

Lösungen mit Exkursen

1

a

1

```
Remove[a]
```

```
A = {{0,a,-1},{0,2,2},{3,1,2}};A//MatrixForm
```

$$\begin{pmatrix} 0 & a & -1 \\ 0 & 2 & 2 \\ 3 & 1 & 2 \end{pmatrix}$$

```
A//TeXForm;
```

```
Det[A]
```

6 + 6 a

```
B = {{1,0,1},{1,1,0},{0,1,1}};B//MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

```
B//TeXForm;
```

```
Det[B]
```

2

```
M = {{3,2,2},{1,1,2},{4,3,4}};M//MatrixForm
```

$$\begin{pmatrix} 3 & 2 & 2 \\ 1 & 1 & 2 \\ 4 & 3 & 4 \end{pmatrix}$$

```
M//TeXForm;
```

```
Det[M]
```

0

2

Det[X] = 0

3

Det[A]

$$6 + 6 a$$

Det[A]/.a->-1

$$0$$

b

A.X.B == M

$$\{\{0, a, -1\}, \{0, 2, 2\}, \{3, 1, 2\}\} \cdot X \cdot \{\{1, 0, 1\}, \{1, 1, 0\}, \{0, 1, 1\}\} = \{\{3, 2, 2\}, \{1, 1, 2\}, \{4, 3, 4\}\}$$

X = Inverse[A].M.Inverse[B]//Simplify//MatrixForm

$$\begin{pmatrix} \frac{7+3a}{6+6a} & \frac{2+a}{2+2a} & \frac{3+a}{6+6a} \\ \frac{2}{1+a} & \frac{3}{2+2a} & \frac{1}{1+a} \\ \frac{-3+a}{2(1+a)} & -\frac{3}{2+2a} & \frac{-1+a}{2(1+a)} \end{pmatrix}$$

X = (Inverse[A].M.Inverse[B]/.a->0)//Simplify//MatrixForm

$$\begin{pmatrix} \frac{7}{6} & 1 & \frac{1}{2} \\ 2 & \frac{3}{2} & 1 \\ -\frac{3}{2} & -\frac{3}{2} & -\frac{1}{2} \end{pmatrix}$$

X = (Inverse[A].M.Inverse[B]/.a->3)//Simplify//MatrixForm

$$\begin{pmatrix} \frac{2}{3} & \frac{5}{8} & \frac{1}{4} \\ \frac{1}{2} & \frac{3}{8} & \frac{1}{4} \\ 0 & -\frac{3}{8} & \frac{1}{4} \end{pmatrix}$$

%//N

$$\{\{0.666667, 0.625, 0.25\}, \{0.5, 0.375, 0.25\}, \{0., -0.375, 0.25\}\}$$

c

A = A/.a->-1; A//MatrixForm

$$\begin{pmatrix} 0 & -1 & -1 \\ 0 & 2 & 2 \\ 3 & 1 & 2 \end{pmatrix}$$

X = {{x11,x12,x13},{x21,x22,x23},{x31,x32,x33}}; X//MatrixForm

$$\begin{pmatrix} x11 & x12 & x13 \\ x21 & x22 & x23 \\ x31 & x32 & x33 \end{pmatrix}$$

A.X//MatrixForm

$$\begin{pmatrix} -x_{21} - x_{31} & -x_{22} - x_{32} & -x_{23} - x_{33} \\ 2 x_{21} + 2 x_{31} & 2 x_{22} + 2 x_{32} & 2 x_{23} + 2 x_{33} \\ 3 x_{11} + x_{21} + 2 x_{31} & 3 x_{12} + x_{22} + 2 x_{32} & 3 x_{13} + x_{23} + 2 x_{33} \end{pmatrix}$$

M.Inverse[B]//MatrixForm

$$\begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{1}{2} \\ 1 & 0 & 1 \\ \frac{5}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$$

(A.X//MatrixForm) == (M.Inverse[B]//MatrixForm)

$$\begin{pmatrix} -x_{21} - x_{31} & -x_{22} - x_{32} & -x_{23} - x_{33} \\ 2 x_{21} + 2 x_{31} & 2 x_{22} + 2 x_{32} & 2 x_{23} + 2 x_{33} \\ 3 x_{11} + x_{21} + 2 x_{31} & 3 x_{12} + x_{22} + 2 x_{32} & 3 x_{13} + x_{23} + 2 x_{33} \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{1}{2} \\ 1 & 0 & 1 \\ \frac{5}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$$

Anhang

Transpose[X][[1]]

{x11, x21, x31}

RowReduce[A.X]//MatrixForm

$$\begin{pmatrix} 1 & 0 & \frac{-3 x_{13} x_{22} + 3 x_{12} x_{23} - 3 x_{13} x_{32} + x_{23} x_{32} + 3 x_{12} x_{33} - x_{22} x_{33}}{3 x_{12} x_{21} - 3 x_{11} x_{22} + 3 x_{12} x_{31} - x_{22} x_{31} - 3 x_{11} x_{32} + x_{21} x_{32}} \\ 0 & 1 & \frac{3 x_{13} x_{21} - 3 x_{11} x_{23} + 3 x_{13} x_{31} - x_{23} x_{31} - 3 x_{11} x_{33} + x_{21} x_{33}}{3 x_{12} x_{21} - 3 x_{11} x_{22} + 3 x_{12} x_{31} - x_{22} x_{31} - 3 x_{11} x_{32} + x_{21} x_{32}} \\ 0 & 0 & 0 \end{pmatrix}$$

RowReduce[M.Inverse[B]//MatrixForm

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -\frac{2}{3} \\ 0 & 0 & 0 \end{pmatrix}$$

LinearSolve[{{1,0,0},{0,1,0},{0,0,1}},{1,2,3}]

{1, 2, 3}

LinearSolve[{{1,0,0},{0,1,0},{0,0,0}},{1,2,3}]

LinearSolve::nosol : Linear equation encountered which has no solution. Mehr...

LinearSolve[{{1, 0, 0}, {0, 1, 0}, {0, 0, 0}}, {1, 2, 3}]

LinearSolve[A,Transpose[M.Inverse[B]][[1]]]

LinearSolve::nosol : Linear equation encountered which has no solution. Mehr...

LinearSolve[{{0, -1, -1}, {0, 2, 2}, {3, 1, 2}}, { $\frac{3}{2}$, 1, $\frac{5}{2}$ }]**LinearSolve[A,Transpose[M.Inverse[B]][[2]]]**

LinearSolve::nosol : Linear equation encountered which has no solution. Mehr...

LinearSolve[{{0, -1, -1}, {0, 2, 2}, {3, 1, 2}}, { $\frac{3}{2}$, 0, $\frac{3}{2}$ }]

```
LinearSolve[A,Transpose[M.Inverse[B]][[3]]]
```

LinearSolve::nosol : Linear equation encountered which has no solution. Mehr...

```
LinearSolve[{{0, -1, -1}, {0, 2, 2}, {3, 1, 2}}, {1/2, 1, 3/2}]
```

2

```
Remove["Global`*"]
```

a Gleichmässig verteilte Streckenlast plus Gegenkraft am Balkenende, allgemeine Gleichung

```
m[x_]:= -q1 (zL-x)^2 + F1 (zL-x); m[x]
```

```
F1 (-x + zL) - q1 (-x + zL)^2
```

```
s[x_] := DSolve[y''[x] == -m[x] / zE / zI, y[x], x] // Flatten; s[x]
```

```
{y[x] -> -\frac{q1 x^4}{2} + x^3 (F1 - 2 q1 zL) - 3 x^2 zL (F1 - q1 zL)} + C[1] + x C[2]}
```

```
s[z]
```

```
{y[z] -> -\frac{q1 z^4}{2} + z^3 (F1 - 2 q1 zL) - 3 z^2 zL (F1 - q1 zL)} + C[1] + z C[2]}
```

```
s[z][[1]][[2]] // FullForm
```

```
Plus[Times[Rational[1, 6], Power[zE, -1],
  Power[zI, -1], Plus[Times[Rational[1, 2], q1, Power[z, 4]],
  Times[Power[z, 3], Plus[F1, Times[-2, q1, zL]]],
  Times[-3, Power[z, 2], zL, Plus[F1, Times[-1, q1, zL]]]], C[1], Times[z, C[2]]]
```

```
s[z][[1]][[2]] /. {C[1]->C1, C[2]->C2}
```

```
C1 + C2 z + \frac{q1 z^4}{2} + z^3 (F1 - 2 q1 zL) - 3 z^2 zL (F1 - q1 zL)}{6 zE zI}
```

```
h[z_]:=s[z][[1]][[2]] /. {C[1]->C1, C[2]->C2} ; h[z]
```

```
C1 + C2 z + \frac{q1 z^4}{2} + z^3 (F1 - 2 q1 zL) - 3 z^2 zL (F1 - q1 zL)}{6 zE zI}
```

```
h[3]
```

DSolve::dsvar : 3 cannot be used as a variable. Mehr...

DSolve::dsvar : 3 cannot be used as a variable. Mehr...

```
\frac{-F1 (-3 + zL) + q1 (-3 + zL)^2}{zE zI}
```

```
h[z]/.z->3
```

```
C1 + 3 C2 + \frac{81 q1}{2} + 27 (F1 - 2 q1 zL) - 27 zL (F1 - q1 zL)}{6 zE zI}
```

```

h1[u_]:=h[z]/.z->u; h1[3]

C1 + 3 C2 +  $\frac{81 q_1}{2} + 27 (F1 - 2 q_1 zL) - 27 zL (F1 - q_1 zL)$ 
              6 zE zI

solv1=Solve[{h1[0]==0,Evaluate[h1'[u]==0]/.u->0},{C1,C2}]/Simplify//Flatten

{C1 -> 0, C2 -> 0}

h2[u_]:=h1[u]/.solv1//Simplify; h2[t]

 $\frac{t^2 (2 F1 (t - 3 zL) + q_1 (t^2 - 4 t zL + 6 zL^2))}{12 zE zI}$ 

Collect[h2[t],{t,t^2,t^3,t^4}]

 $\frac{q_1 t^4}{12 zE zI} + \frac{t^3 (2 F1 - 4 q_1 zL)}{12 zE zI} + \frac{t^2 (-6 F1 zL + 6 q_1 zL^2)}{12 zE zI}$ 

Collect[h2[t],{t,t^2,t^3,t^4}]/Together

 $\frac{2 F1 t^3 + q_1 t^4 - 6 F1 t^2 zL - 4 q_1 t^3 zL + 6 q_1 t^2 zL^2}{12 zE zI}$ 

(Collect[h2[t],{t,t^2,t^3,t^4}] /. q1-> q/2) //Together

 $\frac{4 F1 t^3 + q t^4 - 12 F1 t^2 zL - 4 q t^3 zL + 6 q t^2 zL^2}{24 zE zI}$ 

```

Anhang

```

h2[u_]:=h1[u]/.solv1//Simplify;
h3[t_]=InputForm[h2[u]]/.u->t; {h2[w],h3[w]}

{  $\frac{w^2 (2 F1 (w - 3 zL) + q_1 (w^2 - 4 w zL + 6 zL^2))}{12 zE zI}$  ,
  (w^2 * (2 * F1 * (w - 3 * zL) + q1 * (w^2 - 4 * w * zL + 6 * zL^2))) / (12 * zE * zI) }

h3[t_]=h2[u]/.u->t; {h2[w],h3[w]}

{  $\frac{w^2 (2 F1 (w - 3 zL) + q_1 (w^2 - 4 w zL + 6 zL^2))}{12 zE zI}$  ,  $\frac{w^2 (2 F1 (w - 3 zL) + q_1 (w^2 - 4 w zL + 6 zL^2))}{12 zE zI}$  }

FullForm[h3[t]]

Times[Rational[1, 12], Power[t, 2], Power[zE, -1],
  Power[zI, -1], Plus[Times[2, F1, Plus[t, Times[-3, zL]]],
  Times[q1, Plus[Power[t, 2], Times[-4, t, zL], Times[6, Power[zL, 2]]]]]]]

```

b

q1=q/2

$$\frac{q}{2}$$

Masse in Meter und kg

F1 = 500 9.81 Newton

4905. Newton

hoehe= 6 Zentimeter; A=hoehe^2; zI = A hoehe^2 /12

108 Zentimeter⁴

h3[u]

$$\frac{u^2 (9810. \text{Newton} (u - 3 zL) + \frac{1}{2} q (u^2 - 4 u zL + 6 zL^2))}{1296 zE \text{Zentimeter}^4}$$

A

36 Zentimeter²

zE=210000 Newton /Mimimeter^2

$$\frac{210000 \text{Newton}}{\text{Mimimeter}^2}$$

zL = 4 Meter

4 Meter

q = 800 9.81 Newton /zL

$$\frac{1962. \text{Newton}}{\text{Meter}}$$

h3[t]/.{Zentimeter-> 0.01 Meter, Mimimeter-> 0.001 Meter}

$$\frac{1}{\text{Meter}^2 \text{Newton}} \left(3.67431 \times 10^{-7} t^2 \left(9810. \text{Newton} (-12 \text{Meter} + t) + \frac{981. \text{Newton} (96 \text{Meter}^2 - 16 \text{Meter} t + t^2)}{\text{Meter}} \right) \right)$$

h3[zL]/.{Zentimeter-> 0.01 Meter, Mimimeter-> 0.001 Meter}

-0.18455 Meter

h3[zL]/.{Meter-> 100 Zentimeter, Mimimeter-> 0.1 Zentimeter}

-18.455 Zentimeter

Anhang

q zL^4/zE/zI (* Einheitenvergleich *)

$$\frac{0.022146 \text{Meter}^3 \text{Mimimeter}^2}{\text{Zentimeter}^4}$$

C

h3[t]//Expand//Simplify

$$\frac{(\text{Mimimeter}^2 t^2 (-0.0000865079 \text{Meter}^2 - 0.000021627 \text{Meter} t + 3.6045 \times 10^{-6} t^2))}{(\text{Meter} \text{Zentimeter}^4)}$$

hh=h3[t]//Expand//Simplify

$$\frac{\text{Mimimeter}^2 t^2 (-0.0000865079 \text{ Meter}^2 - 0.000021627 \text{ Meter } t + 3.6045 \times 10^{-6} t^2)}{(\text{Meter Zentimeter}^4)}$$

h3[t] /. t -> u

$$\frac{\left(\text{Mimimeter}^2 u^2 \left(9810. \text{ Newton } (-12 \text{ Meter} + u) + \frac{981. \text{ Newton } (96 \text{ Meter}^2 - 16 \text{ Meter } u + u^2)}{\text{Meter}} \right) \right)}{(272160000 \text{ Newton Zentimeter}^4)}$$

hh[[1]] /. t -> u

$$\frac{1}{\text{Meter}}$$

h4[s_] := hh[[1]] /. t -> s ; h4[u]

$$\frac{1}{\text{Meter}}$$

h4[s_] = Evaluate[h3[t]] /. t -> s

$$\frac{\left(\text{Mimimeter}^2 s^2 \left(9810. \text{ Newton } (-12 \text{ Meter} + s) + \frac{981. \text{ Newton } (96 \text{ Meter}^2 - 16 \text{ Meter } s + s^2)}{\text{Meter}} \right) \right)}{(272160000 \text{ Newton Zentimeter}^4)}$$

D[h4[v], v]

$$\frac{\text{Mimimeter}^2 v^2 \left(9810. \text{ Newton} + \frac{981. \text{ Newton } (-16 \text{ Meter} + 2 v)}{\text{Meter}} \right)}{272160000 \text{ Newton Zentimeter}^4} + \frac{\left(\text{Mimimeter}^2 v \left(9810. \text{ Newton } (-12 \text{ Meter} + v) + \frac{981. \text{ Newton } (96 \text{ Meter}^2 - 16 \text{ Meter } v + v^2)}{\text{Meter}} \right) \right)}{(136080000 \text{ Newton Zentimeter}^4)}$$

D[h4[v], v] /. {v -> zL}

$$\frac{0.000807407 \text{ Meter}^2 \text{ Mimimeter}^2}{\text{Zentimeter}^4}$$

% /. {Meter -> 100 Zentimeter, Mimimeter -> 0.1 Zentimeter}

$$-0.0807407$$

ArcTan[%]

$$-0.080566$$

ArcTan[%] / (1. Degree)

$$-4.61609$$

Remove["Global`*"]

3 Eigenwerte

a

a1

```
v1 = {1,2,1}; v2={3,8,4}; v3={-4,0,3};
k1=1; k2=4; k3=-2;
```

```
Det[{v1,v2,v3}]
```

6

```
M = Transpose[{k1 v1, k2 v2, k3 v3}]. Inverse[Transpose[{v1,v2,v3}]]; M//MatrixForm
```

$$\begin{pmatrix} -8 & \frac{17}{2} & -8 \\ -24 & 29 & -32 \\ -12 & \frac{31}{2} & -18 \end{pmatrix}$$

```
M//InputForm
```

```
{{-8, 17/2, -8}, {-24, 29, -32}, {-12, 31/2, -18}}
```

a2

```
Eigensystem[M]
```

```
{{4, -2, 1}, {{3/4, 2, 1}, {-4/3, 0, 1}, {1, 2, 1}}}
```

b

b1

```
v1 = {1,2}; v2={3,8};
k1=1; k2=4;
```

```
M = Transpose[{k1 v1, k2 v2}]. Inverse[Transpose[{v1,v2}]]; M//MatrixForm
```

$$\begin{pmatrix} -8 & \frac{9}{2} \\ -24 & 13 \end{pmatrix}$$

```
M//TeXForm;
```

```
Eigensystem[M]
```

```
{{4, 1}, {{3/8, 1}, {1/2, 1}}}
```

```
M.{3/8, 1}
```

```
{3/2, 4}
```


M.{1/2, 1}

{ $\frac{1}{2}$, 1}

b2

a[t_]:= +t v1;

a[t]

{t, 2 t}

g[s_]:= {6,3}+s {1,1};

M.g[s]//Simplify

{ $\frac{1}{2} (-69 - 7 s)$, $-105 - 11 s$ }

Solve[g[s]==M.g[s],{t}]

{{}}

b3

(**M.g**[1]-g[1])/(-120)

{ $\frac{3}{8}$, 1}

(**M.g**[0]-g[0])/(-108)

{ $\frac{3}{8}$, 1}

Solve[a[t]==g[s],{t,s}]

{{t → -3, s → -9}}

Solve[a[t]==M.g[s],{t,s}]

{{t → -3, s → -9}}

a[-3]

{-3, -6}

(**M.g**[s]-g[s])/(9+s)/(-12)//Simplify

{ $\frac{3}{8}$, 1}

4 Bewegungen von Punkten, Abstände

a Projiziere die Gerade: g (0), g(1)

```
u = {1,2,1}; a={3,8,4}; b={-4,0,3}; n={0,0,0};
```

```
g[s_]:= {6,3,0}+s{1,1,1};
```

```
g10=g[0]
```

```
{6, 3, 0}
```

```
g20=g[1]
```

```
{7, 4, 1}
```

```
mP = Transpose[{a,b,n}].Inverse[ Transpose[{a,b,u}]]; mP//MatrixForm
```

$$\begin{pmatrix} -3 & \frac{25}{6} & -\frac{16}{3} \\ -8 & \frac{28}{3} & -\frac{32}{3} \\ -4 & \frac{25}{6} & -\frac{13}{3} \end{pmatrix}$$

```
g11=mP.g[0]
```

```
{- 11/2, -20, - 23/2}
```

```
%//N
```

```
{-5.5, -20., -11.5}
```

```
g21=mP.g[1]
```

```
{- 29/3, - 88/3, - 47/3}
```

```
%//N
```

```
{-9.66667, -29.3333, -15.6667}
```

b Drehe die projizierte Gerade (Punkte) um 30 Grad um die z-Achse

```
Remove[d,x]
```

```
d[x_]:= {{Cos[x],-Sin[x],0},{Sin[x],Cos[x],0},{0,0,1}}
```

```
d[30 Degree]//MatrixForm
```

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
%//N
```

```
{0.866025, -0.5, 0.}, {0.5, 0.866025, 0.}, {0., 0., 1.}}
```

```
g12=d[30 Degree].(mP.g[0])
```

$$\left\{ 10 - \frac{11\sqrt{3}}{4}, -\frac{11}{4} - 10\sqrt{3}, -\frac{23}{2} \right\}$$

```
%//N
```

$$\{5.23686, -20.0705, -11.5\}$$

```
g22=d[30 Degree].(mP.g[1])
```

$$\left\{ \frac{44}{3} - \frac{29}{2\sqrt{3}}, -\frac{29}{6} - \frac{44}{\sqrt{3}}, -\frac{47}{3} \right\}$$

```
g22=d[30 Degree].mP.g[1]//Simplify
```

$$\left\{ \frac{1}{6} (88 - 29\sqrt{3}), -\frac{29}{6} - \frac{44}{\sqrt{3}}, -\frac{47}{3} \right\}$$

```
%//N
```

$$\{6.29509, -30.2367, -15.6667\}$$

c Berechne die minimalen Abstände der ursprünglichen Geraden und der bewegten Geraden von 0

```
abst[r_,s_]:=Norm[Cross[r,s]]/Norm[r-s]
```

```
abst[g10,g20]
```

$$3\sqrt{2}$$

```
%//N
```

$$4.24264$$

```
abst[g11,g21]
```

$$5\sqrt{\frac{534}{731}}$$

```
%//N
```

$$4.27348$$

```
abst[g12,g22]
```

$$\frac{\sqrt{1024 + \left(\frac{25}{2} + 12\sqrt{3}\right)^2 + \left(-12 + \frac{25\sqrt{3}}{2}\right)^2}}{\sqrt{\frac{625}{36} + \left(\frac{25}{12} + \frac{44}{\sqrt{3}} - 10\sqrt{3}\right)^2 + \left(-10 + \frac{11\sqrt{3}}{4} + \frac{1}{6}(88 - 29\sqrt{3})\right)^2}}$$

```
%//N
```

$$4.27348$$

d

```
M={g20-g10,g12-g10,g22-g10}; Det[M]/6
```

$$\frac{1}{6} \left(-\frac{85}{2} + \frac{89\sqrt{3}}{4} \right)$$

```
%//N
```

```
-0.660312
```

5 a

```
M=2 {{-8, 17/2, -8}, {-24, 29, -32}, {-12, 31/2, -18}}; M//MatrixForm
```

$$\begin{pmatrix} -16 & 17 & -16 \\ -48 & 58 & -64 \\ -24 & 31 & -36 \end{pmatrix}$$

```
M//TeXForm;
```

```
IdentityMatrix[3]//TeXForm;
```

```
Det[M- t IdentityMatrix[3]]
```

$$-64 + 24t + 6t^2 - t^3$$

```
-64 IdentityMatrix[3] + 24 M + 6 M.M - M.M.M
```

```
{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}
```

```
%//MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$