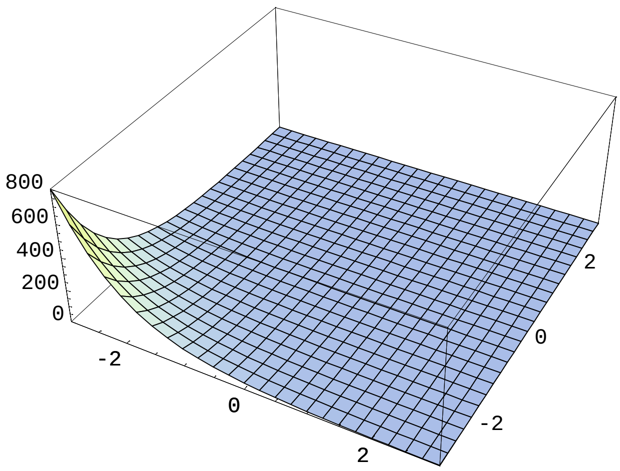
Lösungen

1

```
Remove["Global`*"]

x1 = 1; y1 = 1;  x2 = -1; y2 = 1;  x3 = 1; y3 = -1;  x4 = -1; y4 = -1;
dx = 0.05;  dy = dx;

f[x_, y_, k_] := (2 E^(-k (x + y)) - 0.35) Abs[C0];
Plot3D[f[x, y, 1] /. C0 → 1, {x, -3, 3}, {y, -3, 3}, PlotRange → {-1.5, 800}];
```




```
f[1, 1, 1]
```

-0.0793294 Abs[C0]

```
f[1, 1, 0.86]
```

0.0081323 Abs[C0]

a

```
f[x_, y_] := f[x, y, 0.86];
f[x, y]
```

$(-0.35 + 2\, e^{-0.86\,(x+y)})$ Abs[C0]

```
f1 = f[x, y] /. {x → x1, y → y1}
```

0.0081323 Abs[C0]

```
g[x_, y_, dx_, dy_] := Abs[D[f[x, y], x]] Abs[dx] + Abs[D[f[x, y], y]] Abs[dy];
```

```
{g[x, y, dx, dy], ooooo , ooooo, g[u, v, dv, dv] /. {x → u, y → v}}
```

$\{0.172\, e^{-0.86\, \mathrm{Re}[x+y]}\, \mathrm{Abs}[C0],\ \mathrm{ooooo},\ \mathrm{ooooo},\ 3.44\, e^{-0.86\, \mathrm{Re}[u+v]}\, \mathrm{Abs}[C0]\, \mathrm{Abs}[dv]\}$

```
linFehler = g[x, y, dx, dy] /. {x → x1, y → y1}
```

0.0307994 Abs[C0]

```
{f1, plus - minus, linFehler}
```

{0.0081323 Abs[C0], -minus + plus, 0.0307994 Abs[C0]}

```
Intervall = {f1 - linFehler, f1 + linFehler}
```

{-0.0226671 Abs[C0], 0.0389317 Abs[C0]}

b

```
f1 = f[x, y] /. {x → x1 + dx, y → y1 + dy}
```

-0.0213799 Abs[C0]

(Etwa f1-linFehler ////oooooo//// Negativer Wert physikalisch nicht sinnvoll!)

c

e-Funktion statt linear, je nach Punkt Wachstum sehr verschieden.

2

```
Remove["Global`*"]
```

```
d = 0.3 meter; lae = 12 meter; ρ = 0.9 10^3 kg / meter^3; g = 9.81 meter / sec^2;
h1 = lae / 2 ; h2 = lae / 2 + lae / 2 Sin[60 Degree];
h20 = lae / 2 - lae / 2 Sin[60 Degree]; h3 = 2 lae / 2;
V = d^2 Pi / 4 lae
```

$0.84823\ \mathrm{meter}^3$

```
m = V ρ
```

763.407 kg

```
JJ = 1 / 3 m lae^2
```

$36643.5\ \mathrm{kg}\ \mathrm{meter}^2$

```
g
```

$\frac{9.81\ \mathrm{meter}}{\mathrm{sec}^2}$

```
Ep[h_] = m g h; Ep[h1]
```

$\frac{44934.1\ \text{kg meter}^2}{\text{sec}^2}$

```
Erot[ω_] := 1 / 2 JJ ω^2; Erot[ω]
```

$18321.8\ \text{kg meter}^2\ \omega^2$

a

```
Ep[h1] == Erot[ω]
```

$\frac{44934.1\ \text{kg meter}^2}{\text{sec}^2} == 18321.8\ \text{kg meter}^2\ \omega^2$

```
solv1 = Solve[Ep[h1] == Erot[ω], {ω}] // Flatten
```

$\left\{\omega \to -\frac{1.56605}{\text{sec}},\ \omega \to \frac{1.56605}{\text{sec}}\right\}$

```
ω1 = ω /. solv1[[2]]
```

$\frac{1.56605}{\text{sec}}$

```
v1 = lae ω1
```

$\frac{18.7926\ \text{meter}}{\text{sec}}$

```
v1 1 / 1000 km / meter 60 60 sec / Std
```

$\frac{67.6532\ \text{km}}{\text{Std}}$

b 60 Grad nach unten

```
Ep[h2] == Erot[ω]
```

$\frac{7489.02\ \text{kg meter}\ \left(6\ \text{meter} + 3\sqrt{3}\ \text{meter}\right)}{\text{sec}^2} == 18321.8\ \text{kg meter}^2\ \omega^2$

```
solv2 = Solve[Ep[h2] == Erot[ω], {ω}] // Flatten
```

$\left\{\omega \to -\frac{2.13926}{\text{sec}},\ \omega \to \frac{2.13926}{\text{sec}}\right\}$

```
ω2 = ω /. solv2[[2]]
```

$\frac{2.13926}{\text{sec}}$

```
v2 = lae ω2
```

$\frac{25.6711\ \text{meter}}{\text{sec}}$

```
v2 1 / 1000 km / meter 60 60 sec / Std
```

$\frac{92.416\ \text{km}}{\text{Std}}$

b 60 Grad nach oben

```
Ep[h20] == Erot[ω]
```

$$\frac{7489.02\,\text{kg meter}\,\left(6\,\text{meter} - 3\sqrt{3}\,\text{meter}\right)}{\text{sec}^2} == 18321.8\,\text{kg meter}^2\,\omega^2$$

```
solv2 = Solve[Ep[h20] == Erot[ω], {ω}] // Flatten
```

$$\left\{\omega \to -\frac{0.573213}{\text{sec}},\ \omega \to \frac{0.573213}{\text{sec}}\right\}$$

```
ω2 = ω /. solv2[[2]]
```

$$\frac{0.573213}{\text{sec}}$$

```
v2 = lae ω2
```

$$\frac{6.87855\,\text{meter}}{\text{sec}}$$

```
v2 1 / 1000 km / meter 60 60 sec / Std
```

$$\frac{24.7628\,\text{km}}{\text{Std}}$$

c

```
Ep[h3] == Erot[ω]
```

$$\frac{89868.3\,\text{kg meter}^2}{\text{sec}^2} == 18321.8\,\text{kg meter}^2\,\omega^2$$

```
solv3 = Solve[Ep[h3] == Erot[ω], {ω}] // Flatten
```

$$\left\{\omega \to -\frac{2.21472}{\text{sec}},\ \omega \to \frac{2.21472}{\text{sec}}\right\}$$

```
ω3 = ω /. solv3[[2]]
```

$$\frac{2.21472}{\text{sec}}$$

```
v3 = lae ω3
```

$$\frac{26.5767\,\text{meter}}{\text{sec}}$$

```
v3 1 / 1000 km / meter 60 60 sec / Std
```

$$\frac{95.6761\,\text{km}}{\text{Std}}$$

3

```
Remove["Global`*"]
```

a

```
ρ = 1 Ω / (13.7 meter) (1.5^2 Pi / 4 mm^2)
```

$$\frac{0.128989\ \text{mm}^2\ \Omega}{\text{meter}}$$

```
ρ1 = 0.95 Ω / (13.7 meter) (1.5^2 Pi / 4 mm^2)
```

$$\frac{0.122539\ \text{mm}^2\ \Omega}{\text{meter}}$$

```
ρ2 = 1.05 Ω / (13.7 meter) (1.5^2 Pi / 4 mm^2)
```

$$\frac{0.135438\ \text{mm}^2\ \Omega}{\text{meter}}$$

Eisen, spezifischer Widerstand: 0.1 bis 0.15 mm^2/ meter
==> es könnte Eisen sein --- Flussstahl ---- Rotguss (Bronze)----
Da es aber sich aber um einen Draht handelt, fallen Gussarten weg..

b

```
i = 10 A; U = 230 V; R = U / i /. {V / A → Ω}
```

23 Ω

```
Solve[R == ρ x / (1.5^2 Pi / 4 mm^2), {x}]
```

{{x → 315.1 meter}}

c

```
α = 6.57 10^(-3) / K;
T = 293.16 K; ΔT = 1 K;
R0 = ρ;
ΔR = R0 α ΔT
```

$$\frac{0.000847456\ \text{mm}^2\ \Omega}{\text{meter}}$$

```
{R0, ΔR}
```

$$\left\{\frac{0.128989\ \text{mm}^2\ \Omega}{\text{meter}}, \frac{0.000847456\ \text{mm}^2\ \Omega}{\text{meter}}\right\}$$

```
ΔR / R0 100
```

0.657

4

```
Remove["Global`*"]
```

```
g = 9.81 meter / (sec^2);
v0h = 100 meter / (10 sec);
h1 = -(10 meter + 0.90 meter);
h2 = h1 - 4 meter;
masse = 75 kg;
ρ = 1000 kg / (meter^3);
```

```
vert[t_] = -g t
```

$$-\frac{9.81\,\text{meter}\,t}{\text{sec}^2}$$

```
vvec[t_] := {v0h, vert[t]};
svec[t_] := {v0h t, vert[t] t / 2};
```

```
vert[t] t / 2 == h2
```

$$-\frac{4.905\,\text{meter}\,t^2}{\text{sec}^2} == -14.9\,\text{meter}$$

```
solv1 = Solve[vert[t] t / 2 == h1, {t}] // Flatten
```

{t → -1.49071 sec, t → 1.49071 sec}

```
t1 = t /. solv1[[2]]
```

1.49071 sec

```
solv2 = Solve[vert[t] t / 2 == h2, {t}] // Flatten
```

{t → -1.7429 sec, t → 1.7429 sec}

```
t2 = t /. solv2[[2]]
```

1.7429 sec

a

```
svec[t2]
```

{17.429 meter, -14.9 meter}

```
minSchwimbarlaenge = 17.429046490552107` meter + 10 meter + 4 meter
```

31.429 meter

b

```
vEintauchen = Sqrt[vvec[t1].vvec[t1]] /. {Sqrt[meter^2/sec^2] → meter / sec}
```

$$\frac{17.716\,\text{meter}}{\text{sec}}$$

```
Norm[vvec[t1]]
```

$$17.716\,\text{Abs}\left[\frac{\text{meter}}{\text{sec}}\right]$$

c

```
EkinEintauchen = 1 / 2 masse vEintauchen^2
```

$$\frac{11769.7\ \text{kg meter}^2}{\text{sec}^2}$$

```
EkinEigen = 1 / 2 masse v0h^2
```

$$\frac{3750\ \text{kg meter}^2}{\text{sec}^2}$$

```
EkinEigen / EkinEintauchen 100
```

31.8615

d

```
s1h = v0h t1
```

14.9071 meter

```
s1v = 10 meter; solv3 = Solve[s1v == 1 / 2 g t^2, {t}] // Flatten;
t3 = t /. solv3[[2]]
```

1.42784 sec

```
v0ht3 = s1h / t3
```

$$\frac{10.4403\ \text{meter}}{\text{sec}}$$

```
g hFass + 1 / 2 v0h t3^2
```

$$\frac{9.81\ \text{hFass meter}}{\text{sec}^2} + 10.1937\ \text{meter sec}$$

```
p0 + ρ g hFass + 1 / 2 ρ v0ht3^2 == p0 + ρ g 0 + 1 / 2 ρ 0^2
```

$$\text{p0} + \frac{9810.\ \text{hFass kg}}{\text{meter}^2\ \text{sec}^2} + \frac{54500.\ \text{kg}}{\text{meter sec}^2} == \text{p0}$$

```
Solve[p0 + ρ g hFass + 1 / 2 ρ v0ht3^2 == p0 + ρ g 0 + 1 / 2 ρ 0^2, { hFass}]
```

{{hFass → -5.55556 meter}}

5

```
Remove["Global`*"]
```

```
laeMann = 1.80 meter;
hBrue = 114.8 meter;
laeSeil = 76.8 meter;
hSchaetz = 6 meter;
(* hEnd=24.8 meter; *)
hSchwerp = laeMann / 2;
g = 9.81 meter / (sec^2);
F = 686 newton /. {newton → kg meter / sec^2};
dSeil = 0.02 meter;
masse = F / g
```

69.9286 kg

a

```
hGestreckt = laeSeil + 2 hSchwerp
```

78.6 meter

```
solv1 = Solve[hGestreckt == 1 / 2 g t^2, {t}] // Flatten;
tGestreckt = t /. solv1[[2]]
```

4.00306 sec

```
vGestreckt = g tGestreckt
```

$\frac{39.27\ \text{meter}}{\text{sec}}$

b

```
EkinGestreckt = 1 / 2 masse vGestreckt^2
```

$\frac{53919.6\ \text{kg meter}^2}{\text{sec}^2}$

```
EkinGestreckt /. {kg meter² / sec² → 0.001 kJ}
```

53.9196 kJ

c

```
hVerLae = hBrue - hSchaetz - laeSeil - laeMann
```

30.2 meter

```
hpot = hBrue - hSchaetz - hSchwerp + hSchwerp
```

108.8 meter

```
EFederUnten = masse g hpot
```

$\frac{74636.8\ \text{kg meter}^2}{\text{sec}^2}$

```
{Spannenergie = 1 / 2 Federkonst hVerLae^2,  EFederUnten == Spannenergie}
```

$$\left\{456.02\ \text{Federkonst meter}^2,\ \frac{74636.8\ \text{kg meter}^2}{\text{sec}^2} == 456.02\ \text{Federkonst meter}^2\right\}$$

```
solv3 = Solve[EFederUnten == Spannenergie, {Federkonst}] // Flatten;
Federkonst = Federkonst /. solv3
```

$$\frac{163.67\ \text{kg}}{\text{sec}^2}$$

d

```
Dehnungϵ = hVerLae / laeSeil
```

0.393229

```
Spannungσ = hVerLae Federkonst / (dSeil^2 Pi / 4)
```

$$\frac{1.57335\times10^{7}\ \text{kg}}{\text{meter sec}^2}$$

```
ElastModul = Spannungσ / Dehnungϵ
```

$$\frac{4.00111\times10^{7}\ \text{kg}}{\text{meter sec}^2}$$

```
ElastModul /. {kg / (meter sec^2) → 10^(-6) newton / mm^2}
```

$$\frac{40.0111\ \text{newton}}{\text{mm}^2}$$

e

```
ZuglastMax = hVerLae Federkonst
```

$$\frac{4942.83\ \text{kg meter}}{\text{sec}^2}$$

```
ZuglastMax /. {kg meter / sec^2 → 10^(-3) kNewton}
```

4.94283 kNewton

f

```
FStillstand = masse g;
FFederStillstand = Federkonst h;
FStillstand == FFederStillstand
```

$$\frac{686.\ \text{kg meter}}{\text{sec}^2} == \frac{163.67\ \text{h kg}}{\text{sec}^2}$$

```
solv4 = Solve[FStillstand == FFederStillstand, {h}] // Flatten;
hStillstand = h /. solv4
```

4.19136 meter

```
hStillstandKopfVonUnten = hBrue - laeSeil - hStillstand - laeMann
```

32.0086 meter

6

```
Remove["Global`*"]
```

a

```
R0 = (1 / (1 / 200 + 1 / 200 + 1 / 300 + 1 / 400)) // N
```

63.1579

```
U = 230;
II = U / (1 / (1 / 200 + 1 / 200 + 1 / 300 + 1 / 400)) // N
```

3.64167

```
P = II * U
```

837.583

Eine 1000 W-Sicherung hält. Die 800 W-Sicherung hält nicht.

b

```
solv = NSolve[y == U U / (1 / (1 / (200 + x) + 1 / 200 + 1 / 300 + 1 / 400)) // N, {x}] // Flatten
```

$\left\{x \to \frac{2.0102\times10^{6} - 2400.\,y}{-6877. + 12.\,y}\right\}$

```
x[y_] := x /. solv
```

```
x[y]
```

$\frac{2.0102\times10^{6} - 2400.\,y}{-6877. + 12.\,y}$

```
x[y] /. y → 574
```

57509.1

```
x[y] 10^(-3) kΩ /. y → 574
```

57.5091 kΩ

Positiv, geht.

c

```
x[y] /. y → 570
```

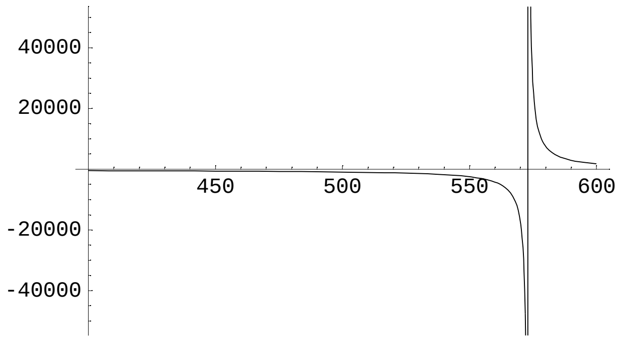
−17356.8

```
x[y] 10^(-3) kΩ /. y → 570
```

−17.3568 kΩ

Negativ, geht nicht.

```
Plot[Evaluate[x[y]], {y, 400, 600}];
```



d

```
U U / (1 / (1 / (200 + x) + 1 / 200 + 1 / 300 + 1 / 400)) // N
```

$52900.\left(0.0108333+\frac{1}{200.+x}\right)$

7

```
Remove["Global`*"]
```

```
V[1] = 120 Liter; p[0] = 1 bar;  p[1] = 12 bar + p[0]; T[0] = 293.15 K;   T[1] = T[0];
ρLuft = 1.204 kg / meter^3;
umr = ( Liter -> 10^(-3) meter^3);
g = 9.81 meter / sec^2
```

$\frac{9.81\,\text{meter}}{\text{sec}^2}$

a

```
p[n] V[n] == const R T[n]
```

p[n] V[n] == const R T[n]

```
solv = Solve[{p[0] V[0] == const R T[0], p[1] V[1] == const R T[0]}, {V[0]}];
V[0] = V[0] /. solv
```

{1560. Liter}

```
ΔV = V[0] - V[1]
```

{1440. Liter}

b

```
m = V[0] ρLuft /. umr
```

{1.87824 kg}

```
F = m g /. {kg meter / sec^2 → newton}
```

{18.4255 newton}

8

```
Remove["Global`*"]
```

a

x kg=1. Masse, y kg=2. Masse, Anteile zusammen 1 kg mit V = 1dm^3

Massengleichung

```
Gleichung1 = (x kg + y kg == 1 kg)
```

kg x + kg y == kg

Wärmegleichung

```
Gleichung2 = (x kg 0.84 kJ / (kg K) + y kg 2.5 kJ / (kg K) == 1 kg kJ / (kg K))
```

$\frac{0.84\,kJ\,x}{K} + \frac{2.5\,kJ\,y}{K} == \frac{kJ}{K}$

Volumengleichung

```
Gleichung3 = (x kg / (2.0 kg / dm^3) + y kg / (0.7 kg / dm^3) == 1 kg / ( kg / dm^3))
```

$0.5\,dm^3\,x + 1.42857\,dm^3\,y == dm^3$

```
Solve[{Gleichung1, Gleichung2, Gleichung3}, {x, y}]
```

{}

(Nicht möglich.)

a2: Varianten

x kg=1. Masse, y kg=2. Masse, Anteile zusammen 1 kg mit V=1dm^3

```
Solve[{Gleichung2, Gleichung3}, {x, y}]
```

```
{{x → 21.4286, y → -6.8}}
```

(Nicht möglich.)

```
{Solve[{Gleichung1, Gleichung2}, {x, y}],
 "und gleichzeitig", Solve[{Gleichung1, Gleichung3}, {x, y}]}
```

```
{{{x → 0.903614, y → 0.0963855}}, und gleichzeitig, {{x → 0.461538, y → 0.538462}}}
```

(Nicht möglich, passt nicht zusammen.)

b

Nicht möglich.

Varianten

Testbereich

```
Remove["Global`*"]
```

1

a Varianten

```
f[x, y] /. {x → x2, y → y2}
```

```
f[x, y] /. {x → x3, y → y3}
```

```
f[x, y] /. {x → x4, y → y4}
```

```
g[x, y, dx, dy] /. {x → x2, y → y2}
```

```
g[x, y, dx, dy] /. {x → x3, y → y3}
```

```
g[x, y, dx, dy] /. {x → x4, y → y4}
```

b Varianten

```
h[u_, v_, du_, dv_] := Max[Abs[Max[f[u, v], f[u + du, v + dv],
     f[u - du, v + dv], f[u + du, v - dv], f[u - du, v - dv]]], Abs[
    Min[f[u, v], f[u + du, v + dv], f[u - du, v + dv], f[u + du, v - dv], f[u - du, v - dv]]]];
h[
 x1,
 y1,
 dx,
 dy]
```

```
h[x2, y2, dx, dy]
```

```
h[x3, y3, dx, dy]
```

```
h[x4, y4, dx, dy]
```