solve[g3[x] == y, {x}] // Flatten
```

```
{x -> 1 - Sqrt[-2 + y], x -> 1 + Sqrt[-2 + y]}
```

```
x = x /. s[[2]]
```

```
1 +  $\sqrt{-2 + y}$ 
```

```
h3[y_] = x /. s ; h3[z]
```

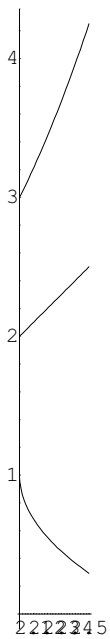
```
1 -  $\sqrt{-2 + z}$ 
```

```
h3[q]
```

```
1 -  $\sqrt{-2 + q}$ 
```

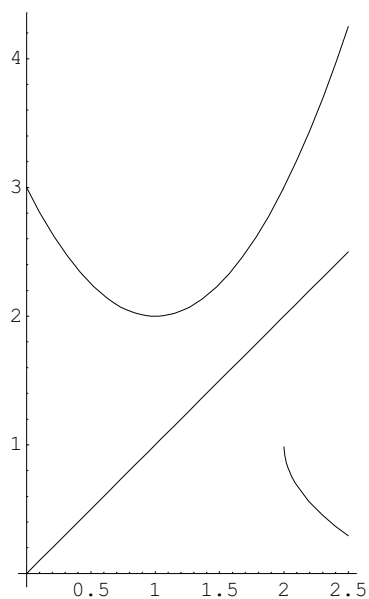
Plot der Funktion und Umkehrfunktion (Skalierung!):

```
p = Plot[{g3[x], x, h3[x]}, {x, 2, 2.5}, AspectRatio -> Automatic];
```



Achtung Skalierungen!

```
ShowPlot[{p, Plot[{g3[x], x, h3[x]}, {x, 0, 2.5}, AspectRatio -> Automatic];}]
```



```
h[w_] := (h1[w] - h2[w]) / h3[w];
h[w]
```

$$\frac{\text{ArcCosh}[w] - \frac{\text{Log}[w]}{\text{Log}[2]}}{1 - \sqrt{-2 + w}}$$

Wo ist h[w] definiert?

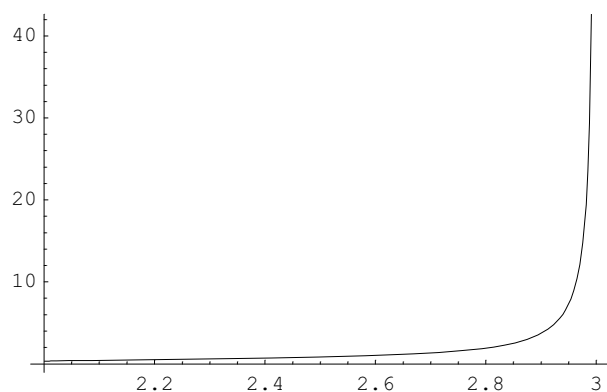
```
Plot[h[w], {w, -1, 3}];
```

```
Plot::plnr : h[w] is not a machine-size real number at w = -1..
```

```
Plot::plnr : h[w] is not a machine-size real number at w = -0.837732.
```

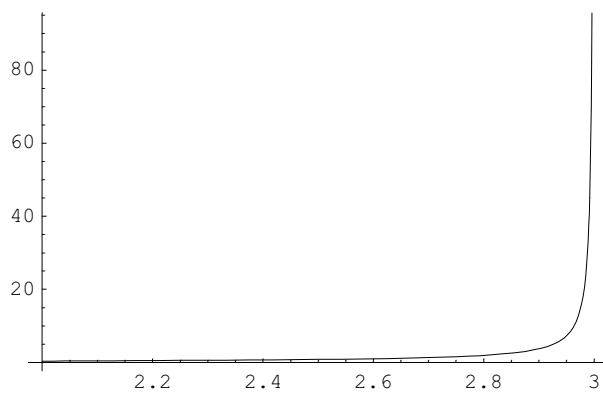
```
Plot::plnr : h[w] is not a machine-size real number at w = -0.660765.
```

```
General::stop : Further output of Plot::plnr will be suppressed during this calculation.
```



Geht nur für w grösser gleich 2. Wieso?

```
Plot[h[w], {w, 2, 3}];
```



### ■ 3. Asymtote, Pole, Nullstellen, Definitionsbereich, ungefährender Wertebereich

```
Remove[x]
```

```
a[x_] := (x^3 - 2 x^2 + 4 x - 5) / (x^2 + 8 x - 6);
```

```
a[x]
```

$$\frac{-5 + 4x - 2x^2 + x^3}{-6 + 8x + x^2}$$

Division:

```
Apart[a[x]]
```

$$-10 + x + \frac{5(-13 + 18x)}{-6 + 8x + x^2}$$

#### ■ Pole:

```
Solve[-6 + 8 x + x^2 == 0, {x}]
```

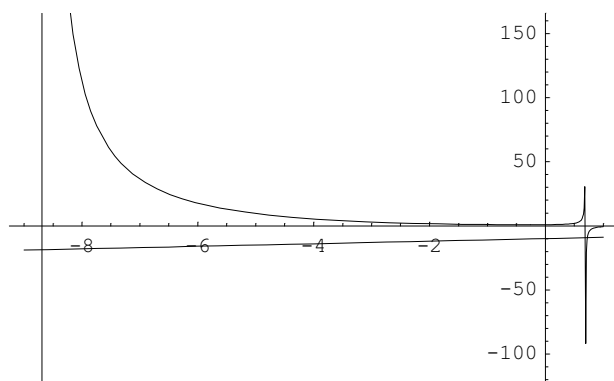
$$\left\{ \left\{ x \rightarrow -4 - \sqrt{22} \right\}, \left\{ x \rightarrow -4 + \sqrt{22} \right\} \right\}$$

```
Solve[-6 + 8 x + x^2 == 0, {x}] // N
```

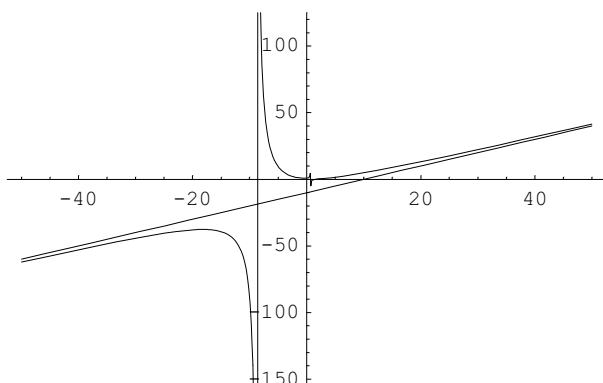
$$\left\{ \left\{ x \rightarrow -8.69042 \right\}, \left\{ x \rightarrow 0.690416 \right\} \right\}$$

### ■ Asymptote:

```
Plot[{-10 + x, a[x]}, {x, -9, 1};
```



```
Plot[{-10 + x, a[x]}, {x, -50, 50};
```



### ■ Nullstellen:

$$\frac{-5 + 4x - 2x^2 + x^3}{-6 + 8x + x^2}$$

```
Solve[ $\frac{-5 + 4x - 2x^2 + x^3}{-6 + 8x + x^2} = 0, \{x\}$ ]
```

$$\left\{ \left\{ x \rightarrow \frac{2}{3} - \frac{8}{3} \left( \frac{2}{79 + 3\sqrt{921}} \right)^{1/3} + \frac{1}{3} \left( \frac{1}{2} (79 + 3\sqrt{921}) \right)^{1/3} \right\}, \right.$$

$$\left\{ x \rightarrow \frac{2}{3} + \frac{4}{3} (1 + i\sqrt{3}) \left( \frac{2}{79 + 3\sqrt{921}} \right)^{1/3} - \frac{1}{6} (1 - i\sqrt{3}) \left( \frac{1}{2} (79 + 3\sqrt{921}) \right)^{1/3} \right\},$$

$$\left\{ x \rightarrow \frac{2}{3} + \frac{4}{3} (1 - i\sqrt{3}) \left( \frac{2}{79 + 3\sqrt{921}} \right)^{1/3} - \frac{1}{6} (1 + i\sqrt{3}) \left( \frac{1}{2} (79 + 3\sqrt{921}) \right)^{1/3} \right\}$$

```
Solve[ $\frac{-5 + 4x - 2x^2 + x^3}{-6 + 8x + x^2} = 0, \{x\}$ ] // N
```

$$\{ \{x \rightarrow 1.52596\}, \{x \rightarrow 0.237021 + 1.79456 i\}, \{x \rightarrow 0.237021 - 1.79456 i\} \}$$

```
Solve[-5 + 4x - 2x^2 + x^3 = 0, {x}] // N
```

$$\{ \{x \rightarrow 1.52596\}, \{x \rightarrow 0.237021 + 1.79456 i\}, \{x \rightarrow 0.237021 - 1.79456 i\} \}$$



Einzig reelle Nullstelle: Etwa 1.525957480649296

### ■ Definitionsbereich

Reelle Zahlen ohne  $x = -4 - \sqrt{22}$ ,  $x = -4 + \sqrt{22}$

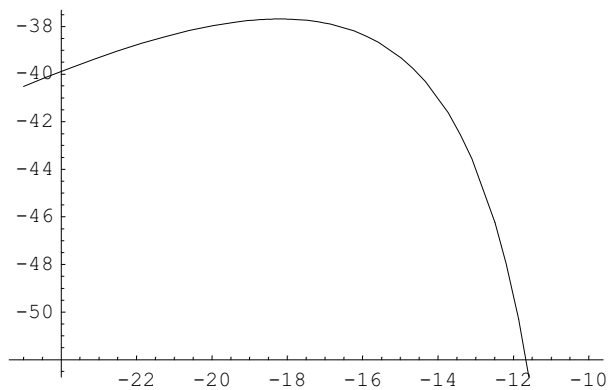
### ■ Wertebereich

$$\text{Solve}\left[y == \frac{-5 + 4x - 2x^2 + x^3}{-6 + 8x + x^2}, \{x\}\right]$$

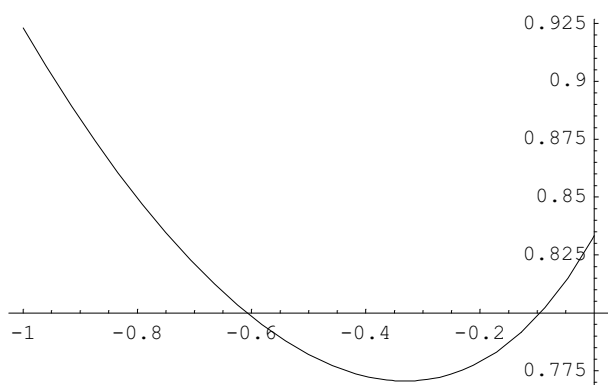
{{}}

x kann nicht als Funktion von y berechnet werden. Graphisch kommt man weiter:

`Plot[a[x], {x, -25, -10}];`



`Plot[a[x], {x, -1, 0}];`



Zwischen ca.  $y = -38$  und  $y = +0.77$  gibt es keine Werte.

## ■ 4. Zusammengesetzte Funktionen

`Remove[u]`

```

u[t_] := ArcCos[t] /; (0 < t && t < Pi/2);
u[t_] := Cos[t] /; (t ≤ 0);
u[t_] := E^t /; (+Pi/2 ≤ t);

```

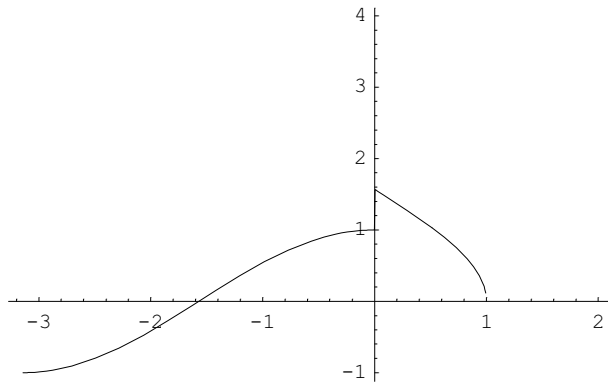
```
Plot[u[x], {x, -Pi, 2}];
```

Plot::plnr : u[x] is not a machine-size real number at x = 1.1477184121076678`.

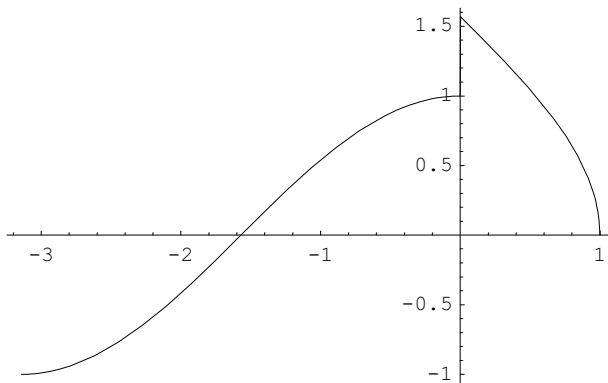
Plot::plnr : u[x] is not a machine-size real number at x = 1.0471985871192238`.

Plot::plnr : u[x] is not a machine-size real number at x = 1.0193749626697324`.

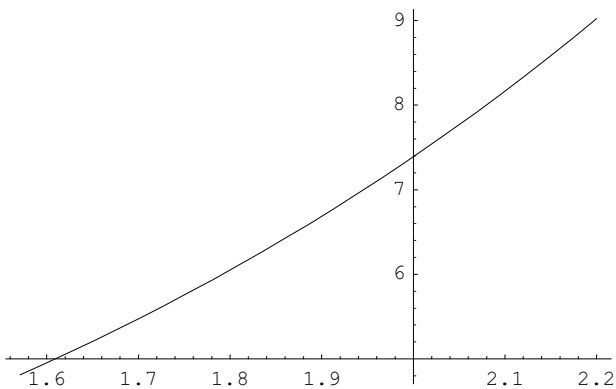
General::stop : Further output of Plot::plnr will be suppressed during this calculation.



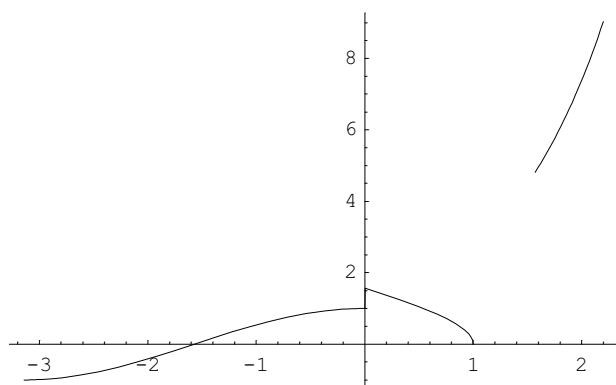
```
p1 = Plot[u[x], {x, -Pi, 1}];
```



```
p2 = Plot[u[x], {x, Pi/2, 2.2}];
```



```
Show[p1, p2];
```

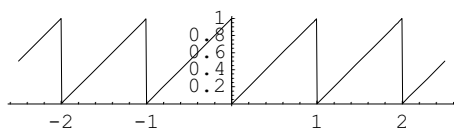


## ■ 5. Rechtecksfunktion:

```
Remove[f, p]
```

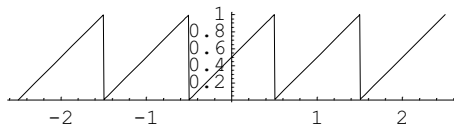
```
f[1, x_] := x - Floor[x];
```

```
p[1] = Plot[f[1, x], {x, -2.5, 2.5}, AspectRatio -> 1/5];
```



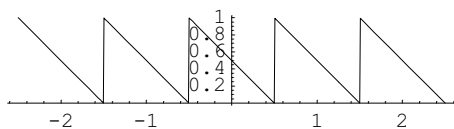
```
f[2, x_] := f[1, x - 0.5];
```

```
p[2] = Plot[f[2, x], {x, -2.5, 2.5}, AspectRatio -> 1/5];
```



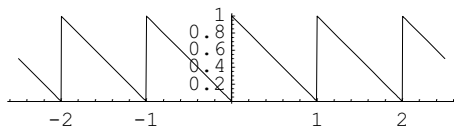
```
f[3, x_] := f[2, -x];
```

```
p[3] = Plot[f[3, x], {x, -2.5, 2.5}, AspectRatio -> 1/5];
```

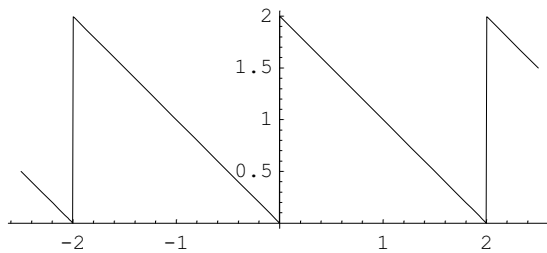


```
f[4, x_] := f[3, x + 0.5];
```

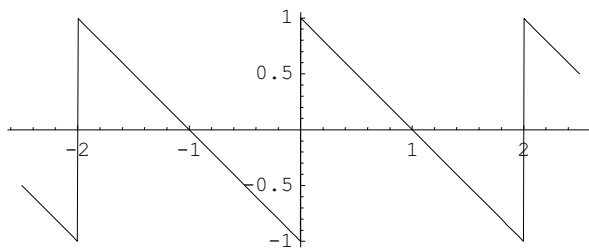
```
p[4] = Plot[f[4, x], {x, -2.5, 2.5}, AspectRatio -> 1/5];
```



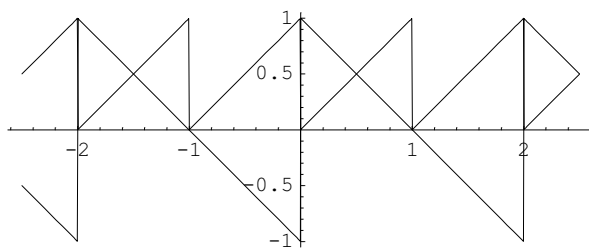
```
f[5, x_] := 2 f[4, 1/2 x];
p[5] = Plot[f[5, x], {x, -2.5, 2.5}, AspectRatio -> 2/5];
```



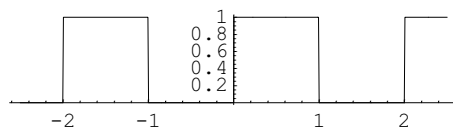
```
f[6, x_] := f[5, x] - 1;
p[6] = Plot[f[6, x], {x, -2.5, 2.5}, AspectRatio -> 2/5];
```



```
Show[p[6], p[1]];
```



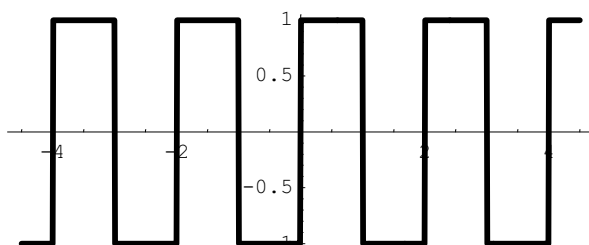
```
f[7, x_] := f[1, x] + f[6, x]; p[7] = Plot[f[7, x], {x, -2.5, 2.5}, AspectRatio -> 1/5];
```



```
f[8, x_] := 2 f[7, x] - 1;
```

```
p[8] =
```

```
Plot[f[8, x], {x, -4.5, 4.5}, PlotStyle -> Thickness[0.01], AspectRatio -> 2/5];
```



## ■ 6. Quadratische Ungleichungen

Wo ist  $-8 - 5x + 2x^2 \geq 5 + x$  ?

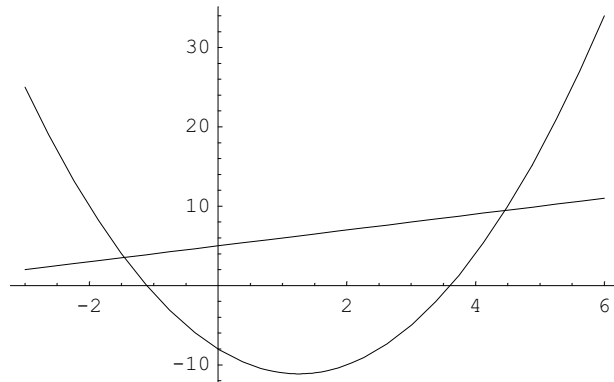
```
Solve[2 x^2 - 5 x - 8 == x + 5, {x}]
```

```
{{x -> 1/2 (3 - sqrt(35))}, {x -> 1/2 (3 + sqrt(35))}}
```

```
Solve[2 x^2 - 5 x - 8 == x + 5, {x}] // N
```

```
{{x -> -1.45804}, {x -> 4.45804}}
```

```
Plot[{2 x^2 - 5 x - 8, x + 5}, {x, -3, 6}];
```



```
2 x^2 - 5 x - 8 >= x + 5 /. x -> 0
```

```
False
```

```
2 x^2 - 5 x - 8 >= x + 5 /. x -> -2
```

```
True
```

```
2 x^2 - 5 x - 8 >= x + 5 /. x -> 5
```

```
True
```