Lösungen Physik

1

```
Remove["Global`*"]

R[R1_, R2_, R3_, R4_, R5_, R6_] :=
 1 / (1 / R1 + 1 / R2 + 1 / (R3 + R4) + 1 / (R5 + R6)) // Together // Simplify

R[R1, R2, R3, R4, R5, R6]
```

$$\frac{R1\ R2\ (R3 + R4)\ (R5 + R6)}{R2\ (R3 + R4)\ (R5 + R6) + R1\ ((R3 + R4)\ (R5 + R6) + R2\ (R3 + R4 + R5 + R6))}$$

```
R[R1_, R4_, R6_] := R[R1, R1, R1, R4, R1, R6];
R[R1, R4, R6]
```

$$\frac{R1\ (R1 + R4)\ (R1 + R6)}{4\ R1^2 + 2\ R4\ R6 + 3\ R1\ (R4 + R6)}$$

```
f[x_, y_, z_, ΔR1_, ΔR4_, ΔR6_] :=
  (Abs[D[R[R1, R4, R6], R1]] ΔR1 + Abs[D[R[R1, R4, R6], R4]] ΔR4 +
     Abs[D[R[R1, R4, R6], R6]] ΔR6) /. {R1 → x, R4 → y, R6 → z};
f[x, y, z, ΔR1, ΔR4, ΔR6]
```

$$\triangle R6\ \mathrm{Abs}\left[-\frac{x\ (x + y)\ (3\ x + 2\ y)\ (x + z)}{(4\ x^2 + 2\ y\ z + 3\ x\ (y + z))^2} + \frac{x\ (x + y)}{4\ x^2 + 2\ y\ z + 3\ x\ (y + z)}\right] + \triangle R4\ \mathrm{Abs}\left[-\frac{x\ (x + y)\ (x + z)\ (3\ x + 2\ z)}{(4\ x^2 + 2\ y\ z + 3\ x\ (y + z))^2} + \frac{x\ (x + z)}{4\ x^2 + 2\ y\ z + 3\ x\ (y + z)}\right] + \triangle R1\ \mathrm{Abs}\left[-\frac{x\ (x + y)\ (x + z)\ (8\ x + 3\ (y + z))}{(4\ x^2 + 2\ y\ z + 3\ x\ (y + z))^2} + \frac{x\ (x + y)}{4\ x^2 + 2\ y\ z + 3\ x\ (y + z)} + \frac{x\ (x + z)}{4\ x^2 + 2\ y\ z + 3\ x\ (y + z)} + \frac{(x + y)\ (x + z)}{4\ x^2 + 2\ y\ z + 3\ x\ (y + z)}\right]$$

a

```
r1a = R[2, 2, 2, 10, 2, 10] // N

0.857143

r1 = R[2, 10, 10] // N

0.857143
```

b

```
dr1 = f[2, 10, 10, 0.05, 0.15, 0.15]
```

0.0204082

```
dr1u = R[2 - 0.05, 10 - 0.15, 10 - 0.15] // N
```

0.836727

```
r1 - dr1
```

0.836735

```
r1 - dr1u
```

0.0204156

```
dr1o = R[2 + 0.05, 10 + 0.15, 10 + 0.15] // N
```

0.877544

```
r1 + dr1
```

0.877551

```
dr1o - r1
```

0.020401

Lineare Fehleraproximation und extreme Werte stimmen überein.

c, d

```
R[2, 10, R6]
```

$$\frac{12\,(2+\text{R6})}{38+13\,\text{R6}}$$

```
D[R[2, 10, R6], R6] // Simplify
```

$$\frac{144}{(38+13\,\text{R6})^2}$$

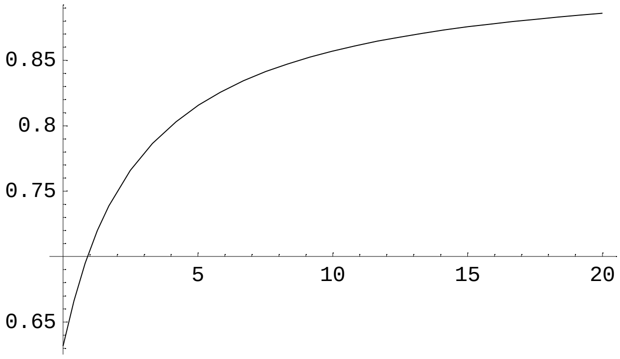
```
Evaluate[Simplify[D[R[2, 10, R6], R6]] == 0]
```

$$\frac{144}{(38+13\,\text{R6})^2} == 0$$

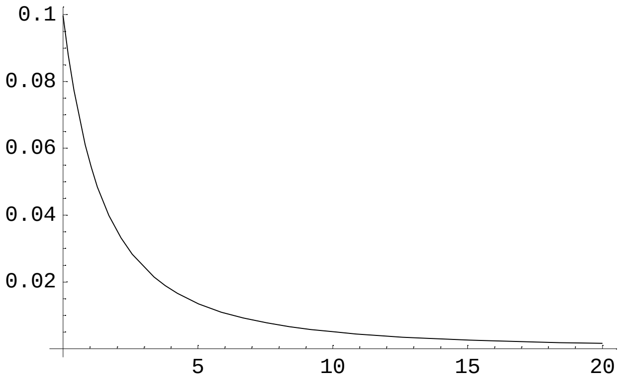
```
Solve[Evaluate[Simplify[D[R[2, 10, R6], R6]] == 0], {R6}] // Flatten
```

{}

```
Plot[{R[2, 10, R6]}, {R6, 0, 20}];
```




```
Plot[Evaluate[Simplify[D[R[2, 10, R6], R6]]], {R6, 0, 20}];
```



R total ist minimal für R6 = 0 und maximal für R6 = unendlich.

```
R[2, 10, 0]
```

$\frac{12}{19}$

```
R[2, 10, 0] // N
```

0.631579

```
R[2, 10, Infinity]
```

$\frac{12}{13}$

```
R[2, 10, Infinity] // N
```

0.923077

e

```
Solve[R[2, 2, 2, 10, 2, R6] == 2.8, {R6}]
```

{{R6 → -3.37705}}

```
Solve[R[2, 10, R6] == 2.8, {R6}]
```

{{R6 → -3.37705}}

```
Solve[R[2, 10, R6] == 2.8, {R6}]
```

```
{{R6 → -3.37705}}
```

Keine Lösung. R6 kann nicht negativ sein!

2

```
Remove["Global`*"]
```

```
m = 2.6 10^(-3); g = 9.81;  v0 = 340;
v[t_] := {Sin[45 Degree] v0 , Cos[45 Degree] v0 - g t};
v[t] // N
```

```
{240.416, 240.416 - 9.81 t}
```

```
solv2 = Solve[v[t][[2]] == 0, {t}] // Flatten
```

```
{t → 24.5073}
```

```
t1 = t /. solv2
```

```
24.5073
```

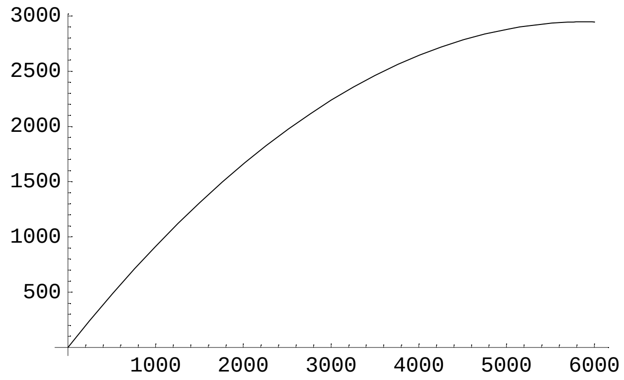
```
t2 = 2 t1
```

```
49.0145
```

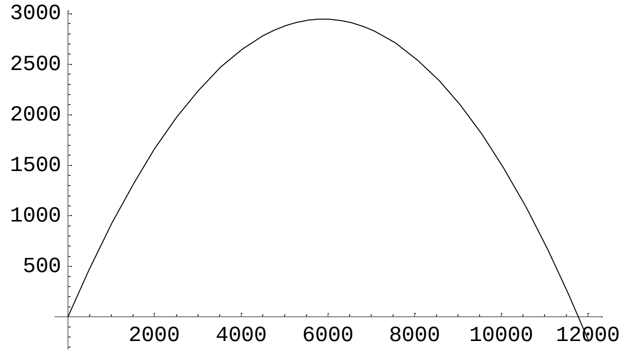
```
s[t_] := {Sin[45 Degree] v0 t, Cos[45 Degree] v0 t - 1 / 2 g t^2};
s[t] // N
```

```
{240.416 t, 240.416 t - 4.905 t^2}
```

```
ParametricPlot[s[t], {t, 0, 25}];
```



```
ParametricPlot[s[t], {t, 0, 50}];
```



a

```
vertMax = s[t1][[2]]
```

```
2945.97
```

b

```
horMax = s[t2][[1]]
```

```
11783.9
```

Die 20 km werden längst nicht erreicht.

c

```
en = 1 / 2 m v0^2  (* Symmetrie *)
```

```
150.28
```

```
en = 1 / 2 2.6 10^(-3) 340^2
```

```
150.28
```

3

```
Remove["Global`*"]
```

```
h = 1.5; m = 1; r = 0.02; α = 30 Degree; g = 9.81; J = 1 / 2 m r^2
```

```
0.0002
```

```
s = h / Sin[α]
```

```
3.
```

```
G = m g
```

9.81

```
F = G Sin[α]
```

4.905

```
a = F / m
```

4.905

a

```
ePot = m g h
```

14.715

```
eKinRot[v_] := 1 / 2 m v^2 + 1 / 2 J (v / r) ^2
```

```
eKinRot[v] == ePot
```

$0.75\, v^2 == 14.715$

```
solv[3] = Solve[eKinRot[v] == ePot, {v}]
```

{{v → -4.42945}, {v → 4.42945}}

```
vEnd = v /. solv[3][[2]]
```

4.42945

```
eRot = 1 / 2 J (vEnd / r) ^2
```

4.905

```
eKin = 1 / 2 m vEnd^2
```

9.81

```
eKin + eRot == ePot
```

True

b

```
ωEnd = vEnd / r
```

221.472

```
ωEnd == 2 Pi / Tu
```

$221.472 == \frac{2\pi}{Tu}$

```
solv[31] = Solve[ωEnd == 2 Pi / Tu, {Tu}] // Flatten;  Tu = Tu /. solv[31]
```

0.0283701

```
touren / 60 == 1 / Tu
```

$\frac{\text{touren}}{60} == 35.2484$

```
Solve[touren / 60 == 1 / Tu, {touren}]
```

```
{{touren → 2114.91}}
```

c

```
vEnd == aw tE
```

```
4.42945 == aw tE
```

```
gl1 = (s == 1 / 2 aw tE^2);
gl2 = (vEnd == aw tE);
solv3c = Solve[{gl1, gl2}, {aw, tE}] // Flatten
```

```
{aw → 3.27, tE → 1.35457}
```

```
tE == t /. solv3c
```

```
1.35457 == t
```

d

```
Δl = 0.04; l = 0.2;
```

```
FKin == cD Δl
```

```
FKin == 0.04 cD
```

```
eKin == 1 / 2 cD Δl^2 // Solve
```

```
{{cD → 12262.5}}
```

e

```
Remove[v]
```

Modellierung:

m = V1 ϱ = r1^2 Pi laenge ϱ
2m = V2 ϱ = r2^2 Pi laenge ϱ = 2 r1^2 Pi laenge ϱ
==> r2^2 = 2 r1^2

Jtotal = 1/2 (2m) r2^2 = m r2^2 = J + 1/2 m r1^2
J = m r2^2 - 1/2 m r1^2 = m 2 r1^2 - 1/2 m r1^2 = 3/2 m r1^2

Kontrolle mit Tabelle:
J = 1/2 m (r2^1+r1^2) = 1/2 m (2r1^2+r1^2) = 3/2 m r1^2

```
JNeu = 3 J
```

0.0006

```
eKinRotNeu[v_] := 1 / 2 m v^2 + 1 / 2 JNeu (v / r) ^2
```

```
eKinRotNeu[v] == ePot
```

$1.25\, v^2 == 14.715$

```
solv[3] = Solve[eKinRotNeu[v] == ePot, {v}]
```

{{v → -3.43103}, {v → 3.43103}}

```
vEndNeu = v /. solv[3][[2]]
```

3.43103

```
eRotNeu = 1 / 2 JNeu (vEndNeu / r) ^2
```

8.829

4

```
Remove["Global`*"]
```

```
d = 0.05; s1 = 0.1; temp1 = 273.15 + 18; temp2 = 273.15 + 100; p1 = 1;
V[s_] := d^2 Pi / 4 s;
V[s1]
```

0.00019635

a

```
solv41 = Solve[p1 / temp1 == p2 / temp2, {p2}] // Flatten
```

{p2 → 1.28164}

```
p2 = p2 /. solv41
```

1.28164

b

```
solv42 = Solve[p2 V[s1] == p1 V[s2] , {s2}] // Flatten
```

{s2 → 0.128164}

```
s2 = s2 /. solv42
```

0.128164

5

```
Remove["Global`*"]

m = 2; M = 5; s = 2; g = 9.81;
```

a

```
solv51 = Solve[a (m + M) == m g, {a}] // Flatten

{a → 2.80286}

a = a /. solv51

2.80286

solv52 = Solve[1 / 2 a t^2 == s, {t}] // Flatten

{t → -1.19462, t → 1.19462}

tEnd = t /. solv52[[2]]

1.19462
```

b

```
vEnd = a tEnd

3.34835

eKinm = 1 / 2 m vEnd^2

11.2114

Solve[m g s == (m + M) / 2 v^2, {v}]

{{v → -3.34835}, {v → 3.34835}}
```

c

```
solv53 =
 Solve[{1 / 2 M vEnd^2 == 1 / 2 M v1^2 + 1 / 2 M v2^2, M vEnd == M v1 + M v2}, {v1, v2}] // Chop

{{v1 → 0, v2 → 3.34835}, {v1 → 3.34835, v2 → 0}}
```

d

```
eKin2 = 1 / 2 M v2^2 /. solv53[[1]][[2]]

28.0286
```

6

a

```
Remove["Global`*"]

m = 1; h = 1; s = 0.02; g = 9.81;

ePot = m g h;
gl1 = (eKin == ePot);
gl2 = (eKin == 1 / 2 m v^2);
ePot

9.81

solv61 = Solve[{gl1, gl2}, {v}] // Flatten

{v → -4.42945, v → 4.42945}

vEnd = v /. solv61[[2]]

4.42945

a = vEnd / t

4.42945
-------
   t

s == 1 / 2 a t^2

0.02 == 2.21472 t

solv62 = Solve[s == 1 / 2 a t^2, {t}] // Flatten

{t → 0.00903047}

tVer = t /. solv62

0.00903047

imp = m vEnd

4.42945

F = imp / tVer

490.5
```

b

```
Remove["Global`*"]

u = 230 volt; temp2 = 100 K; temp1 = 20 K; V = 0.1^3 m^3;
t = 30 sec; ϱ = 1 kg / (0.1^3 m^3); spW = 4187 J / (kg K);
```

```
W[i_] = u i t /. {sec volt → J / amp}
```

$\frac{6900\, i\, J}{amp}$

```
Solve[W[i] == spW (temp2 - temp1) V ρ, {i}]
```

{{i → 48.5449 amp}}

c

```
Remove["Global`*"]
```

```
k = 6.674 10^-11; mE = 5.9736 10^24 ; m = 892; tU = 6.4 60 60; rErde = 6371 10^3
```

6371000

```
v = 2 r Pi / tU; Fz[r_] := m v^2 / r; Fz[r]
```

0.0000663376 r

```
F1[r_] := k m mE / r^2; F1[r]
```

$\frac{3.55621\times 10^{17}}{r^2}$

```
solv6c = Solve[F1[r] == Fz[r], {r}] // Flatten
```

$\{r \to -8.75076\times 10^6 - 1.51568\times 10^7\, i,\ r \to -8.75076\times 10^6 + 1.51568\times 10^7\, i,\ r \to 1.75015\times 10^7\}$

```
rU = r /. solv6c[[3]]
```

1.75015×10^7

```
rU - rErde
```

1.11305×10^7

Anderer Weg

```
Remove["Global`*"]
```

```
k = 6.674 10^-11 ; mE = 5.9736 10^24 ; m = 892 ; tU = 6.4 60 60 ; rErde = 6371 10^3 ;
```

```
gl1 = (m r ω^2 == γ M m / r^2 /. {γ M / r^2 → gr, ω → (2 Pi) / T}) // Simplify
```

$\frac{4\pi^2 r}{T} == gr\, T$

```
solve6c1 = Solve[{γ M == gr r^2 /. {γ M -> g rErde^2}}, {gr}] // Flatten
```

$\{gr \to \frac{40589641000000\, g}{r^2}\}$

```
gr = gr /. solve6c1
```

$\frac{40589641000000\, g}{r^2}$

```
solv6c2 = (Solve[gl1, {r}] /. {g -> 9.81, T → tU}) // Flatten
```

{r → −8.74715 × 10^6 + 1.51505 × 10^7 ⅈ, r → −8.74715 × 10^6 − 1.51505 × 10^7 ⅈ, r → 1.74943 × 10^7}

```
r = r /. solv6c2[[3]]; r - rErde
```

1.11233 × 10^7

d

```
Remove["Global`*"]
```

```
s = 0.25; λ = 2 s; v = 337;
```

```
Solve[v == λ f, {f}]
```

{{f → 674.}}

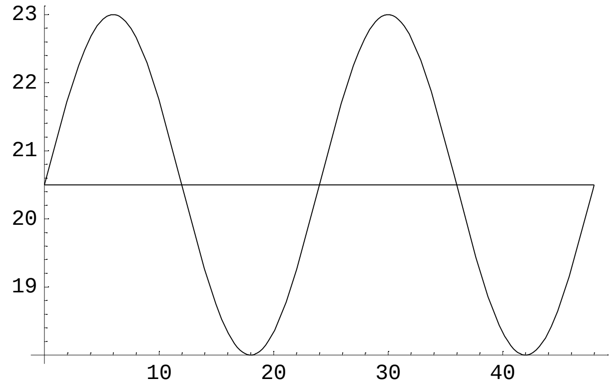
Das ist etwa e gegen f

e

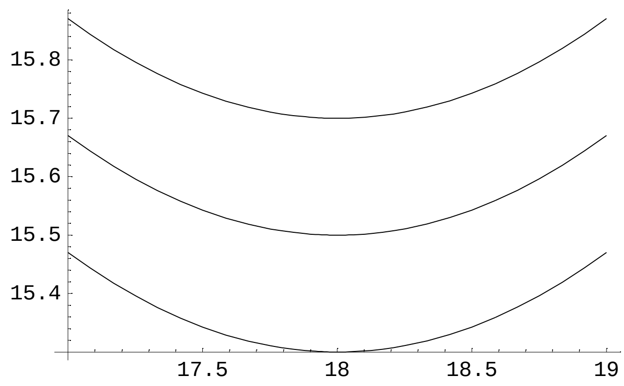
```
Remove["Global`*"]
```

```
ϑ1 = 20.5; ϑ2 = 2.6; Δϑ1 = 0.3; Δϑ2 = 0.3; t1 = 18; Δt = 1;
```

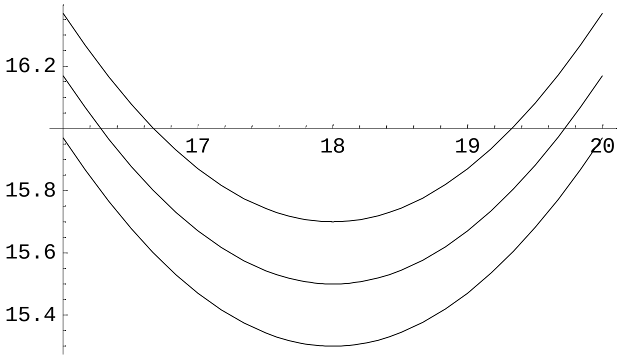
```
f[t_, ϑ1_, ϑ2_] := ϑ1 + ϑ2 Sin[2 Pi t / 24];
Plot[{f[t, 20.5, 2.5], 20.5}, {t, 0, 48}];
```



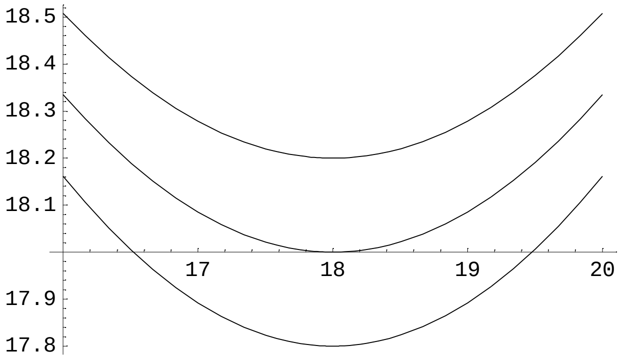

```
Plot[{f[t, 20.5, 5], f[t, 20.7, 5], f[t, 20.3, 5]}, {t, 17, 19}];
```



```
Plot[{f[t, 20.5, 5], f[t, 20.7, 5], f[t, 20.3, 5]}, {t, 16, 20}];
```




```
Plot[{f[t, 20.5, 2.5], f[t, 20.5, 2.3], f[t, 20.5, 2.7]}, {t, 16, 20}];
```




```
M1 = {f[t, 20.5, 2.6], f[t, 20.2, 2.3],
   f[t, 20.8, 2.3], f[t, 20.2, 2.9], f[t, 20.8, 2.9]} /. {t → 18 }

{17.9, 17.9, 18.5, 17.3, 17.9}

M2 = {f[t, 20.5, 2.6], f[t, 20.2, 2.3],
   f[t, 20.8, 2.3], f[t, 20.2, 2.9], f[t, 20.8, 2.9]}  /. {t → 17 }

{17.9886, 17.9784, 18.5784, 17.3988, 17.9988}

M3 = {f[t, 20.5, 2.6], f[t, 20.2, 2.3],
   f[t, 20.8, 2.3], f[t, 20.2, 2.9], f[t, 20.8, 2.9]}  /. {t → 19}

{17.9886, 17.9784, 18.5784, 17.3988, 17.9988}

m1 = Min[Union[M1, M2, M3]]

17.3

m2 = Max[Union[M1, M2, M3]]

18.5784

(Max[Union[M1, M2, M3]] - Min[Union[M1, M2, M3]]) / 2

0.639185

f1[t_, h1_, h2_, Δt_, Δh1_, Δh2_] := (Abs[Evaluate[D[x + y Sin[2 Pi z / 24], x]]] Δh1 +
     Abs[Evaluate[D[x + y Sin[2 Pi z / 24], y]]] Δh2 +
     Abs[Evaluate[D[x + y Sin[2 Pi z / 24], z]]] Δt ) /. {x → h1, y → h2, z → t};
```

```
diff = f1[18, 20.5, 2.6, 2, 0.1, 0.2]
```

```
0.3
```

```
val = f[18, 20.5, 2.6]
```

```
17.9
```

```
m3 = val - diff
```

```
17.6
```

```
m4 = val + diff
```

```
18.2
```

```
{m1, m3, val, m4, m2}
```

```
{17.3, 17.6, 17.9, 18.2, 18.5784}
```

f

```
Remove["Global`*"]
```

```
v1 = 0; h1 = 1; h2 = 0; g = 9.81;
```

```
gl = (ϱ v1^2 / 2 + ϱ g h1 + p == ϱ v2^2 / 2 + ϱ g h2 + p);
solv6f = Solve[gl, {v2}]
```

```
{{v2 → -4.42945}, {v2 → 4.42945}}
```

```
v2 = v2 /. solv6f[[2]]
```

```
4.42945
```

Anderer Weg:

```
Sqrt[2 g h1]
```

```
4.42945
```

Oben muss soviel Wasser reinkommen wie unten rausgeht. Da mit den Rohrdurchmessern auch die Volumen pro Zeit, also die Geschwindikgeiten gleich sind, ist oben v =4.429...