

# Lösungen

1

```
rem; Remove["Global`*"]
```

```
M = {{2, -2, 3}, {-3, 1, 3}, {-2, -1, 2}};
```

```
MatrixForm[M]
```

$$\begin{pmatrix} 2 & -2 & 3 \\ -3 & 1 & 3 \\ -2 & -1 & 2 \end{pmatrix}$$

```
MM = M + M;
```

```
MatrixForm[MM]
```

$$\begin{pmatrix} 4 & -4 & 6 \\ -6 & 2 & 6 \\ -4 & -2 & 4 \end{pmatrix}$$

■ a

```
Det[M]
```

25

```
Det[MM]
```

200

■ b

Det[M] ungleich 0, Inverse existiert

■ c

```
{Inverse[M] // MatrixForm, Inverse[M] // N // MatrixForm}
```

$$\left\{ \begin{pmatrix} \frac{1}{5} & \frac{1}{25} & -\frac{9}{25} \\ 0 & \frac{2}{5} & -\frac{3}{5} \\ \frac{1}{5} & \frac{6}{25} & -\frac{4}{25} \end{pmatrix}, \begin{pmatrix} 0.2 & 0.04 & -0.36 \\ 0. & 0.4 & -0.6 \\ 0.2 & 0.24 & -0.16 \end{pmatrix} \right\}$$

## ■ d

```
{Inverse[Transpose[M]] // MatrixForm, Inverse[Transpose[M]] // N // MatrixForm}
```

$$\left\{ \begin{pmatrix} \frac{1}{5} & 0 & \frac{1}{5} \\ \frac{1}{25} & \frac{2}{5} & \frac{6}{25} \\ -\frac{9}{25} & -\frac{3}{5} & -\frac{4}{25} \end{pmatrix}, \begin{pmatrix} 0.2 & 0. & 0.2 \\ 0.04 & 0.4 & 0.24 \\ -0.36 & -0.6 & -0.16 \end{pmatrix} \right\}$$

```
{Transpose[Inverse[M]] // MatrixForm, Transpose[Inverse[M]] // N // MatrixForm}
```

$$\left\{ \begin{pmatrix} \frac{1}{5} & 0 & \frac{1}{5} \\ \frac{1}{25} & \frac{2}{5} & \frac{6}{25} \\ -\frac{9}{25} & -\frac{3}{5} & -\frac{4}{25} \end{pmatrix}, \begin{pmatrix} 0.2 & 0. & 0.2 \\ 0.04 & 0.4 & 0.24 \\ -0.36 & -0.6 & -0.16 \end{pmatrix} \right\}$$

```
{Inverse[M] // MatrixForm, Inverse[Transpose[M]] // MatrixForm}
```

$$\left\{ \begin{pmatrix} \frac{1}{5} & \frac{1}{25} & -\frac{9}{25} \\ 0 & \frac{2}{5} & -\frac{3}{5} \\ \frac{1}{5} & \frac{6}{25} & -\frac{4}{25} \end{pmatrix}, \begin{pmatrix} \frac{1}{5} & 0 & \frac{1}{5} \\ \frac{1}{25} & \frac{2}{5} & \frac{6}{25} \\ -\frac{9}{25} & -\frac{3}{5} & -\frac{4}{25} \end{pmatrix} \right\}$$

Die Transponierte der Inversen ist die Inverse der Transponierten.

## ■ e

```
Inverse[Transpose[M]] == Transpose[Inverse[M]]
```

```
True
```

## ■ f

```
IM = Inverse[M];
```

```
IM7 = IM.IM.IM.IM.IM.IM.IM; IM7 // MatrixForm (* exakt *)
```

$$\begin{pmatrix} -\frac{823454}{1220703125} & -\frac{3516614}{6103515625} & \frac{974151}{6103515625} \\ -\frac{245403}{244140625} & -\frac{659023}{1220703125} & -\frac{493293}{1220703125} \\ -\frac{244574}{1220703125} & \frac{1395591}{6103515625} & -\frac{9278194}{6103515625} \end{pmatrix}$$

```
IM7 // N // MatrixForm (* nicht exakt *)
```

$$\begin{pmatrix} -0.000674574 & -0.000576162 & 0.000159605 \\ -0.00100517 & -0.000539872 & -0.000404106 \\ -0.000200355 & 0.000228654 & -0.00152014 \end{pmatrix}$$

```
Det[IM]
```

$$\frac{1}{25}$$

```
Det[IM]^7
```

$$\frac{1}{6103515625}$$

**Det[IM7]**

$$\frac{1}{6103515625}$$

### ■ g

**OP0 = {{2}, {5}, {8}}; OP0 // MatrixForm**

$$\begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix}$$

**OP1 = M.OP0; OP1 // MatrixForm**

$$\begin{pmatrix} 18 \\ 23 \\ 7 \end{pmatrix}$$

**{OP2 = Inverse[M].OP0; OP2 // MatrixForm,  
OP2 = Inverse[M].OP0; OP2 // N // MatrixForm}**

$$\left\{ \begin{pmatrix} -\frac{57}{25} \\ -\frac{14}{5} \\ \frac{8}{25} \end{pmatrix}, \begin{pmatrix} -2.28 \\ -2.8 \\ 0.32 \end{pmatrix} \right\}$$

**{Norm[OP2 - OP1], Norm[OP2 - OP1] // N}**

$$\left\{ \frac{\sqrt{700963}}{25}, 33.4894 \right\}$$

### ■ h

**M2 = M.M; M2 // MatrixForm**

$$\begin{pmatrix} 4 & -9 & 6 \\ -15 & 4 & 0 \\ -5 & 1 & -5 \end{pmatrix}$$

**M2.OP2 == OP1**

True

### ■ i

**Eigenvalues[M] // N**

{3.83195, 0.584025 + 2.48657 i, 0.584025 - 2.48657 i}

**Det[M - λ IdentityMatrix[3]] == 0**

$$25 - 11\lambda + 5\lambda^2 - \lambda^3 = 0$$

**Solve[Det[M - λ IdentityMatrix[3]] == 0, {λ}]**

$$\left\{ \left\{ \lambda \rightarrow \frac{1}{3} \left( 5 - \frac{8}{(215 + 9\sqrt{577})^{1/3}} + (215 + 9\sqrt{577})^{1/3} \right) \right\}, \right.$$

$$\left. \left\{ \lambda \rightarrow \frac{5}{3} + \frac{4(1 + i\sqrt{3})}{3(215 + 9\sqrt{577})^{1/3}} - \frac{1}{6}(1 - i\sqrt{3})(215 + 9\sqrt{577})^{1/3} \right\}, \right.$$

$$\left. \left\{ \lambda \rightarrow \frac{5}{3} + \frac{4(1 - i\sqrt{3})}{3(215 + 9\sqrt{577})^{1/3}} - \frac{1}{6}(1 + i\sqrt{3})(215 + 9\sqrt{577})^{1/3} \right\} \right\}$$

**Eigenvectors[M] // N**

{{-3.07369, 4.31542, 1.}, {0.322557 - 0.770142 i, 0.770862 - 0.946281 i, 1.},  
{0.322557 + 0.770142 i, 0.770862 + 0.946281 i, 1.}}

■ **Aufgabe kann auch anders interpretiert werden:**

**Eigenvalues[M2] // N**

{14.6838, -5.84192 + 2.90443 i, -5.84192 - 2.90443 i}

**Eigenvectors[M2] // N**

{{-3.07369, 4.31542, 1.}, {0.322557 - 0.770142 i, 0.770862 - 0.946281 i, 1.},  
{0.322557 + 0.770142 i, 0.770862 + 0.946281 i, 1.}}

## 2

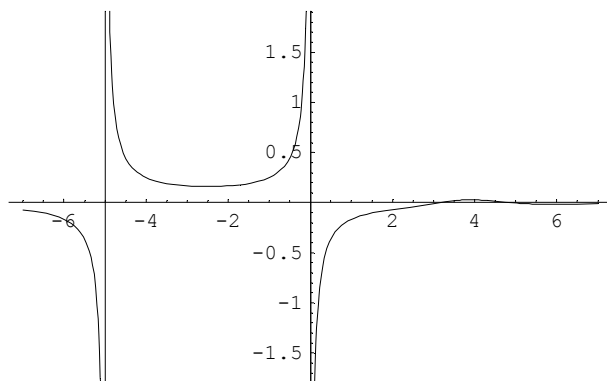
**rem; Remove["Global`\*"]**

**f[x\_] := (2 E^(-(x - 4)^2) - 1) / (x (x + 5)); f[x]**

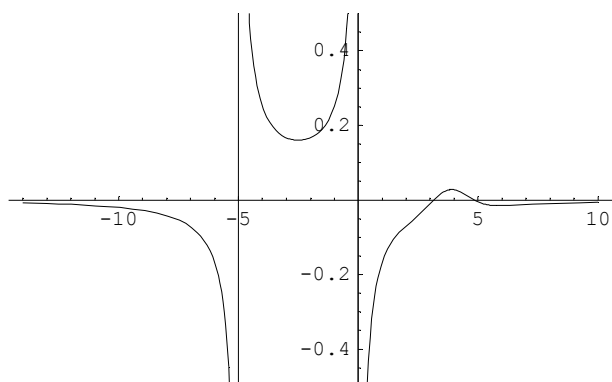
$$\frac{-1 + 2 e^{-(4+x)^2}}{x(5+x)}$$

■ **a**

**Plot[f[x], {x, -7, 7}];**



```
Plot[f[x], {x, -14, 10}, PlotRange -> {-0.5, 0.5}];
```



■ b

```
f'[x]
```

$$-\frac{-1 + 2 e^{-(4+x)^2}}{x (5+x)^2} - \frac{-1 + 2 e^{-(4+x)^2}}{x^2 (5+x)} - \frac{4 e^{-(4+x)^2} (-4+x)}{x (5+x)}$$

```
f'[x] /. x -> -1
```

$$\frac{1}{4} \left(1 - \frac{2}{e^{25}}\right) + \frac{1}{16} \left(-1 + \frac{2}{e^{25}}\right) - \frac{5}{e^{25}}$$

```
steig = (f'[x] /. x -> -1) // N
```

```
0.1875
```

```
ArcTan[steig]
```

```
0.185348
```

```
ArcTan[steig] / Degree // N
```

```
10.6197
```

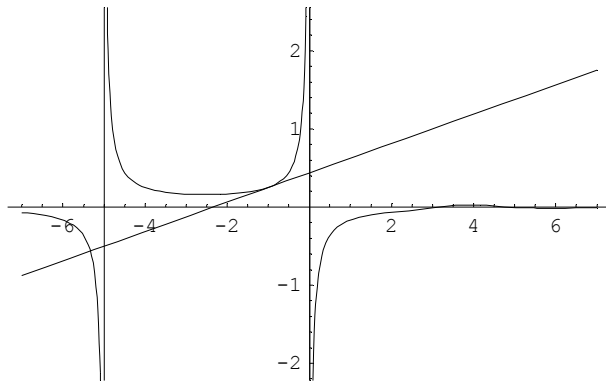
```
g[x_] := f[-1] + steig (x - (-1)); g[x]
```

$$\frac{1}{4} \left(1 - \frac{2}{e^{25}}\right) + 0.1875 (1+x)$$

```
g[x] // N
```

```
0.25 + 0.1875 (1. + x)
```

```
Plot[{f[x], g[x]}, {x, -7, 7}, Epilog -> {PointSize[0.03], Point[{-1, f[-1]}]}];
```



■ c

■ c1

```
Solve[f[x] == 0, {x}]
```

```
{{x -> 4 - Sqrt[Log[2]}, {x -> 4 + Sqrt[Log[2]]}}
```

```
solv1 = NSolve[f[x] == 0, {x}]
```

```
{{x -> 3.16745}, {x -> 4.83255}}
```

```
{x1 = x /. solv1[[1]], x2 = x /. solv1[[2]]}
```

```
{3.16745, 4.83255}
```

■ c2

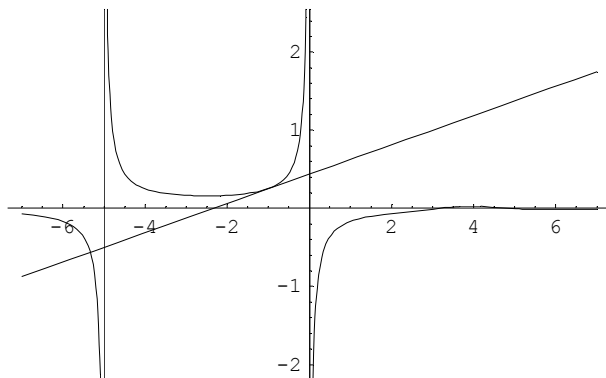
```
solv2 = Solve[1/f[x] == 0, {x}]
```

```
{{x -> -5}, {x -> 0}}
```

```
{x3 = x /. solv2[[1]], x4 = x /. solv2[[2]]}
```

```
{-5, 0}
```

```
Plot[{f[x], g[x]}, {x, -7, 7}, Epilog -> {PointSize[0.03], Point[{-1, f[-1]}],  
Point[{x1, f[x1]}], Point[{x2, f[x2]}], Point[{x3, 0}], Point[{x4, 0}]}];
```



## ■ d

**f'[x]**

$$-\frac{-1+2e^{-(4+x)^2}}{x(5+x)^2} - \frac{-1+2e^{-(4+x)^2}}{x^2(5+x)} - \frac{4e^{-(4+x)^2}(-4+x)}{x(5+x)}$$

**solv3 = FindRoot[f'[x] == 0, {x, -3}]**

{x → -2.5}

**x5 = x /. solv3**

-2.5

**solv4 = FindRoot[f'[x] == 0, {x, 4}]**

{x → 3.90875}

**x6 = x /. solv4**

3.90875

**solv5 = FindRoot[f'[x] == 0, {x, 5}]**

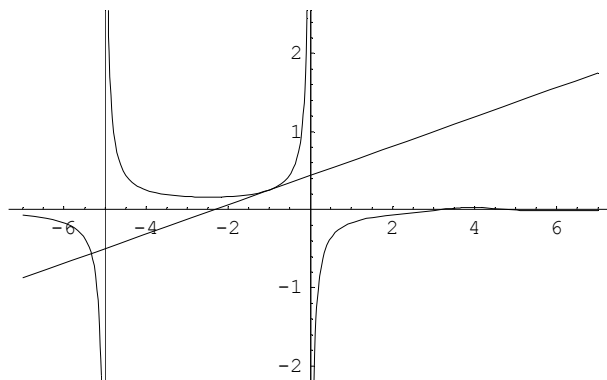
{x → 5.84439}

**x7 = x /. solv5**

5.84439

**Plot[{f[x], g[x]}, {x, -7, 7},**

**Epilog → {PointSize[0.03], Point[{-1, f[-1]}], Point[{x1, f[x1]}],**  
**Point[{x2, f[x2]}], Point[{x3, 0}], Point[{x4, 0}], Point[{x5, f[x5]}],**  
**Point[{x6, f[x6]}], Point[{x7, f[x7]}]}];**



## ■ e

**solv6 = FindRoot[Evaluate[f''[x] == 0], {x, 2}]**

{x → 2.20542}

```

solv7 = FindRoot[Evaluate[f''[x] == 0], {x, 3}]
{x → 3.00981}

solv8 = FindRoot[Evaluate[f''[x] == 0], {x, 4}]
{x → 4.55421}

solv9 = FindRoot[Evaluate[f''[x] == 0], {x, 7}]
{x → 6.52488}

x8 = x /. solv6
2.20542

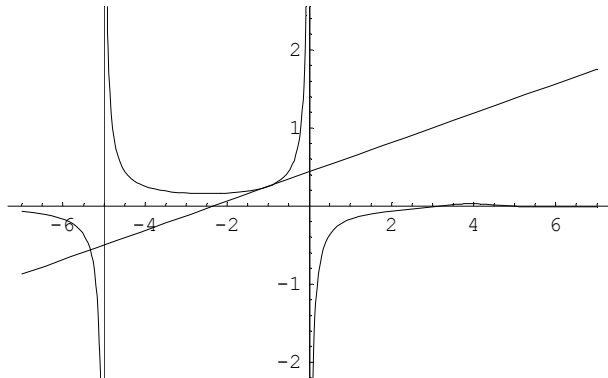
x9 = x /. solv7
3.00981

x10 = x /. solv8
4.55421

x11 = x /. solv9
6.52488

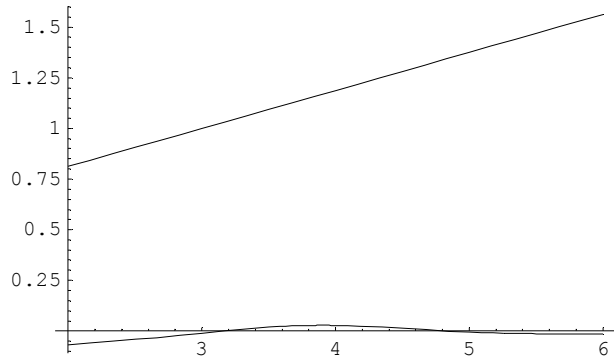
Plot[{f[x], g[x]}, {x, -7, 7},
  Epilog → {PointSize[0.03], Point[{-1, f[-1]}], Point[{x1, f[x1]}],
    Point[{x2, f[x2]}], Point[{x3, 0}], Point[{x4, 0}], Point[{x5, f[x5]}],
    Point[{x6, f[x6]}], Point[{x7, f[x7]}], Point[{x8, f[x8]}],
    Point[{x9, f[x9]}], Point[{x10, f[x10]}], Point[{x11, f[x11]}]}];

```





```
Plot[{f[x], g[x]}, {x, 2, 6},
  Epilog -> {PointSize[0.03], Point[{-1, f[-1]}], Point[{x1, f[x1]}],
    Point[{x2, f[x2]}], Point[{x3, 0}], Point[{x4, 0}], Point[{x5, f[x5]}],
    Point[{x6, f[x6]}], Point[{x7, f[x7]}], Point[{x8, f[x8]}],
    Point[{x9, f[x9]}], Point[{x10, f[x10]}], Point[{x11, f[x11]}]}];
```



### ■ f

```
Limit[f[x], x -> Infinity]
```

0

```
Limit[f[x], x -> -Infinity]
```

0

### ■ g

```
ser = Series[f[x], {x, 4, 3}]
```

$$\frac{1}{36} - \frac{13(x-4)}{1296} - \frac{2459(x-4)^2}{46656} + \frac{32435(x-4)^3}{1679616} + O[x-4]^4$$

```
N[%]
```

$$0.0277778 - 0.0100309(x-4.) - 0.0527049(x-4.)^2 + 0.019311(x-4.)^3 + O[x-4.]^4$$

```
serN = Series[f[x], {x, 4, 3}] // Normal
```

$$\frac{1}{36} - \frac{13(-4+x)}{1296} - \frac{2459(-4+x)^2}{46656} + \frac{32435(-4+x)^3}{1679616}$$

```
N[%]
```

$$0.0277778 - 0.0100309(-4.+x) - 0.0527049(-4.+x)^2 + 0.019311(-4.+x)^3$$

```
ExpandAll[Evaluate[serN]]
```

$$-\frac{13196}{6561} + \frac{23419x}{17496} - \frac{9953x^2}{34992} + \frac{32435x^3}{1679616}$$

```
N[%]
```

$$-2.01128 + 1.33853x - 0.284436x^2 + 0.019311x^3$$

## ■ h

```
Integrate[- $\frac{13196}{6561} + \frac{23419 x}{17496} - \frac{9953 x^2}{34992} + \frac{32435 x^3}{1679616}$ , {x, 3, 5}]
```

```
 $\frac{1429}{69984}$ 
```

```
int1 = Integrate[-2.0112787684804148` + 1.3385345221764975` x -  
0.284436442615455` x2 + 0.019310961553116904` x3, {x, 3, 5}]
```

```
0.020419
```

## ■ i

```
int2 = NIntegrate[f[x], {x, 3, 5}]
```

```
0.0276849
```

```
Abs[int1 - int2]
```

```
0.00726595
```

## 3

```
rem; Remove["Global`*"]
```

```
x1 = 0; x2 = Pi; y1 = 0; y2 = 3 Pi / 2;
```

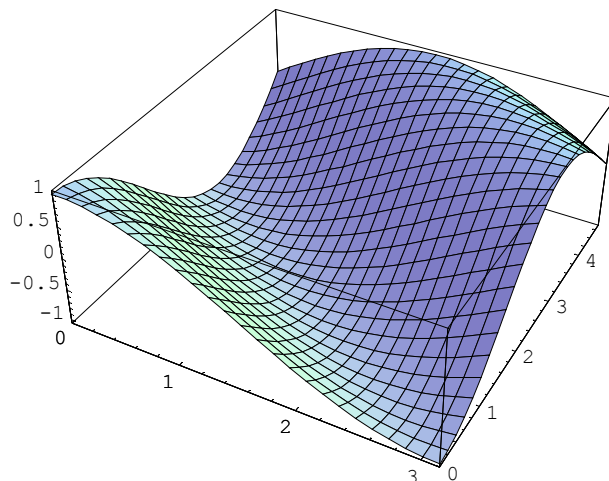
```
f1[x_, y_] := Cos[x + y];
```

```
f2[x_, y_] := Cos[x] + Sin[y];
```

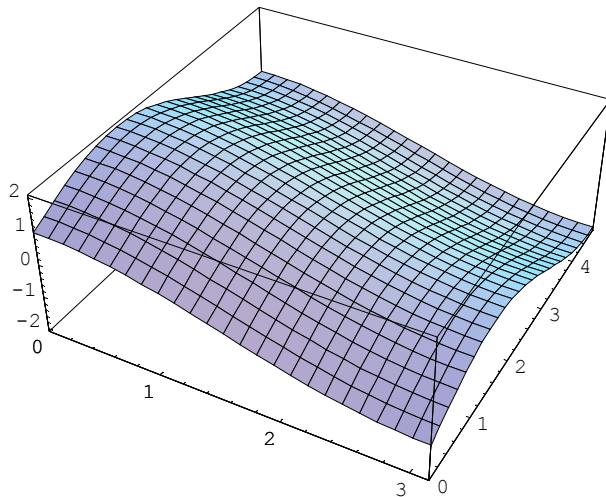
```
f3[x_, y_] := Cos[x] Sin[y];
```

## ■ a

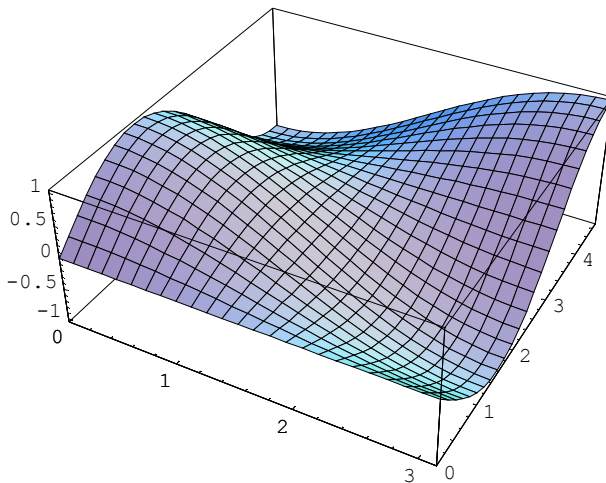
```
Plot3D[f1[x, y], {x, x1, x2}, {y, y1, y2}];
```



```
Plot3D[f2[x, y], {x, x1, x2}, {y, y1, y2}];
```



```
Plot3D[f3[x, y], {x, x1, x2}, {y, y1, y2}];
```



## ■ b

```
A1[y_] := Integrate[f1[x, y], {x, x1, x2}]; A1[y]
```

```
-2 Sin[y]
```

```
A2[y_] := Integrate[f2[x, y], {x, x1, x2}]; A2[y]
```

```
 $\pi$  Sin[y]
```

```
A3[y_] := Integrate[f3[x, y], {x, x1, x2}]; A3[y]
```

```
0
```

## ■ c

```
V1 = Integrate[A1[y], {y, y1, y2}]
```

```
-2
```

```
V2 = Integrate[A2[y], {y, y1, y2}]
```

```
 $\pi$ 
```

```
V3 = Integrate[A3[y], {y, y1, y2}]
```

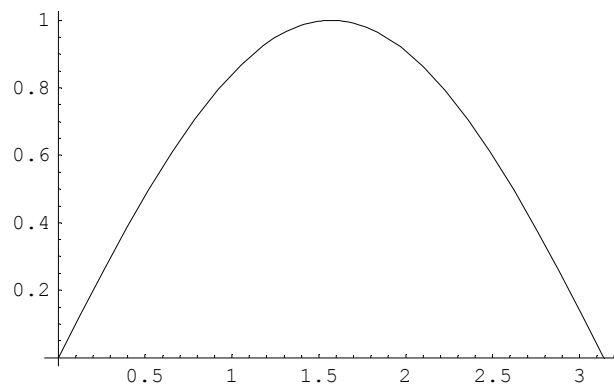
```
0
```

### ■ d

```
D[f2[x, y], x] == 0
```

```
-Sin[x] == 0
```

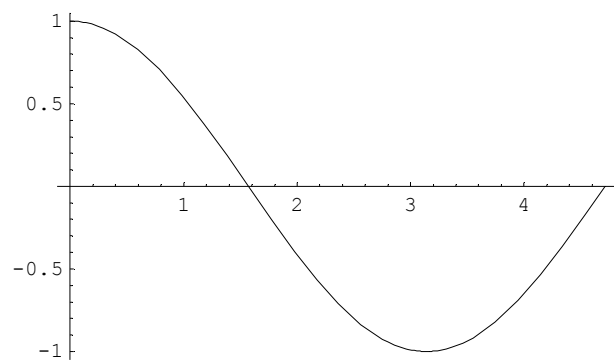
```
Plot[Sin[x], {x, x1, x2}];
```



```
D[f2[x, y], y] == 0
```

```
Cos[y] == 0
```

```
Plot[Cos[y], {y, y1, y2}];
```



```
Cos[Pi / 2]
```

```
0
```

```
Cos[3 Pi / 2]
```

```
0
```

```
f2[0, Pi / 2]
```

```
2
```

```
f2[0, 3 Pi / 2]
```

```
0
```

```
f2[Pi, Pi / 2]
```

```
0
```

```
f2[Pi, 3 Pi / 2]
```

```
-2
```

Minimalstelle bei {Pi, 3Pi/2}

## 4

```
rem; Remove["Global`*"]
```

```
{d / 2 == x4 Sqrt[2] / 2, h^2 == 4^2 - (d / 2)^2}
```

$$\left\{ \frac{d}{2} = \frac{x4}{\sqrt{2}}, h^2 = 16 - \frac{d^2}{4} \right\}$$

```
solv1 = Solve[d / 2 == x4 Sqrt[2] / 2, {d}] // Flatten
```

```
{d -> Sqrt[2] x4}
```

```
d[x4_] = d /. solv1; d[k]
```

```
Sqrt[2] k
```

```
d[x4]
```

```
Sqrt[2] x4
```

$$h^2 = 16 - \frac{d[x4]^2}{4}$$

$$h^2 = 16 - \frac{x4^2}{2}$$

```
solv2 = Solve[h^2 == 16 - \frac{x4^2}{2}, {h}] // Flatten
```

$$\left\{ h \rightarrow -\frac{\sqrt{32 - x4^2}}{\sqrt{2}}, h \rightarrow \frac{\sqrt{32 - x4^2}}{\sqrt{2}} \right\}$$

```
h[x4_] = h /. solv2[[2]]
```

$$\frac{\sqrt{32 - x4^2}}{\sqrt{2}}$$

```
h[k]
```

$$\frac{\sqrt{32 - k^2}}{\sqrt{2}}$$

■ a

**V4[x4\_] := x4^2 h[x4] / 3; V4[x4]**

$$\frac{x4^2 \sqrt{32 - x4^2}}{3 \sqrt{2}}$$

**V4'[x4]**

$$-\frac{x4^3}{3 \sqrt{2} \sqrt{32 - x4^2}} + \frac{1}{3} \sqrt{2} x4 \sqrt{32 - x4^2}$$

**solv3 = Solve[Evaluate[V4'[x4] == 0], {x4}] // Flatten**

$$\left\{ x4 \rightarrow 0, x4 \rightarrow -\frac{8}{\sqrt{3}}, x4 \rightarrow \frac{8}{\sqrt{3}} \right\}$$

**N[%]**

$$\{x4 \rightarrow 0., x4 \rightarrow -4.6188, x4 \rightarrow 4.6188\}$$

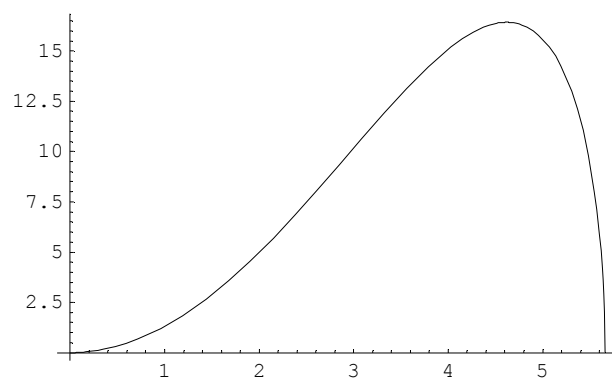
**xMax = x4 /. solv3[[3]]**

$$\frac{8}{\sqrt{3}}$$

**V4[k]**

$$\frac{k^2 \sqrt{32 - k^2}}{3 \sqrt{2}}$$

**Plot[V4[x4], {x4, 0, 2 4 / Sqrt[2]}];**



x0 → 4.6188

■ b

**V4[x4]**

$$\frac{x4^2 \sqrt{32 - x4^2}}{3 \sqrt{2}}$$

**V4[x4] /. x4 -> xMax**

$$\frac{256}{9\sqrt{3}}$$

**N[%]**

16.4224

## ■ C

**rem; Remove["Global`\*"]**

**{d^2 == x3^2 - (x3/2)^2, s^2 + (x3/2)^2 == 4^2}**

$$\left\{d^2 == \frac{3x3^2}{4}, s^2 + \frac{x3^2}{4} == 16\right\}$$

**solv1 = Solve[d^2 == x3^2 - (x3/2)^2, {d}] // Flatten**

$$\left\{d \rightarrow -\frac{\sqrt{3}x3}{2}, d \rightarrow \frac{\sqrt{3}x3}{2}\right\}$$

**d[x3\_] = d /. solv1[[2]]; d[k]**

$$\frac{\sqrt{3}k}{2}$$

**d[x3]**

$$\frac{\sqrt{3}x3}{2}$$

**solv2 = Solve[s^2 + (x3/2)^2 == 4^2, {s}] // Flatten**

$$\left\{s \rightarrow -\frac{1}{2}\sqrt{64 - x3^2}, s \rightarrow \frac{\sqrt{64 - x3^2}}{2}\right\}$$

**s[x3\_] = s /. solv2[[2]]; s[k]**

$$\frac{\sqrt{64 - k^2}}{2}$$

**s[x3]**

$$\frac{\sqrt{64 - x3^2}}{2}$$

**h[x3\_] := Sqrt[s[x3]^2 - (d[x3]/3)^2]; h[k]**

$$\sqrt{-\frac{k^2}{12} + \frac{1}{4}(64 - k^2)}$$

**h[x3]**

$$\sqrt{-\frac{x3^2}{12} + \frac{1}{4}(64 - x3^2)}$$

**V3[x3\_] := x3 d[x3] / 2 h[x3] / 3; V3[k]**

$$\frac{k^2 \sqrt{-\frac{k^2}{12} + \frac{1}{4} (64 - k^2)}}{4 \sqrt{3}}$$

**V3[x3]**

$$\frac{x3^2 \sqrt{-\frac{x3^2}{12} + \frac{1}{4} (64 - x3^2)}}{4 \sqrt{3}}$$

**V3'[x3]**

$$-\frac{x3^3}{12 \sqrt{3} \sqrt{-\frac{x3^2}{12} + \frac{1}{4} (64 - x3^2)}} + \frac{x3 \sqrt{-\frac{x3^2}{12} + \frac{1}{4} (64 - x3^2)}}{2 \sqrt{3}}$$

**solv3 = Solve[Evaluate[V3'[x3] == 0], {x3}] // Flatten**

**{x3 → 0, x3 → -4 √2, x3 → 4 √2}**

**N[%]**

**{x3 → 0., x3 → -5.65685, x3 → 5.65685}**

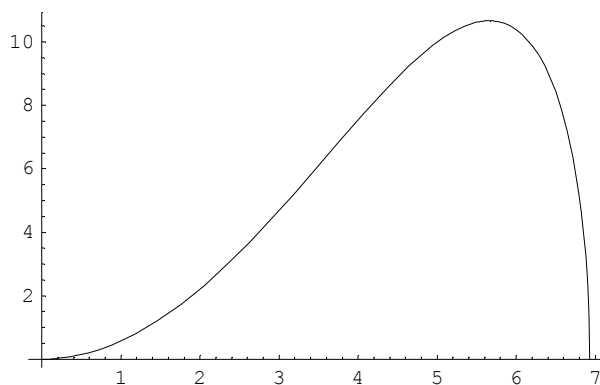
**xMax = x3 /. solv3[[3]]**

**4 √2**

**V3[k]**

$$\frac{k^2 \sqrt{-\frac{k^2}{12} + \frac{1}{4} (64 - k^2)}}{4 \sqrt{3}}$$

**Plot[V3[x3], {x3, 0, 2 Sqrt[4^2 - 2^2]}];**



■ **d**

**V3[xMax]**

$$\frac{32}{3}$$

**N[%]**

**10.6667**



## 5

```
rem; Remove["Global`*"]
```

```
f[x_] := 1/2 (x - 2) (x + 3) E^x; f[y]
```

$$\frac{1}{2} e^y (-2 + y) (3 + y)$$

```
h[x_] := f[x] E^x; h[x]
```

$$\frac{1}{2} e^{2x} (-2 + x) (3 + x)$$

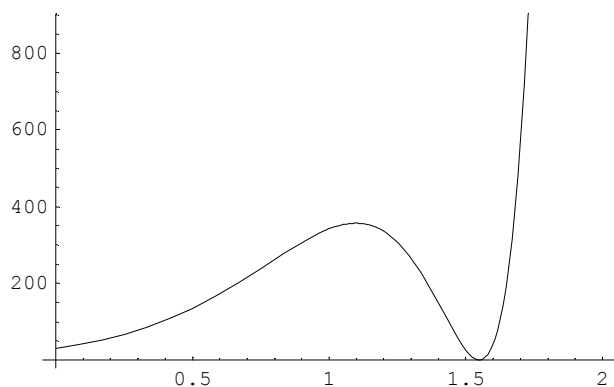
```
1 + Evaluate[h'[y]]^2 // Simplify
```

$$1 + \frac{1}{4} e^{4y} (-11 + 4y + 2y^2)^2$$

```
(1 + Evaluate[h'[u]]^2) /. u -> 2
```

$$1 + \frac{25 e^8}{4}$$

```
Plot[(1 + Evaluate[h'[u]]^2) /. u -> t, {t, 0, 2}];
```

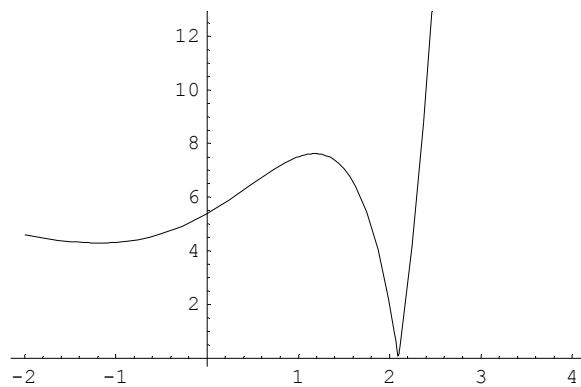


## ■ a

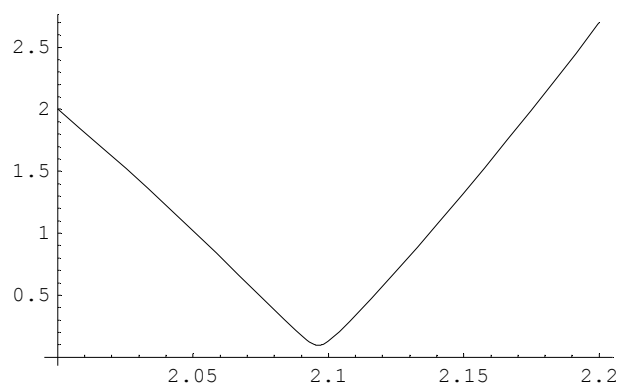
```
dist[x_] := Sqrt[(x - 2)^2 + (f[x] - 2)^2]; dist[x]
```

$$\sqrt{(-2 + x)^2 + \left(-2 + \frac{1}{2} e^x (-2 + x) (3 + x)\right)^2}$$

```
Plot[dist[x], {x, -2, 4}];
```



```
Plot[dist[x], {x, 2, 2.2}];
```



```
dist'[x] // Simplify
```

$$\frac{4(-2+x) - 4e^x(-5+3x+x^2) + e^{2x}(30-23x-8x^2+4x^3+x^4)}{4\sqrt{(-2+x)^2 + \left(2 - \frac{1}{2}e^x(-6+x+x^2)\right)^2}}$$

```
solv = FindRoot[Evaluate[D[dist[k], k] /. k -> x], {x, 2.1}]
```

```
{x -> 2.09627}
```

```
x1 = x /. solv
```

```
2.09627
```

```
dist[x1]
```

```
0.0963626
```

■ b

```
Sqrt[1 + Evaluate[h'[x]]^2]
```

$$\sqrt{1 + \left(\frac{1}{2}e^{2x}(-2+x) + \frac{1}{2}e^{2x}(3+x) + e^{2x}(-2+x)(3+x)\right)^2}$$

```
laengel = NIntegrate[Sqrt[1 + Evaluate[h'[x]]^2], {x, 0, 2}]
```

```
42.5487
```

## ■ c

```

s[t_] := {t, t^2, Sqrt[1+t]}; s[t]
{t, t^2, sqrt[1+t]}

norm[v_] := Sqrt[v.v]; D[s[t], t].D[s[t], t]
1 + 4 t^2 +  $\frac{1}{4(1+t)}$ 

len = NIntegrate[Evaluate[D[s[t], t].D[s[t], t]] /. t -> u, {u, 0, 2}]
12.9413

```

## ■ d

```

v1 = {-1, 2, -3}; a1 = {0, 1, -2};
v2 = {1, -2, 2}; a2 = {1, 0, 1};

V = Det[{v2 - v1, a1, a2}] / Norm[Cross[a1, a2]]
 $\frac{5}{\sqrt{6}}$ 

N[%]
2.04124

```

## 6

```

rem; Remove["Global`*"]

a = {-1, 2, -3}; b = {0, 1, -2}; c = Cross[a, b]
{-1, -2, -1}

```

## ■ a

```

X = Transpose[{a, b, c}]; X // MatrixForm

$$\begin{pmatrix} -1 & 0 & -1 \\ 2 & 1 & -2 \\ -3 & -2 & -1 \end{pmatrix}$$


Eλ = {{2, 0, 0}, {0, -1, 0}, {0, 0, 3}}; Eλ // MatrixForm

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$


```

```
M = X.Eλ.Inverse[X]; M // MatrixForm
```

$$\begin{pmatrix} \frac{13}{6} & \frac{1}{3} & \frac{1}{6} \\ -\frac{11}{3} & \frac{11}{3} & \frac{7}{3} \\ \frac{49}{6} & -\frac{5}{3} & -\frac{11}{6} \end{pmatrix}$$

```
M // N // MatrixForm
```

$$\begin{pmatrix} 2.16667 & 0.333333 & 0.166667 \\ -3.66667 & 3.66667 & 2.33333 \\ 8.16667 & -1.66667 & -1.83333 \end{pmatrix}$$

## ■ b

```
M.M - Transpose[M].Y.M + M == IdentityMatrix[3];
```

```
M.M - IdentityMatrix[3] + M == Transpose[M].Y.M;
```

```
Y = (Inverse[Transpose[M]].(M.M - IdentityMatrix[3] + M).Inverse[M] // MatrixForm)
```

$$Y = \begin{pmatrix} \frac{4589}{216} & -\frac{1081}{108} & -\frac{1}{216} \\ -\frac{649}{108} & \frac{215}{54} & \frac{53}{108} \\ -\frac{1729}{216} & \frac{485}{108} & \frac{53}{216} \end{pmatrix}$$

```
Inverse[Transpose[M]].(M.M - IdentityMatrix[3] + M).Inverse[M] // N // MatrixForm
```

$$\begin{pmatrix} 21.2454 & -10.0093 & -0.00462963 \\ -6.00926 & 3.98148 & 0.490741 \\ -8.00463 & 4.49074 & 0.24537 \end{pmatrix}$$

## ■ c

```
detQ = (1 / Det[M]) ^ 50
```

$$\frac{1}{808281277464764060643139600456536293376}$$

```
detQ // N
```

$$1.23719 \times 10^{-39}$$

```
detQ 10^39
```

$$\frac{1818989403545856475830078125}{1470255078792914101801469952}$$

```
detQ 10^39 // N
```

$$1.23719$$

## ■ d

**M5 = M.M.M.M.M;**

**M5 // MatrixForm**

$$\begin{pmatrix} \frac{403}{6} & \frac{211}{3} & \frac{211}{6} \\ \frac{79}{3} & \frac{551}{3} & \frac{277}{3} \\ \frac{739}{6} & \frac{145}{3} & \frac{139}{6} \end{pmatrix}$$

**M5 // N // MatrixForm**

$$\begin{pmatrix} 67.1667 & 70.3333 & 35.1667 \\ 26.3333 & 183.667 & 92.3333 \\ 123.167 & 48.3333 & 23.1667 \end{pmatrix}$$

**v = Transpose[{b + x b - x^2 b - x^3 b}] // MatrixForm**

$$\begin{pmatrix} 0 \\ 1 + x - x^2 - x^3 \\ -2 - 2x + 2x^2 + 2x^3 \end{pmatrix}$$

**M5.v // Simplify // MatrixForm**

$$\begin{pmatrix} 0 \\ (-1 + x) (1 + x)^2 \\ -2 (-1 + x) (1 + x)^2 \end{pmatrix}$$

**M5.v // Expand // MatrixForm**

$$\begin{pmatrix} 0 \\ -1 - x + x^2 + x^3 \\ 2 + 2x - 2x^2 - 2x^3 \end{pmatrix}$$

Kommetar: Eigenvektoren ändern nur die Länge!

## ■ e

**Dφ[φ\_] := {{Cos[φ], -Sin[φ], 0}, {Sin[φ], Cos[φ], 0}, {0, 0, 1}};**

**MatrixForm[Dφ[φ]]**

$$\begin{pmatrix} \text{Cos}[\varphi] & -\text{Sin}[\varphi] & 0 \\ \text{Sin}[\varphi] & \text{Cos}[\varphi] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

## ■ f

**U = Inverse[M].Dφ[φ].M // Simplify; MatrixForm[U]**

$$\begin{pmatrix} \frac{1}{216} (-49 + 265 \text{Cos}[\varphi] + 348 \text{Sin}[\varphi]) & \frac{1}{108} (5 - 5 \text{Cos}[\varphi] - 189 \text{Sin}[\varphi]) & \frac{1}{216} (11 - 1 \\ \frac{1}{108} (833 - 833 \text{Cos}[\varphi] - 606 \text{Sin}[\varphi]) & \frac{1}{54} (-85 + 139 \text{Cos}[\varphi] + 423 \text{Sin}[\varphi]) & \frac{1}{108} (-187 \\ \frac{1}{216} (-2695 + 2695 \text{Cos}[\varphi] + 2652 \text{Sin}[\varphi]) & \frac{1}{108} (275 - 275 \text{Cos}[\varphi] - 1611 \text{Sin}[\varphi]) & \frac{1}{216} (605 - \end{pmatrix}$$

**Eigenvalues[U]**

$\{1, \cos[\varphi] - i \sin[\varphi], \cos[\varphi] + i \sin[\varphi]\}$

Reell für  $\sin[\varphi]=0$ , also für  $\varphi = n\pi$ ,  $n =$  ganze Zahl.

## ■ g

**X = Transpose[{a, b, c}]; X // MatrixForm**

$$\begin{pmatrix} -1 & 0 & -1 \\ 2 & 1 & -2 \\ -3 & -2 & -1 \end{pmatrix}$$

**EX = {{1, 0, 0}, {0, 1, 0}, {0, 0, -1}}; ; MatrixForm[EX]**

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

**S = X.EX.Inverse[X]; S // MatrixForm**

$$\begin{pmatrix} \frac{2}{3} & -\frac{2}{3} & -\frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \end{pmatrix}$$

**S // N // MatrixForm**

$$\begin{pmatrix} 0.666667 & -0.666667 & -0.333333 \\ -0.666667 & -0.333333 & -0.666667 \\ -0.333333 & -0.666667 & 0.666667 \end{pmatrix}$$

**e1 = {1, 0, 0}; e2 = {0, 1, 0}; e3 = {0, 0, 1};**

**OA = Transpose[{e1 + e2 + e3}]; OA // MatrixForm**

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

**S.OA // MatrixForm**

$$\begin{pmatrix} -\frac{1}{3} \\ -\frac{5}{3} \\ -\frac{1}{3} \end{pmatrix}$$

**S.OA // N // MatrixForm**

$$\begin{pmatrix} -0.333333 \\ -1.666667 \\ -0.333333 \end{pmatrix}$$


---

## 7

**rem; Remove["Global`\*"]**

```
f[x_] := x Cos[(x + x^2) Pi] - x x^x; f[x]
```

```
-x1+x + x Cos[π (x + x2) ]
```

```
h[x_] := 2 x Sin[x^2 + 1] + x E^(2 x^2) + Cos[x] - x Sin[x]; h[x]
```

```
e2x2 x + Cos[x] - x Sin[x] + 2 x Sin[1 + x2]
```

### ■ a

```
f'[x]
```

```
Cos[π (x + x2) ] - x1+x (  $\frac{1+x}{x} + \text{Log}[x]$  ) - π x (1 + 2 x) Sin[π (x + x2) ]
```

```
f'[x] // Simplify
```

```
Cos[π x (1 + x) ] - xx (1 + x + x Log[x]) - π x (1 + 2 x) Sin[π x (1 + x) ]
```

### ■ b

```
steig = (f'[x] /. x -> 1) // Simplify
```

```
-1
```

```
ArcTan[steig]
```

```
 $-\frac{\pi}{4}$ 
```

```
ArcTan[steig] // N
```

```
-0.785398
```

```
ArcTan[steig] / Degree // N
```

```
-45.
```

### ■ c

```
Integrate[h[x], x]
```

```
 $\frac{e^{2x^2}}{4} + x \text{Cos}[x] - \text{Cos}[1 + x^2]$ 
```