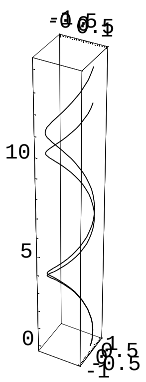
Lösungen Modulprüfung Analysis E+M 07

1E+M

a

```
In[1]:= Remove["Global`*"]

In[2]:= v[t_]:= {Cos[t],Sin[t],t};
        w[t_]:= {Cos[t],Sin[t],t+(t/10)^2};

In[4]:= p1 = ParametricPlot3D[v[t],{t,0,4Pi},DisplayFunction->Identity];
        p2 = ParametricPlot3D[w[t],{t,0,4Pi},DisplayFunction->Identity];
        Show[p2,p1,DisplayFunction->$DisplayFunction];
```



b

```
In[7]:= D[v[t],t]/.t->0

Out[7]= {0, 1, 1}

In[8]:= D[w[t],t]/.t->0

Out[8]= {0, 1, 1}
```

```
In[9]:= ArcCos[D[v[t],t].D[w[t],t]/Norm[D[v[t],t]]/Norm[D[w[t],t]]]/.t->0
Out[9]= 0
```

c

```
In[10]:= D[v[t],t]/.t->4Pi
Out[10]= {0, 1, 1}
In[11]:= D[w[t],t]/.t->4Pi
```

Out[11]= $\left\{0, 1, 1 + \frac{2\pi}{25}\right\}$

```
In[12]:= N[%]
Out[12]= {0., 1., 1.25133}
In[13]:= ArcCos[D[v[t],t].D[w[t],t]/Norm[D[v[t],t]]/Norm[D[w[t],t]]]/.t->4Pi
```

Out[13]= $\text{ArcCos}\left[\frac{2 + \frac{2\pi}{25}}{\sqrt{2\left(1 + \left(1 + \frac{2\pi}{25}\right)^2\right)}}\right]$

```
In[14]:= N[%]
Out[14]= 0.111175
In[15]:= %/Degree
Out[15]= 6.36985
```

d

```
In[16]:= len1 = Integrate[Evaluate[Sqrt[D[v[t],t].D[v[t],t]]],{t,0,4Pi}]
```

Out[16]= $4\sqrt{2}\,\pi$

```
In[17]:= N[%]
Out[17]= 17.7715
In[18]:= len2 = Integrate[Evaluate[Sqrt[D[w[t],t].D[w[t],t]]],{t,0,4Pi}]
```

Out[18]= $\frac{1}{50}\left(4\pi\sqrt{2\left(625 + 50\pi + 2\pi^2\right)} + 50\left(\sqrt{2\left(625 + 50\pi + 2\pi^2\right)} - 25\left(\sqrt{2} + \text{ArcSinh}[1] + \text{Log}\left[\frac{25}{25 + 2\pi + \sqrt{2\left(625 + 50\pi + 2\pi^2\right)}}\right]\right)\right)\right)$

```
In[19]:= N[%]
Out[19]= 18.9308
In[20]:= NIntegrate[Evaluate[Sqrt[D[w[t],t].D[w[t],t]]],{t,0,4Pi}]
Out[20]= 18.9308
```

e

In[21]:= **Integrate[Evaluate[D[v[t],t].D[w[t],t] * Sqrt[D[v[t],t].D[v[t],t]]],{t,0,4Pi}]**

Out[21]= $8\sqrt{2}\,\pi + \frac{4\sqrt{2}\,\pi^2}{25}$

In[22]:= **N[%]**

Out[22]= 37.7763

In[23]:= **NIntegrate[Evaluate[D[v[t],t].D[w[t],t] * Sqrt[D[v[t],t].D[v[t],t]]],{t,0,4Pi}]**

Out[23]= 37.7763

2E+M

In[24]:= **Remove["Global`*"];**

a

In[25]:= **f[x_,y_]:= Sin[x+y] + Cos[x-y]**

In[26]:= **D[f[x,y],x] == 0**

Out[26]= Cos[x + y] − Sin[x − y] == 0

In[27]:= **D[f[x,y],y] == 0**

Out[27]= Cos[x + y] + Sin[x − y] == 0

In[28]:= **Solve[Evaluate[{D[f[x,y],x] == 0, D[f[x,y],y] == 0}],{x,y}]**

```
Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. Mehr…
```

Out[28]= $\{\{y \to -\frac{3\pi}{4}, x \to -\frac{3\pi}{4}\}, \{y \to -\frac{3\pi}{4}, x \to \frac{\pi}{4}\}, \{y \to -\frac{\pi}{4}, x \to -\frac{\pi}{4}\}, \{y \to -\frac{\pi}{4}, x \to \frac{3\pi}{4}\}, \{y \to \frac{\pi}{4}, x \to -\frac{3\pi}{4}\}, \{y \to \frac{\pi}{4}, x \to \frac{\pi}{4}\}, \{y \to \frac{3\pi}{4}, x \to -\frac{\pi}{4}\}, \{y \to \frac{3\pi}{4}, x \to \frac{3\pi}{4}\}\}$

In[29]:= **N[%]**

Out[29]= {{y → −2.35619, x → −2.35619}, {y → −2.35619, x → 0.785398},
{y → −0.785398, x → −0.785398}, {y → −0.785398, x → 2.35619},
{y → 0.785398, x → −2.35619}, {y → 0.785398, x → 0.785398},
{y → 2.35619, x → −0.785398}, {y → 2.35619, x → 2.35619}}

b

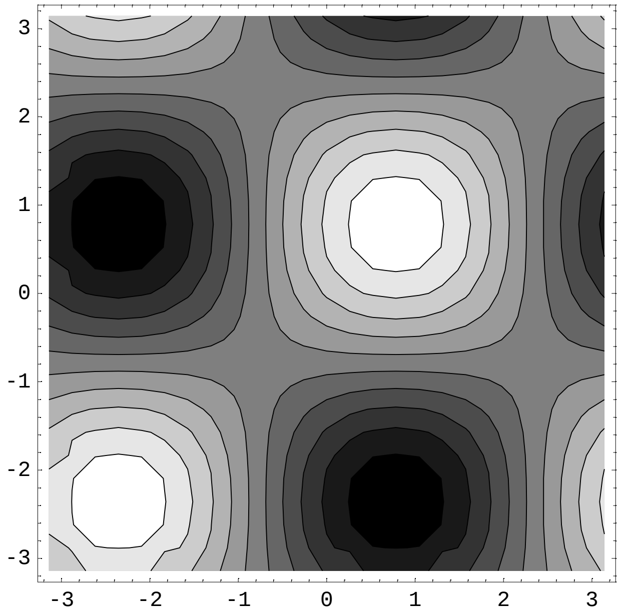
In[30]:= **f[x,y] //TrigFactor**

Out[30]= (Cos[x] + Sin[x]) (Cos[y] + Sin[y])

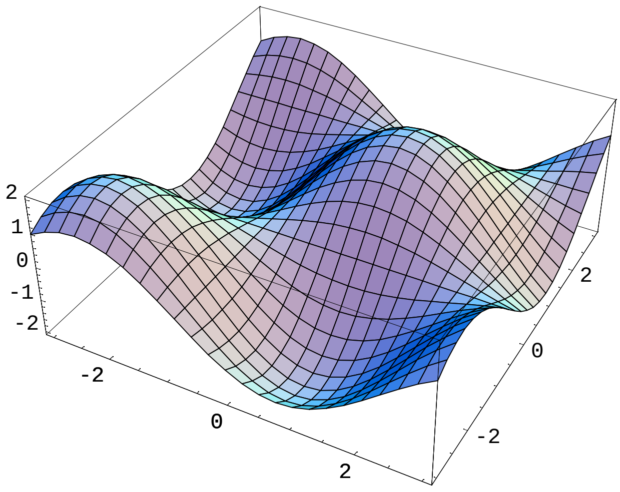
```
In[31]:= Sin[x+Pi/4] Sqrt[2]//TrigFactor //Simplify

Out[31]= Cos[x] + Sin[x]

In[32]:= (Sin[x+Pi/4] Sqrt[2])(Sin[y+Pi/4] Sqrt[2])//TrigFactor //Simplify

Out[32]= (Cos[x] + Sin[x]) (Cos[y] + Sin[y])

In[33]:= ( (Sin[x+Pi/4] Sqrt[2])(Sin[y+Pi/4] Sqrt[2])//TrigFactor //Simplify ) == (
         f[x,y] //TrigFactor )

Out[33]= True
```

c

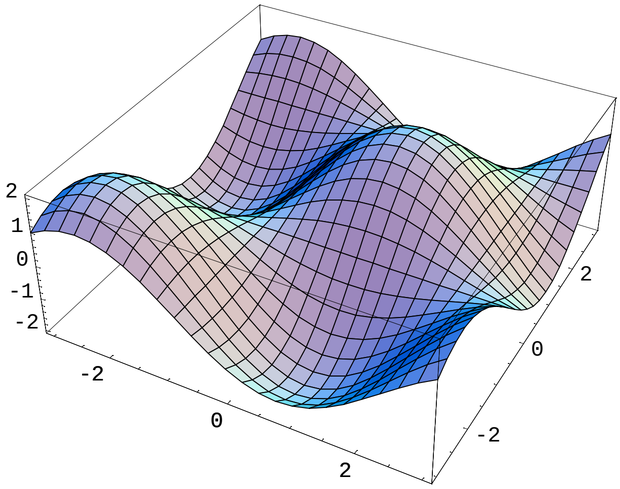
```
In[34]:= ContourPlot[f[x,y],{x,-Pi,Pi},{y,-Pi,Pi}];
```




```
In[35]:= Plot3D[f[x,y],{x,-Pi,Pi},{y,-Pi,Pi}];
```



```
In[36]:= p1= Plot3D[Sin[x+Pi/4] 2 Sin[y+Pi/4],{x,-Pi,Pi},{y,-Pi,Pi}];
```




```
In[37]:= (* Maximum mit Differentialrechnung? *)

In[38]:= D[Sin[x+Pi/4] 2 Sin[y+Pi/4],x]
```

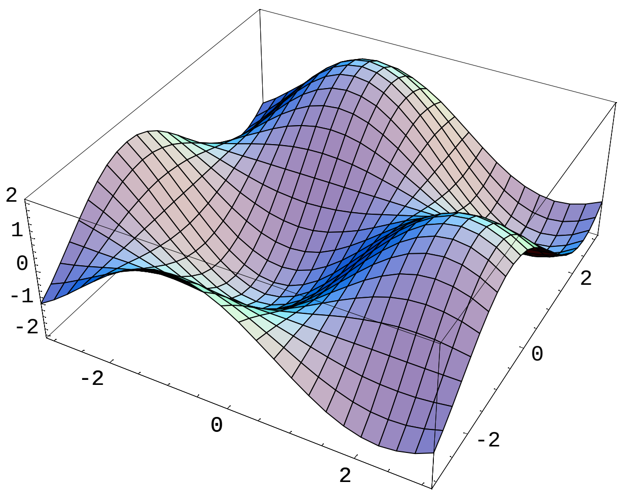
Out[38]= $2\,\mathrm{Cos}\left[\frac{\pi}{4}+x\right]\,\mathrm{Sin}\left[\frac{\pi}{4}+y\right]$

```
In[39]:= Solve[Evaluate[{D[Sin[x+Pi/4] 2 Sin[y+Pi/4],x]==0, D[Sin[x+Pi/4] 2
         Sin[y+Pi/4],y]==0}],{x,y}]

         Solve::ifun : Inverse functions are being used by Solve, so some
            solutions may not be found; use Reduce for complete solution information. Mehr…
```

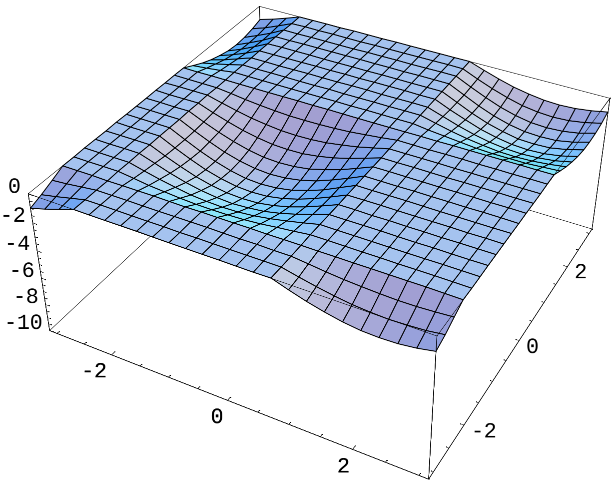
Out[39]= $\left\{\left\{y\to-\frac{3\pi}{4},\ x\to-\frac{3\pi}{4}\right\},\ \left\{y\to-\frac{3\pi}{4},\ x\to\frac{\pi}{4}\right\},\ \left\{y\to-\frac{\pi}{4},\ x\to-\frac{\pi}{4}\right\},\ \left\{y\to-\frac{\pi}{4},\ x\to\frac{3\pi}{4}\right\},\ \left\{y\to\frac{\pi}{4},\ x\to-\frac{3\pi}{4}\right\},\ \left\{y\to\frac{\pi}{4},\ x\to\frac{\pi}{4}\right\},\ \left\{y\to\frac{3\pi}{4},\ x\to-\frac{\pi}{4}\right\},\ \left\{y\to\frac{3\pi}{4},\ x\to\frac{3\pi}{4}\right\}\right\}$

d

```
In[40]:= h[x_,y_]:= Sin[x+y]-Cos[x-y];
         Plot3D[h[x,y],{x,-Pi,Pi},{y,-Pi,Pi}];
```



```
In[42]:= Plot3D[h[x,y],{x,-Pi,Pi},{y,-Pi,Pi},PlotRange->{-10,0}];
```




```
In[43]:= grad2[v_]:={D[v,x],D[v,y]};
         grad2[f[x,y]]

Out[44]= {Cos[x + y] - Sin[x - y], Cos[x + y] + Sin[x - y]}

In[45]:= grad2[h[x,y]]

Out[45]= {Cos[x + y] + Sin[x - y], Cos[x + y] - Sin[x - y]}

In[46]:= ?Reduce
```

Reduce[expr, vars] reduces the statement expr by solving equations or inequalities for vars and eliminating quantifiers. Reduce[expr, vars, dom] does the reduction over the domain dom. Common choices of dom are Reals, Integers and Complexes. Mehr...

```
In[47]:= Reduce[Evaluate[grad2[f[x,y]] == λ grad2[h[x,y]] &&  grad2[h[x,y]]==0],{x,y}]
```

Out[47]= (C[1] | C[2]) ∈ Integers &&

$$\left(\left(x == \frac{1}{2}\left(-\frac{\pi}{2} + 2\pi C[1]\right) + \pi C[2] \;\&\&\; y == \frac{1}{2}\left(-\frac{\pi}{2} + 2\pi C[1]\right) - \pi C[2]\right) \;||\right.$$

$$\left(x == \frac{1}{2}\left(\frac{\pi}{2} + 2\pi C[1]\right) + \pi C[2] \;\&\&\; y == \frac{1}{2}\left(\frac{\pi}{2} + 2\pi C[1]\right) - \pi C[2]\right) \;||$$

$$\left(x == \frac{1}{2}\left(-\frac{\pi}{2} + 2\pi C[1]\right) + \frac{1}{2}(\pi + 2\pi C[2]) \;\&\&\right.$$

$$\left. y == \frac{1}{2}\left(-\frac{\pi}{2} + 2\pi C[1]\right) + \frac{1}{2}(-\pi - 2\pi C[2])\right) \;||$$

$$\left.\left(x == \frac{1}{2}\left(\frac{\pi}{2} + 2\pi C[1]\right) + \frac{1}{2}(\pi + 2\pi C[2]) \;\&\&\; y == \frac{1}{2}\left(\frac{\pi}{2} + 2\pi C[1]\right) + \frac{1}{2}(-\pi - 2\pi C[2])\right)\right)$$

```
In[48]:= N[%]

Out[48]= (C[1] | C[2]) ∈ Integers && ((x == 0.5 (-1.5708 + 6.28319 C[1]) + 3.14159 C[2] &&
            y == 0.5 (-1.5708 + 6.28319 C[1]) - 3.14159 C[2]) ||
          (x == 0.5 (1.5708 + 6.28319 C[1]) + 3.14159 C[2] &&
            y == 0.5 (1.5708 + 6.28319 C[1]) - 3.14159 C[2]) ||
          (x == 0.5 (-1.5708 + 6.28319 C[1]) + 0.5 (3.14159 + 6.28319 C[2]) &&
            y == 0.5 (-1.5708 + 6.28319 C[1]) + 0.5 (-3.14159 - 6.28319 C[2])) ||
          (x == 0.5 (1.5708 + 6.28319 C[1]) + 0.5 (3.14159 + 6.28319 C[2]) &&
            y == 0.5 (1.5708 + 6.28319 C[1]) + 0.5 (-3.14159 - 6.28319 C[2])))

In[49]:= Reduce[{grad2[f[x,y]] == λ grad2[h[x,y]],  grad2[h[x,y]]==0}]

Out[49]= Sin[x - y] == 0 && Cos[x + y] == 0
```

In[50]:= **Solve[{grad2[f[x,y]] == λ grad2[h[x,y]], grad2[h[x,y]]==0},{x,y}]**

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. Mehr…

Out[50]= $\{\{y \to -\frac{3\pi}{4}, x \to -\frac{3\pi}{4}\}, \{y \to -\frac{3\pi}{4}, x \to \frac{\pi}{4}\}, \{y \to -\frac{\pi}{4}, x \to -\frac{\pi}{4}\}, \{y \to -\frac{\pi}{4}, x \to \frac{3\pi}{4}\}, \{y \to \frac{\pi}{4}, x \to -\frac{3\pi}{4}\}, \{y \to \frac{\pi}{4}, x \to \frac{\pi}{4}\}, \{y \to \frac{3\pi}{4}, x \to -\frac{\pi}{4}\}, \{y \to \frac{3\pi}{4}, x \to \frac{3\pi}{4}\}\}$

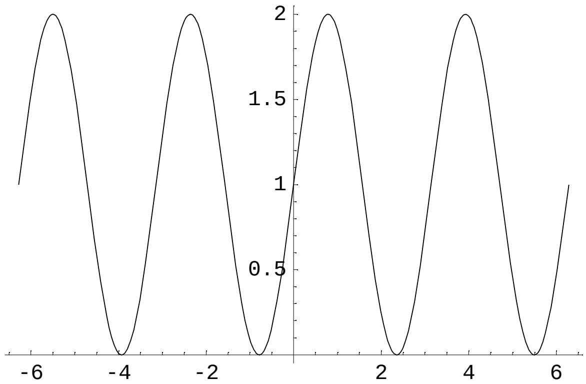
In[51]:= **N[%]**

Out[51]= {{y → −2.35619, x → −2.35619}, {y → −2.35619, x → 0.785398}, {y → −0.785398, x → −0.785398}, {y → −0.785398, x → 2.35619}, {y → 0.785398, x → −2.35619}, {y → 0.785398, x → 0.785398}, {y → 2.35619, x → −0.785398}, {y → 2.35619, x → 2.35619}}

In[52]:= $\mathbf{f}[\frac{3\pi}{4}, \frac{3\pi}{4}]$

Out[52]= 0

In[53]:= **Plot[f[x,x],{x,-2Pi,2Pi}];**



In[54]:= **Solve[h[x,y]==0,{y}]**

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. Mehr…

Out[54]= $\{\{y \to -\frac{3\pi}{4}\}, \{y \to -\frac{\pi}{4}\}, \{y \to \frac{\pi}{4}\}, \{y \to \frac{3\pi}{4}\}\}$

In[55]:= **N[%]**

Out[55]= {{y → −2.35619}, {y → −0.785398}, {y → 0.785398}, {y → 2.35619}}

In[56]:= **Expand[TrigExpand[Sin[x+y]]
/Cos[y]/Sin[x]]==Expand[TrigExpand[Cos[x-y]]/Cos[y]/Sin[x]]**

Out[56]= 1 + Cot[x] Tan[y] == Cot[x] + Tan[y]

In[57]:= **Solve[1+(Cot[x]-1) Tan[y] == Cot[x],{Tan[y]}]**

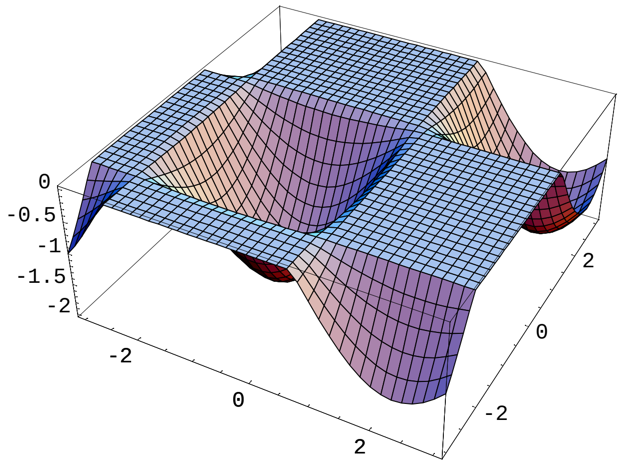
Out[57]= {{Tan[y] → 1}}

In[58]:= **(* Alternativen *)**

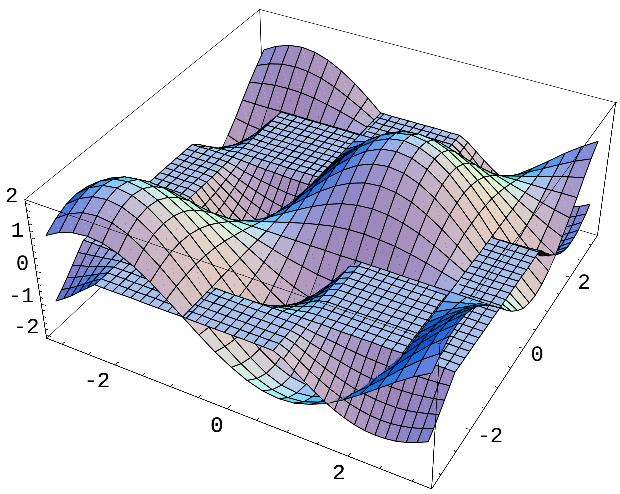
In[59]:= **Cos[y]==0**

Out[59]= Cos[y] == 0

```
In[60]:= Cos[x]==0

Out[60]= Cos[x] == 0

In[61]:= h1[x_,y_]:= If[h[x,y] <= 0,  h[x,y], 0];

In[62]:= p2= Plot3D[h1[x,y],{x,-Pi,Pi},{y,-Pi,Pi},PlotPoints->40];
```




```
In[63]:= Show[p1,p2];
```



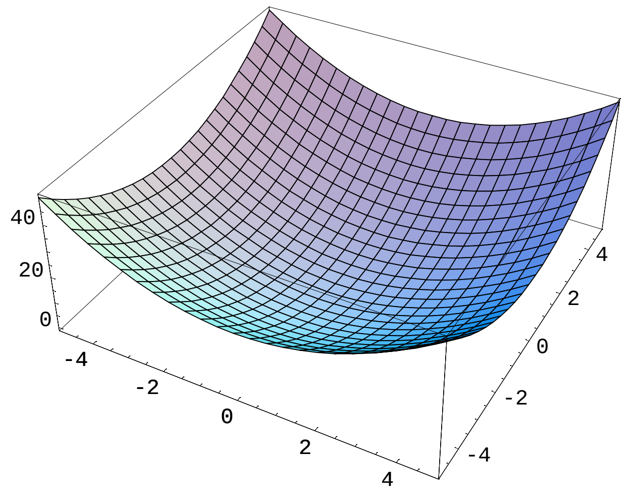
3E+M

```
In[64]:= Remove["Global`*"];
```

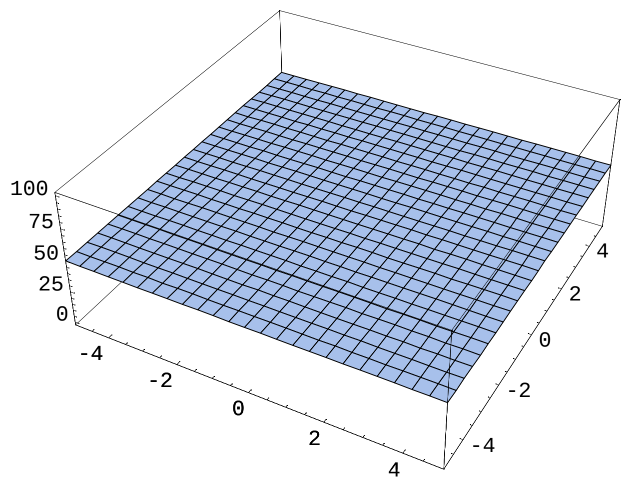
a

```
In[65]:= f[x_,y_]:= x^2+y^2
```

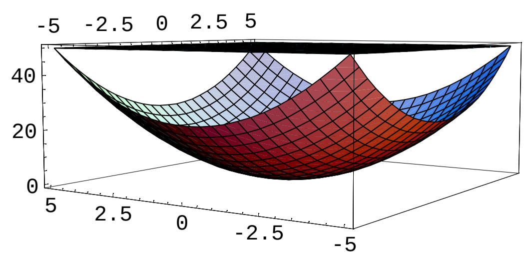
```
In[66]:= p1 = Plot3D[f[x,y],{x,-5,5},{y,-5,5}];
```



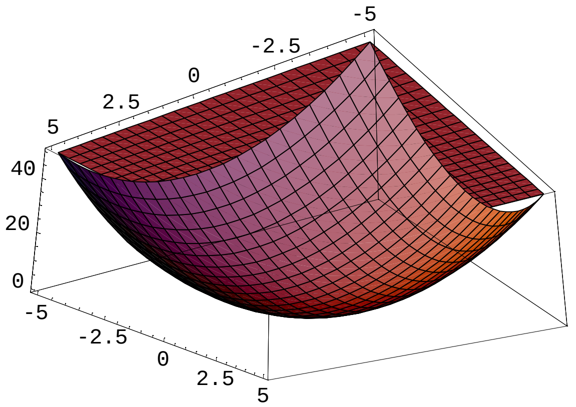

```
In[67]:= p2 = Plot3D[50,{x,-5,5},{y,-5,5}];
```



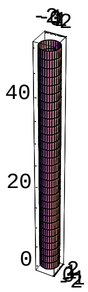

```
In[68]:= Show[p2,p1,ViewPoint->{-2.418, -1.626, 0.262}];
```



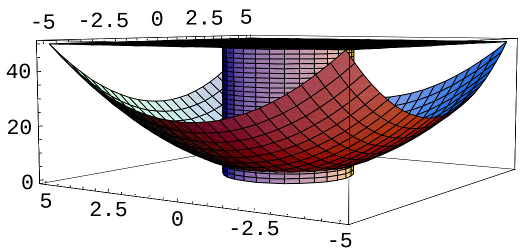
```
In[69]:= Show[p2,p1,ViewPoint->{-2.245, -1.510, -1.114}];
```




```
In[70]:= p3=ParametricPlot3D[{2 Cos[t],2 Sin[t],h},{t,0,2Pi},{h,0,50}];
```




```
In[71]:= Show[p2,p1,p3,ViewPoint->{-2.418, -1.626, 0.262}];
```



b

```
In[72]:= Integrate[50-f[x,y],{x,-5,5},{y,-5,5}]
```

Out[72]= $\frac{10000}{3}$

```
In[73]:= N[%]
```

Out[73]= 3333.33

c

In[74]:= **f[x,5]**

Out[74]= $25 + x^2$

In[75]:= **len = Integrate[Evaluate[Sqrt[1+D[f[x,5],x]^2]],{x,-5,5}]**

Out[75]= $5\sqrt{101} + \frac{\text{ArcSinh}[10]}{2}$

In[76]:= **N[%]**

Out[76]= 51.7485

d

In[77]:= **Integrate[Sqrt[1+D[Sqrt[x],x]^2] 2 Sqrt[x] Pi,{x,0,t},GenerateConditions→False]**

Out[77]= $\frac{1}{6}\pi\left(-1 + \sqrt{1+4\,t} + 4\,t\sqrt{1+4\,t}\right)$

In[78]:= **Integrate[Sqrt[1+D[Sqrt[x],x]^2] 2 Sqrt[x] Pi,{x,0,2}]**

Out[78]= $\frac{13\pi}{3}$

In[79]:= **N[%]**

Out[79]= 13.6136

In[80]:= **Integrate[Sqrt[1+D[Sqrt[x],x]^2] 2 Sqrt[x] Pi,{x,0,4}]**

Out[80]= $\frac{1}{6}\left(-1 + 17\sqrt{17}\right)\pi$

In[81]:= **N[%]**

Out[81]= 36.1769

e

In[82]:= **v[x_,y_]:={x,y,f[x,y]};**
Norm[Cross[D[v[x,y],x],D[v[x,y],y]]]

Out[83]= $\sqrt{1 + 4\,\text{Abs}[x]^2 + 4\,\text{Abs}[y]^2}$

In[84]:= **Sqrt[Cross[D[v[x,y],x],D[v[x,y],y]].Cross[D[v[x,y],x],D[v[x,y],y]]]**

Out[84]= $\sqrt{1 + 4\,x^2 + 4\,y^2}$

In[85]:= **Integrate[Evaluate[Sqrt[Cross[D[v[x,y],x],D[v[x,y],y]].Cross[D[v[x,y],x],D[v[x,y],y]]]], {x,-5,5}]**

Out[85]= $\frac{1}{4}\left(20\sqrt{101+4\,y^2} - (1+4\,y^2)\,\text{Log}\left[-10+\sqrt{101+4\,y^2}\right] + (1+4\,y^2)\,\text{Log}\left[10+\sqrt{101+4\,y^2}\right]\right)$

In[86]:=
```
Integrate[Integrate[Evaluate[Sqrt[Cross[D[v[x,y],x],D[v[x,y],y]].Cross[D[v[x,y],x],D[v[x,y],y]]]], {x,-5,5}], {y,-5,5}]
```

Out[86]= $\frac{1}{6}\left(200\sqrt{201}+1030\,\text{ArcSinh}\left[\frac{10}{\sqrt{101}}\right]-2\,\text{ArcTan}\left[\frac{100}{\sqrt{201}}\right]-515\,\text{Log}\left[-10+\sqrt{201}\right]+515\,\text{Log}\left[10+\sqrt{201}\right]\right)$

In[87]:=
```
N[%]
```

Out[87]= 773.504

In[88]:=
```
NIntegrate[Evaluate[Sqrt[Cross[D[v[x,y],x],D[v[x,y],y]].Cross[D[v[x,y],x],D[v[x,y],y]]]], {x,-5,5}, {y,-5,5}]
```

Out[88]= 773.504

In[89]:=
```
Solve[h == r^2,{r}]
```

Out[89]= $\{\{r\to-\sqrt{h}\},\{r\to\sqrt{h}\}\}$

In[90]:=
```
Solve[h == 2^2,{h}]
```

Out[90]= $\{\{h\to4\}\}$

4E+M

a

In[91]:=
```
Series[Sin[x],{x,2Pi,10}]
```

Out[91]= $(x-2\pi)-\frac{1}{6}(x-2\pi)^3+\frac{1}{120}(x-2\pi)^5-\frac{(x-2\pi)^7}{5040}+\frac{(x-2\pi)^9}{362880}+O[x-2\pi]^{11}$

In[92]:=
```
N[%]
```

Out[92]= $(x-6.28319)-0.166667(x-6.28319)^3+0.00833333(x-6.28319)^5-0.000198413(x-6.28319)^7+2.75573\times10^{-6}(x-6.28319)^9+O[x-6.28319]^{11}$

b

In[93]:=
```
u[t_]:=(Series[Sin[x],{x,2Pi,10}]//Normal)/.x->1/t
```

In[94]:=
```
u[t]
```

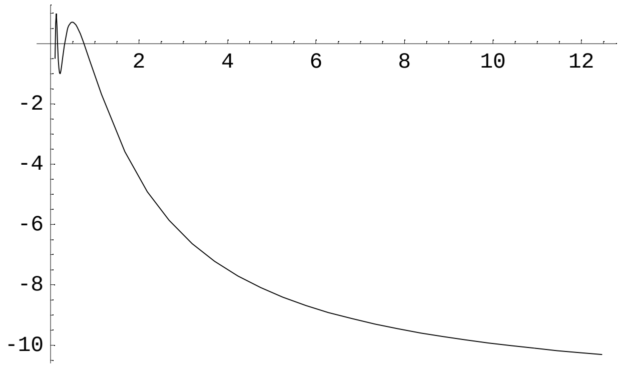
Out[94]= $-2\pi-\frac{1}{6}\left(-2\pi+\frac{1}{t}\right)^3+\frac{1}{120}\left(-2\pi+\frac{1}{t}\right)^5-\frac{(-2\pi+\frac{1}{t})^7}{5040}+\frac{(-2\pi+\frac{1}{t})^9}{362880}+\frac{1}{t}$

In[95]:=
```
N[%]
```

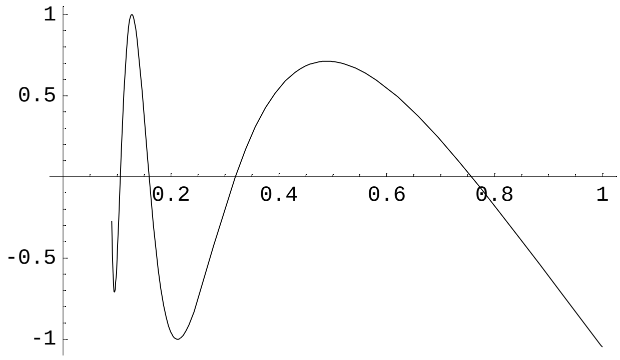
Out[95]= $-6.28319-0.166667\left(-6.28319+\frac{1}{t}\right)^3+0.00833333\left(-6.28319+\frac{1}{t}\right)^5-0.000198413\left(-6.28319+\frac{1}{t}\right)^7+2.75573\times10^{-6}\left(-6.28319+\frac{1}{t}\right)^9+\frac{1}{t}$

c

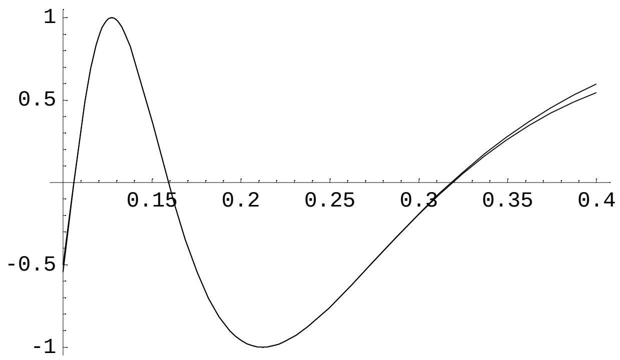
In[96]:= **Plot[u[t],{t,0.1,4Pi-0.1}];**



In[97]:= **Plot[u[t],{t,0.09,1}];**



In[98]:= **Plot[{u[t],Sin[1/t]},{t,0.1,0.4}];**



d

In[99]:= **u[0.4]**

Out[99]= 0.546435

In[100]:=

```
Sin[1/0.4]
```

Out[100]=

0.598472

In[101]:=

```
Sin[1/0.4]-u[0.4]
```

Out[101]=

0.0520373

In[102]:=

```
100 (Sin[1/0.4]-u[0.4])/Sin[1/0.4]
```

Out[102]=

8.69502

e

In[103]:=

```
su[x_]:=Integrate[u[t],{t,0.1,x},GenerateConditions→False]// Expand;
su[x]
```

Out[104]=

$$-0.455678 - \frac{3.44466\times 10^{-7}}{x^8} + \frac{0.0000222619}{x^7} - \frac{0.000619683}{x^6} + \frac{0.00973848}{x^5} - \frac{0.0962504}{x^4} + \frac{0.646482}{x^3} - \frac{3.27139}{x^2} + \frac{15.0796}{x} - 11.8996\, x + 20.988\, \mathrm{Log}[x]$$

In[105]:=

```
N[%]
```

Out[105]=

$$-0.455678 - \frac{3.44466\times 10^{-7}}{x^8} + \frac{0.0000222619}{x^7} - \frac{0.000619683}{x^6} + \frac{0.00973848}{x^5} - \frac{0.0962504}{x^4} + \frac{0.646482}{x^3} - \frac{3.27139}{x^2} + \frac{15.0796}{x} - 11.8996\, x + 20.988\, \mathrm{Log}[x]$$

In[106]:=

```
(* Terme mit Koeffizienten absolut grösser 0.1 *)
```

In[107]:=

```
tab1=Union[Table[If[Abs[su[x][[n]][[1]]]>0.1,su[x][[n]],0],{n,2,11}],
{If[Abs[su[x][[1]]]>0.1,su[x][[1]],0]}]
```

Out[107]=

$$\left\{-0.455678,\ 0,\ \frac{0.646482}{x^3},\ -\frac{3.27139}{x^2},\ \frac{15.0796}{x},\ -11.8996\, x,\ 20.988\, \mathrm{Log}[x]\right\}$$

In[108]:=

```
Apply[Plus,tab1]
```

Out[108]=

$$-0.455678 + \frac{0.646482}{x^3} - \frac{3.27139}{x^2} + \frac{15.0796}{x} - 11.8996\, x + 20.988\, \mathrm{Log}[x]$$

5E+M

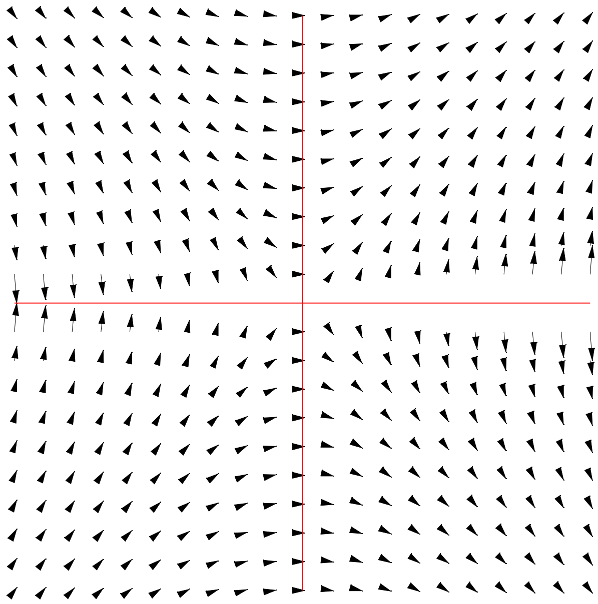
a

In[109]:=

```
Remove["Global`*"];
<<Graphics`PlotField`
```

In[111]:=

```
g1=PlotVectorField[{1, 16/12 x/y},{x,-2,2,0.2},{y,0.2,2,0.2},Epilog→
{Hue[1],Line[{{-2,0},{2,0}}],Line[{{0,-2},{0,2}}]},AspectRatio→Automatic,
DisplayFunction->Identity];
g2=PlotVectorField[{1, 16/12 x/y},{x,-2,2,0.2},{y,-2,-0.2,0.2},Epilog→
{Hue[1],Line[{{-2,0},{2,0}}],Line[{{0,-2},{0,2}}]},AspectRatio→Automatic,
DisplayFunction->Identity];
Show[g1,g2, DisplayFunction->$DisplayFunction];
```



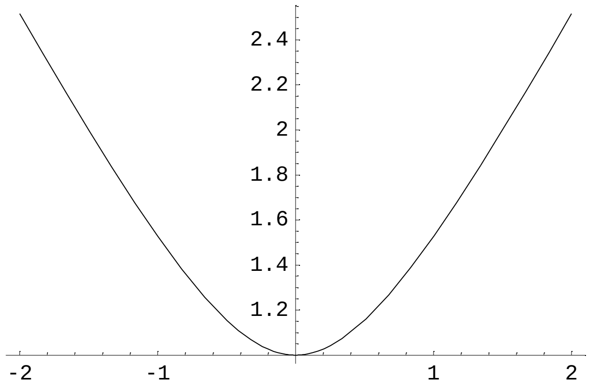
b

In[114]:=

```
Remove[x,y];
solv = Flatten[DSolve[{12 y'[x] - 16 x/y[x] == 0, y[0]==1},y,x]];
y = y/.solv;
Print["y(x) = ",Simplify[y[x]]];
g3=Plot[y[x],{x,-2,2}];
```

DSolve::bvnul : For some branches of the general solution, the given boundary conditions lead to an empty solution. Mehr…

$y(x) = \sqrt{1 + \frac{4 x^2}{3}}$



In[119]:=

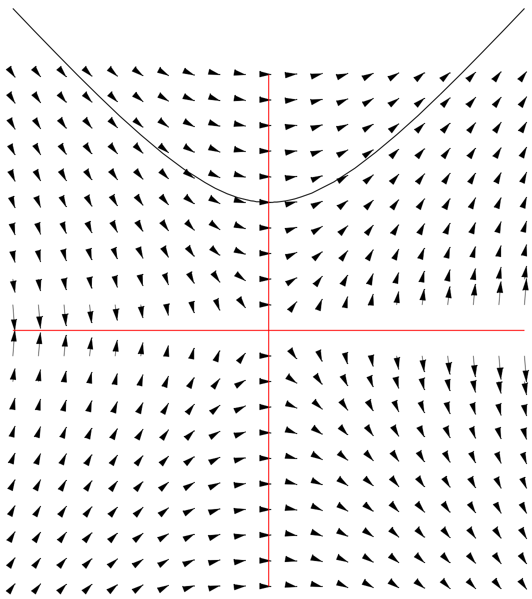
```
y[x]
```

Out[119]=

$\frac{\sqrt{3 + 4 x^2}}{\sqrt{3}}$

In[120]:=

```
Show[g1,g2,g3, DisplayFunction->$DisplayFunction];
```



c

In[121]:=

```
Limit[y[x]/x,x->Infinity]
```

Out[121]=

$$\frac{2}{\sqrt{3}}$$

In[122]:=

```
(* ===> Linear *)
```

6E+M

In[123]:=

```
Remove["Global`*"];
```

a

In[124]:=

```
solv1 = Flatten[DSolve[{y''[t] + 2 y'[t]+ y[t] == 0},y,t]]
```

Out[124]=

$\{y \to \text{Function}[\{t\}, e^{-t} C[1] + e^{-t} t C[2]]\}$

b

In[125]:=

```
 y[t_]:= a t^2 E^(-t);
(y''[t] +2 y'[t]+ y[t] //Simplify) == E^(-t)
```

Out[126]=

$2 a e^{-t} == e^{-t}$

In[127]:=

```
Remove["Global`*"];
solv2 = Flatten[DSolve[{y''[t] +2 y'[t]+ y[t] == E^(-t)},y,t]];
y = y/.solv2;
y[t]
```

Out[130]=

$$\frac{1}{2} e^{-t} t^2 + e^{-t} C[1] + e^{-t} t C[2]$$

c

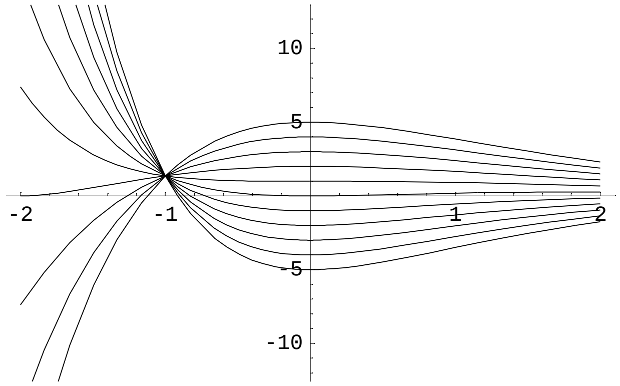
In[131]:=

```
y[0]
```

Out[131]=

$C[1]$

In[132]:=
```
y'[t]/.t->0
```
Out[132]=
$-C[1] + C[2]$

In[133]:=
```
(* Unendlich viele Möglichkeiten mit 2 Variablen C1, C2 *)
```
In[134]:=
```
Remove["Global`*"];
y[t_,c1_,c2_]:= E^(-t)(1/2 t^2+c1+c2 t);
Plot[Evaluate[Table[y[t,c1,c1],{c1,-5,5}]],{t,-2,2}];
```


In[137]:=
```
(* Kurven durch Ursprung *)
```
In[138]:=
```
Solve[Evaluate[{y[0,c1,c2]==0, (D[y[t,c1,c2],t]/.t->0)==0}],{c1,c2}]
```
Out[138]=
$\{\{c1 \to 0, c2 \to 0\}\}$

In[139]:=
```
(* Eine Lösung, Eindeutikeit *)
```

d

In[140]:=
```
Remove["Global`*"];
solv2 = Flatten[DSolve[{y''[t] +2 y'[t]+ y[t] == E^(-t)},y,t]];
y = y/.solv2;
Solve[Evaluate[D[y[t],t]==0],{t}]
```
Out[143]=
$\left\{\left\{t \to 1 - C[2] - \sqrt{1 - 2\,C[1] + C[2]^2}\right\}, \left\{t \to 1 - C[2] + \sqrt{1 - 2\,C[1] + C[2]^2}\right\}\right\}$

In[144]:=
```
(*  Unendlich viele Möglichkeiten  *)
```
In[145]:=
```
% /. C[1]->0
```
In[146]:=
```
(*  Nuch unendlich viele Möglichkeiten  *)
```

In[147]:=
```
% /. C[2]->0
```

In[148]:=
```
(*  Noch 2 Möglichkeiten  *)
```

e

In[149]:=
```
y[-1] == E/2
```

Out[149]=
$$\frac{e}{2} + e\,C[1] - e\,C[2] == \frac{e}{2}$$

In[150]:=
```
y[0] == 1
```

Out[150]=
$$C[1] == 1$$

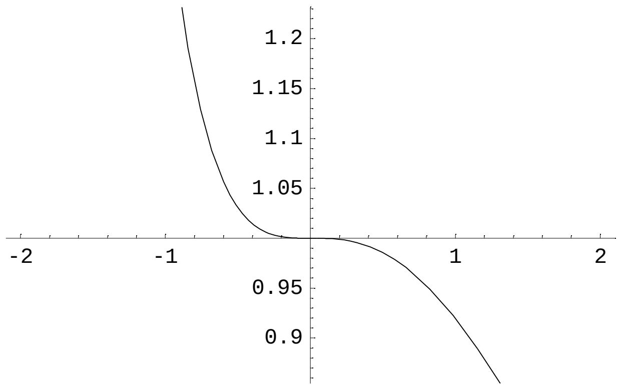
In[151]:=
```
Solve[{y[-1] == E/2,y[0] == 1},{C[1],C[2]}]
```

Out[151]=
$$\{\{C[2] \to 1,\ C[1] \to 1\}\}$$

In[152]:=
```
z[t_]:=(y[t]/.{C[1]->1,C[2]->1})//Evaluate;
z[t]
```

Out[153]=
$$e^{-t} + e^{-t}\,t + \frac{1}{2}\,e^{-t}\,t^2$$

In[154]:=
```
Plot[z[t],{t,-2,2}];
```



In[155]:=
```
len= NIntegrate[Evaluate[Sqrt[1+D[z[t],t]^2]],{t,-1,1}]
```

Out[155]=
2.12024

7E+M

In[156]:=
```
Remove["Global`*"];
```

a

In[157]:=
```
solv = Flatten[DSolve[{y''[x] +3 y'[x] - 4 y[x] == 0, y[0]==1, y'[0]==0},y,x]]
```

Out[157]=
$\{y \to \text{Function}[\{x\}, \frac{1}{5} e^{-4x} (1 + 4 e^{5x})]\}$

In[158]:=
```
Solve[x^2+3x-4==0,{x}]
```

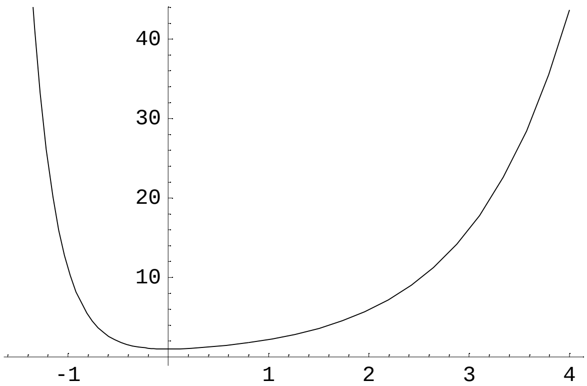
Out[158]=
$\{\{x \to -4\}, \{x \to 1\}\}$

In[159]:=
$\frac{1}{5} e^{-4x} (1 + 4 e^{5x})$ // **Simplify**

Out[159]=
$\frac{e^{-4x}}{5} + \frac{4 e^{x}}{5}$

b

In[160]:=
p1 = Plot$[\frac{1}{5} e^{-4x} (1 + 4 e^{5x}), \{x, -1.5, 4\}]$;



c

In[161]:=

```
solv = Flatten[DSolve[{y''[x] +3 y'[x] - 4 y[x] == 1, y[0]==1, y'[0]==0},y,x]]
```

Out[161]=

$$\left\{y \to \text{Function}\left[\{x\}, \frac{1}{4} e^{-4x} (1 - e^{4x} + 4 e^{5x})\right]\right\}$$

In[162]:=

$$\frac{1}{4} e^{-4x} (1 - e^{4x} + 4 e^{5x}) \; // \; \text{Simplify}$$

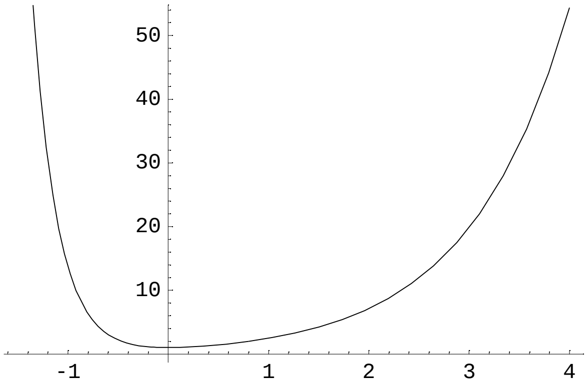
Out[162]=

$$-\frac{1}{4} + \frac{e^{-4x}}{4} + e^{x}$$

d

In[163]:=

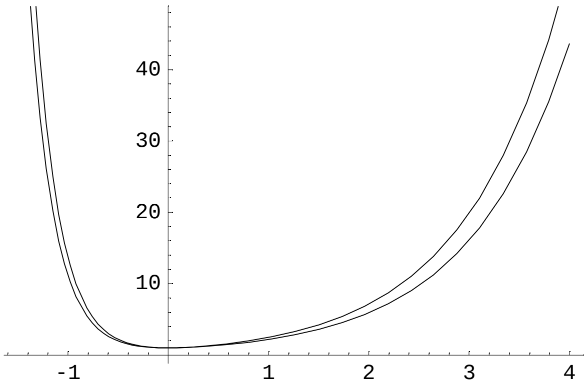
$$\text{p2} = \text{Plot}\left[\frac{1}{4} e^{-4x} (1 - e^{4x} + 4 e^{5x}), \{x, -1.5, 4\}\right];$$



e

In[164]:=

```
Show[p1,p2];
```



In[165]:=

$$d[x\_] := \frac{1}{4} e^{-4x} (1 - e^{4x} + 4 e^{5x}) - \frac{1}{5} e^{-4x} (1 + 4 e^{5x}) \text{ // Simplify;}$$

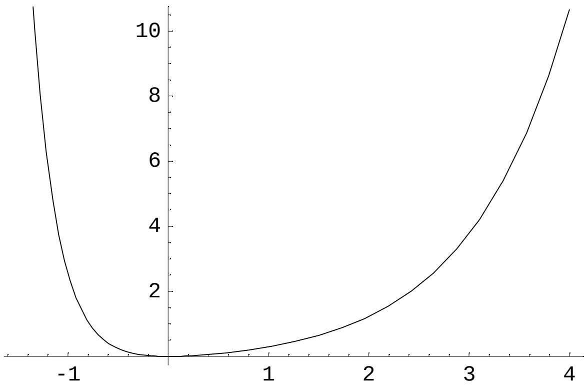
d[x]

Out[166]=

$$\frac{1}{20} (-5 + e^{-4x} + 4 e^{x})$$

In[167]:=

```
p3 = Plot[d[x], {x, -1.5, 4}];
```



In[168]:=

```
(* Anteile von E^x und E^(-x) ähnlich. *)
```

8E+M

a

In[169]:=

```
Sum[1/k! x^k,{k,0,Infinity}] -1-x
```

Out[169]=

$$-1 + e^{x} - x$$

In[170]:=

```
Integrate[Evaluate[Sum[1/k! x^k,{k,0,Infinity}] -1-x],{x,0,t}]
```

Out[170]=

$$-1 + e^{t} - t - \frac{t^2}{2}$$

In[171]:=

```
1/k! x^k
```

Out[171]=

$$\frac{x^k}{k!}$$

In[172]:=

```
Integrate[1/k! x^k,x]//Simplify
```

Out[172]=

$$\frac{x^{1+k}}{k! + k\,k!}$$

In[173]:=
```
(* Majorante ist Potenzreihe der e-Funktion - e^x-1-x-x^2/2 -C *)
```

b Diff., Indentität

In[174]:=
```
(Sin[x/2]+Cos[x/2])^2 // Expand // TrigReduce
```

Out[174]=
```
1 + Sin[x]
```

In[175]:=
```
Cos[x] == Integrate[1-(Sin[x/2]+Cos[x/2])^2,x]
```

Out[175]=
```
True
```

In[176]:=
```
(* Die Identität ist aber falsch, da links eine bestimmte Funktion steht, rechts
aber ein unbestimmtes Integral. Richtig nur für bestimmtes Integral mit spezieller
Wahl der Grenzen. *)
```

c Partielle Integration

In[177]:=
```
Integrate[(x-n+1)E^x,x] == E^x (x-n)
```

Out[177]=
```
True
```

In[178]:=
```
(* Die Identität ist aber falsch, da links eine bestimmte Funktion steht, rechts
aber ein unbestimmtes Integral. Richtig nur für bestimmtes Integral mit spezieller
Wahl der Grenzen. *)
```

d Diff, Indentität

In[179]:=
```
D[Log[(x Sqrt[x])]-Log[(x Sqrt[x-n])],{x,1}]//Simplify
```

Out[179]=

$$\frac{n}{2\, n\, x - 2\, x^2}$$

In[180]:=
```
Limit[1/Evaluate[D[Log[(x Sqrt[x])]-Log[(x Sqrt[x-n])],{x,1}]//Simplify], {x ->n}]
```

Out[180]=
```
{0}
```

In[181]:=
```
Limit[Evaluate[1/D[Log[(x Sqrt[x])]-Log[(x Sqrt[x-n])],{x,1}]//Simplify], {x
->n^2}]
```

Out[181]=

$$\{-2\,(-1+n)\, n^2\}$$

e

In[182]:=

```
q = Sqrt[2];
sinE[x_]:=E^(x/q) Sin[x/q];
cosE[x_]:=E^(x/q) Cos[x/q];
```

In[185]:=

```
Integrate[Integrate[sinE[x],x],x]
```

Out[185]=

$$-e^{\frac{x}{\sqrt{2}}} \text{Cos}\left[\frac{x}{\sqrt{2}}\right]$$

In[186]:=

```
Integrate[Integrate[cosE[x],x],x]
```

Out[186]=

$$e^{\frac{x}{\sqrt{2}}} \text{Sin}\left[\frac{x}{\sqrt{2}}\right]$$

In[187]:=

```
Integrate[Integrate[sinE[x],x],x] == -cosE[x]
```

Out[187]=

True

In[188]:=

```
Integrate[Integrate[cosE[x],x],x] == sinE[x]
```

Out[188]=

True