Lösungen 1

1

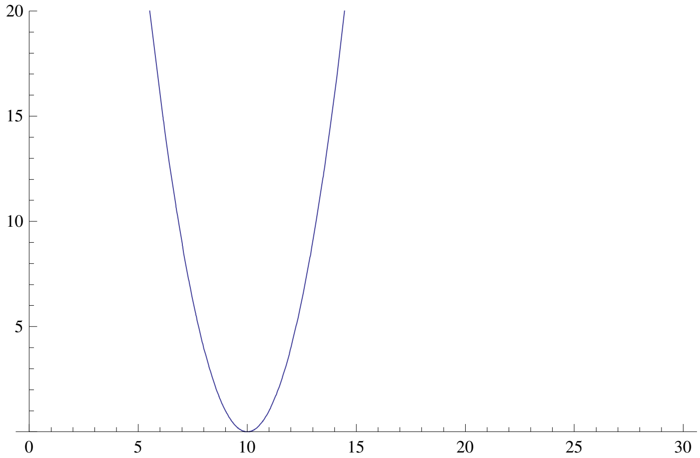
▪ a

```
x1 = 30; y1 = 10; z1 = 20;
```

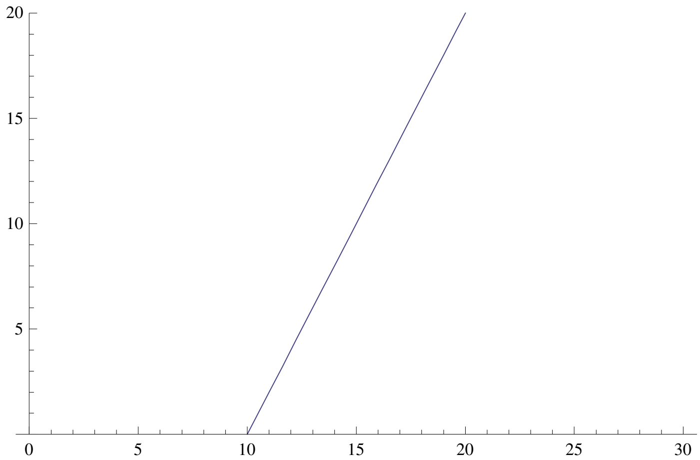
```
f[x_] := (x - 10) ^2
```

```
f1[x_] := 10 - (x - 10) ^2
```

```
Plot[f[x], {x, 0, 30}, PlotRange → {0, 20}]
```




```
Plot[2 x - 20, {x, 0, 30}, PlotRange → {0, 20}]
```




```
Solve[(x - (y + 20) / 2) ^2 == 0, {y}] // ExpandAll
```

{{y → -20 + 2 x}, {y → -20 + 2 x}}

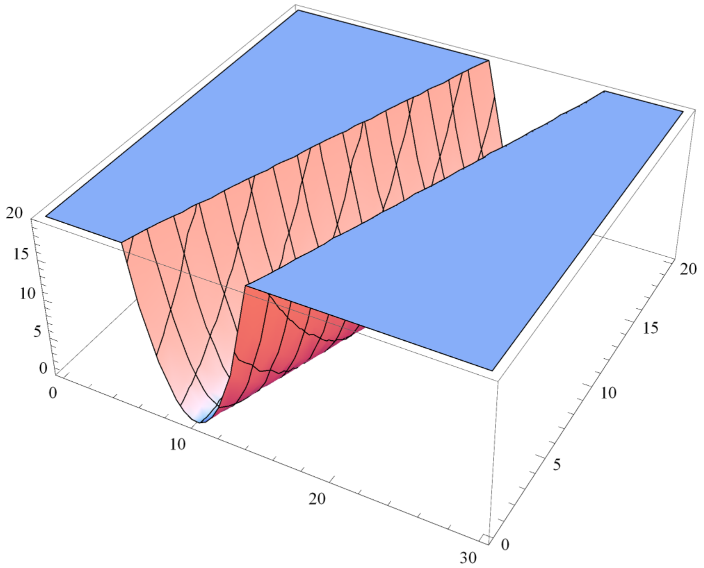
```
Solve[(x - (0 + 20) / 2) ^2 == z, {z}] // ExpandAll
```

$\{\{z \to 100 - 20\,x + x^2\}\}$

```
f[x] // Expand
```

$100 - 20\,x + x^2$

```
x1 = 30; y1 = 10; z1 = 20;
Plot3D[(x - (y + 20) / 2) ^2 , {x, 0, 30}, {y, 0, 20}, PlotRange → {0, 20}]
```



- **b**

```
solv1 = Solve[f[x] == 20, {x}]
```

$\{\{x \to 2(5-\sqrt{5})\}, \{x \to 2(5+\sqrt{5})\}\}$

```
x01 = x /. solv1[[1]]
```

$2(5-\sqrt{5})$

```
x02 = x /. solv1[[2]]
```

$2(5+\sqrt{5})$

```
solv2 = NSolve[f[x] == 20, {x}] // Flatten
```

{x → 5.52786, x → 14.4721}

```
x1 = x /. solv2[[1]]
```

5.52786

```
x2 = x /. solv2[[2]]
```

14.4721

```
len = Integrate[Evaluate[Sqrt[1 + (f '[u]) ^2] /. u → x], {x, x01, x02}]
```

$\frac{1}{2}\left(36\sqrt{5} + \text{ArcSinh}\left[4\sqrt{5}\right]\right)$

```
len = NIntegrate[Evaluate[Sqrt[1 + (f1 '[u]) ^2] /. u → x], {x, x1, x2}]
```

41.6929

- **c**

```
VQ = 30 × 20 × 10
```

6000

```
VPex = Integrate[20 - f[x], {x, x01, x02}]
```

$\frac{160\sqrt{5}}{3}$

```
VP = NIntegrate[20 - f[x], {x, x01, x02}]
```

119.257

```
VN = VP y1
```

1192.57

- **d**

```
Solve[2 x - 20 == y, {x}]
```

$\left\{\left\{x \to \frac{20 + y}{2}\right\}\right\}$

```
xx[y_] := (y + 20) / 2;  {xx[0], xx[y1]}
```

{10, 15}

```
len1 = Sqrt[y1^2 + (xx[0] - xx[y1])^2]
```

$5\sqrt{5}$

```
% // N
```

11.1803

```
Solve[VPex y1 == r^2 Pi len1, {r}]
```

$\left\{\left\{r \to -8\sqrt{\frac{5}{3\pi}}\right\}, \left\{r \to 8\sqrt{\frac{5}{3\pi}}\right\}\right\}$

```
solv3 = Solve[VN == r^2 Pi len1, {r}] // Flatten
```

{r → -5.82692, r → 5.82692}

```
r1 = r /. solv3[[2]]
```

5.82692

2

```
LaplaceTransform[xxx, t, s]
```

$\frac{xxx}{s}$

```
InverseLaplaceTransform[xxx, s, t]
```

xxx DiracDelta[t]

- **a**

```
LaplaceTransform[Sin[3 t ] + Sinh[2 t] - t^2, t, s]
```

$-\frac{2}{s^3} + \frac{2}{-4 + s^2} + \frac{3}{9 + s^2}$

- **b**

```
LaplaceTransform[2 - t + DiracDelta[t], t, s]
```

$1 - \frac{1}{s^2} + \frac{2}{s}$

c

```
LaplaceTransform[(2 - E^(-t)) Sin[t], t, s]
```

$$\frac{2}{1+s^2} - \frac{1}{1+(1+s)^2}$$

d

```
LaplaceTransform[E^(1 + 2 t) E^(1 - t), t, s]
```

$$\frac{e^2}{-1+s}$$

e

```
InverseLaplaceTransform[s / (4 s^2 - 2), s, t] // ExpandAll
```

$$\frac{1}{8} e^{-\frac{t}{\sqrt{2}}} + \frac{1}{8} e^{-\frac{t}{\sqrt{2}}+\sqrt{2}\,t}$$

```
InverseLaplaceTransform[s / (4 s^2 - 2), s, t]
```

$$\frac{1}{8} e^{-\frac{t}{\sqrt{2}}} \left(1 + e^{\sqrt{2}\,t}\right)$$

f

```
InverseLaplaceTransform[1 / (s^4 - 1), s, t]
```

$$-\frac{e^{-t}}{4} + \frac{e^t}{4} - \frac{\text{Sin}[t]}{2}$$

```
Apart[1 / (s^4 - 1)]
```

$$\frac{1}{4(-1+s)} - \frac{1}{4(1+s)} - \frac{1}{2(1+s^2)}$$

```
InverseLaplaceTransform[Apart[1 / (s^4 - 1)], s, t]
```

$$-\frac{e^{-t}}{4} + \frac{e^t}{4} - \frac{\text{Sin}[t]}{2}$$

g

```
InverseLaplaceTransform[2 + 1 / (2 s) + 1 / (s - 1)^2, s, t]
```

$$\frac{1}{2} + e^t\, t + 2\, \text{DiracDelta}[t]$$

3

```
Remove["Global`*"]
```

a

```
solv = DSolve[{y''[t] + y'[t] == c + Cos[t] - t, y[0] == 1, y'[0] == 0}, y, t] // Flatten
```

$$\left\{y \to \text{Function}\left[\{t\}, \frac{1}{2} e^{-t} \left(3 + 2c - 2c\, e^t + 2 e^t t + 2c\, e^t t - e^t t^2 - e^t \text{Cos}[t] + e^t \text{Sin}[t]\right)\right]\right\}$$

```
u[t_, c_] := Evaluate[(y[t] /. %)]
```

```
u[t, c]
```

$$\frac{1}{2} e^{-t} \left(3 + 2c - 2c\, e^t + 2 e^t t + 2c\, e^t t - e^t t^2 - e^t \text{Cos}[t] + e^t \text{Sin}[t]\right)$$

```
u[t2, c3]
```

$$\frac{1}{2} e^{-t2} \left(3 + 2\,c3 - 2\,c3\,e^{t2} + 2\,e^{t2}\,t2 + 2\,c3\,e^{t2}\,t2 - e^{t2}\,t2^2 - e^{t2}\,\text{Cos}[t2] + e^{t2}\,\text{Sin}[t2]\right)$$

```
u[t, 0] // ExpandAll
```

$$\frac{3\,e^{-t}}{2} + t - \frac{t^2}{2} - \frac{\text{Cos}[t]}{2} + \frac{\text{Sin}[t]}{2}$$

b

```
u[10, c]
```

$$\frac{3 + 2\,c - 80\,e^{10} + 18\,c\,e^{10} - e^{10}\,\text{Cos}[10] + e^{10}\,\text{Sin}[10]}{2\,e^{10}}$$

```
Solve[u[10, c] == 1]
```

$$\left\{\left\{c \to \frac{-3 + 82\,e^{10} + e^{10}\,\text{Cos}[10] - e^{10}\,\text{Sin}[10]}{2\left(1 + 9\,e^{10}\right)}\right\}\right\}$$

$$\frac{-3 + 82\,e^{10} + e^{10}\,\text{Cos}[10] - e^{10}\,\text{Sin}[10]}{2\left(1 + 9\,e^{10}\right)} \;//\; \text{ExpandAll}$$

$$-\frac{3}{2 + 18\,e^{10}} + \frac{82\,e^{10}}{2 + 18\,e^{10}} + \frac{e^{10}\,\text{Cos}[10]}{2 + 18\,e^{10}} - \frac{e^{10}\,\text{Sin}[10]}{2 + 18\,e^{10}}$$

```
N[%]
```

4.53913

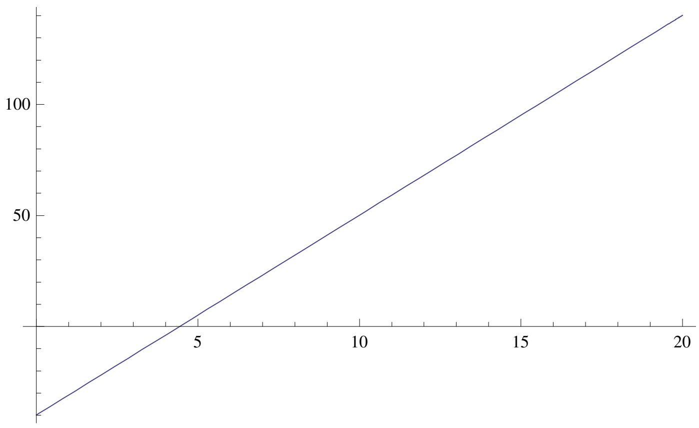
c

$$\text{yc}[t\_, c\_] := \frac{1}{2} e^{-t} \left(3 + 2\,c - 2\,c\,e^{t} + 2\,e^{t}\,t + 2\,c\,e^{t}\,t - e^{t}\,t^2 - e^{t}\,\text{Cos}[t] + e^{t}\,\text{Sin}[t]\right)$$

```
Solve[yc[10, c] == 1, {c}]
```

$$\left\{\left\{c \to \frac{-3 + 82\,e^{10} + e^{10}\,\text{Cos}[10] - e^{10}\,\text{Sin}[10]}{2\left(1 + 9\,e^{10}\right)}\right\}\right\}$$

```
Plot[yc[10, c], {c, 0, 20}]
```



4

```
Remove["Global`*"]
```

```
LaplaceTransform[y'''[t] - 2 y'[t] - y[t], t, s] /.
 {LaplaceTransform[y[t], t, s] → Y[s], y[0] → -1, y'[0] → 0, y''[0] → 0}
```

$s^2 - Y[s] + s^3 Y[s] - 2 (1 + s Y[s])$

```
LaplaceTransform[Sinh[t], t, s]
```

$\frac{1}{-1 + s^2}$

```
Solve[s^2 - Y[s] + s^3 Y[s] - 2 (1 + s Y[s]) == 1/(-1 + s^2), {Y[s]}]
```

$\left\{\left\{Y[s] \to \frac{1 - s - s^2}{(-1 + s) (1 + s)^2}\right\}\right\}$

```
InverseLaplaceTransform[(1 - s - s^2)/((-1 + s) (1 + s)^2), s, t]
```

$-\frac{1}{4} e^{-t} \left(3 + e^{2 t} + 2 t\right)$

5

```
Remove["Global`*"]
```

```
LaplaceTransform[y'[t] + y[t] - z[t], t, s] /. {LaplaceTransform[y[t], t, s] → Y[s],
  LaplaceTransform[z[t], t, s] → Z[s], y[0] → 1, z[0] → 1}
```

$-1 + Y[s] + s Y[s] - Z[s]$

```
LaplaceTransform[Sin[t], t, s]
```

$\frac{1}{1 + s^2}$

```
LaplaceTransform[y[t] + z'[t] + z[t], t, s] /. {LaplaceTransform[y[t], t, s] → Y[s],
  LaplaceTransform[z[t], t, s] → Z[s], y[0] → 1, z[0] → 1}
```

$-1 + Y[s] + Z[s] + s Z[s]$

```
LaplaceTransform[E^(-t), t, s]
```

$\frac{1}{1 + s}$

```
Solve[{-1 + Y[s] + s Y[s] - Z[s] == 1/(1 + s^2), -1 + Y[s] + Z[s] + s Z[s] == 1/(1 + s)}, {Y[s], Z[s]}]
```

$\left\{\left\{Y[s] \to -\frac{-4 - 5 s - 5 s^2 - 3 s^3 - s^4}{2 + 4 s + 5 s^2 + 5 s^3 + 3 s^4 + s^5}, Z[s] \to -\frac{-s - s^2 - s^3}{\left(1 + s^2\right) \left(2 + 2 s + s^2\right)}\right\}\right\}$

```
-(-4 - 5 s - 5 s^2 - 3 s^3 - s^4)/(2 + 4 s + 5 s^2 + 5 s^3 + 3 s^4 + s^5) // Apart
```

$\frac{1}{1 + s} + \frac{3 - s}{5 \left(1 + s^2\right)} + \frac{4 + s}{5 \left(2 + 2 s + s^2\right)}$

```
y[t] = Simplify[InverseLaplaceTransform[1/(1 + s) + (3 - s)/(5 (1 + s^2)) + (4 + s)/(5 (2 + 2 s + s^2)), s, t] /.
    Assumptions → (t ∈ Reals)] // ExpandAll
```

$e^{-t} + \left(\frac{1}{10} + \frac{3 i}{10}\right) e^{(-1-i) t} + \left(\frac{1}{10} - \frac{3 i}{10}\right) e^{(-1+i) t} - \frac{\text{Cos}[t]}{5} + \frac{3 \text{Sin}[t]}{5}$

$$e^{-t} - \left(\frac{1}{10} + \frac{i}{10}\right) e^{(-1-i)\,t} \left((-2 - i) + (1 + 2\,i)\,e^{2\,i\,t}\right) + \frac{1}{5}\,(-\text{Cos}[t] + 3\,\text{Sin}[t]) \;//\; \text{ExpToTrig} \;//\; \text{Simplify}$$

$$\frac{1}{5}\,(-\text{Cos}[t] + 3\,\text{Sin}[t] + \text{Cosh}[t]\,(5 + \text{Cos}[t] + 3\,\text{Sin}[t]) - (5 + \text{Cos}[t] + 3\,\text{Sin}[t])\,\text{Sinh}[t])$$

$$\text{vx}[t\_] := \frac{1}{5}\,(-\text{Cos}[t] + 3\,\text{Sin}[t] + \text{Cosh}[t]\,(5 + \text{Cos}[t] + 3\,\text{Sin}[t]) - (5 + \text{Cos}[t] + 3\,\text{Sin}[t])\,\text{Sinh}[t])$$

$$\frac{-s - s^2 - s^3}{(1 + s^2)\,(2 + 2\,s + s^2)} \;//\; \text{Apart}$$

$$\frac{1 - 2\,s}{5\,(1 + s^2)} + \frac{-2 - 3\,s}{5\,(2 + 2\,s + s^2)}$$

$$z[t] = \text{InverseLaplaceTransform}\left[\frac{1 - 2\,s}{5\,(1 + s^2)} + \frac{-2 - 3\,s}{5\,(2 + 2\,s + s^2)},\ s,\ t\right]$$

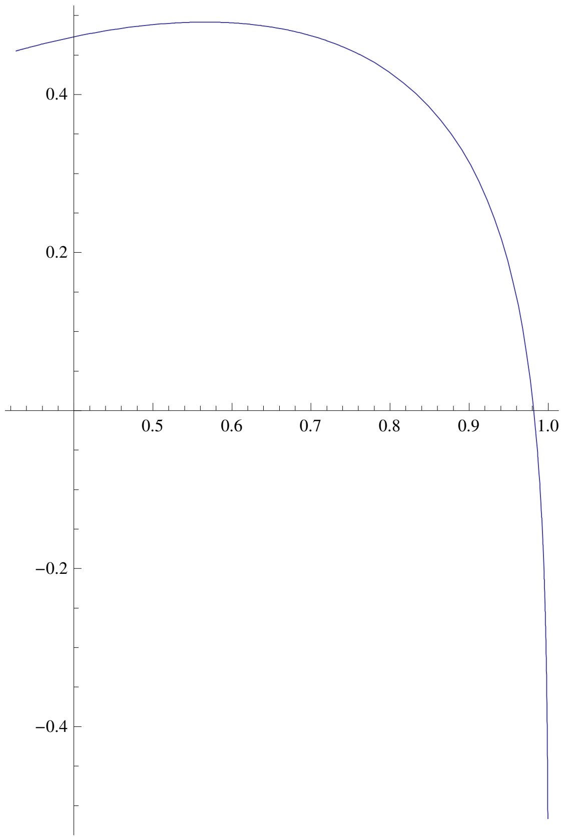
$$\left(-\frac{1}{10} + \frac{i}{10}\right) e^{(-1-i)\,t} \left((2 + i) + (1 + 2\,i)\,e^{2\,i\,t}\right) + \frac{1}{5}\,(-2\,\text{Cos}[t] + \text{Sin}[t])$$

$$\left(-\frac{1}{10} + \frac{i}{10}\right) e^{(-1-i)\,t} \left((2 + i) + (1 + 2\,i)\,e^{2\,i\,t}\right) + \frac{1}{5}\,(-2\,\text{Cos}[t] + \text{Sin}[t]) \;//\; \text{ExpToTrig} \;//\; \text{Simplify}$$

$$\frac{1}{5}\,(-2\,\text{Cos}[t] + \text{Sin}[t] + \text{Cosh}[t]\,(-3\,\text{Cos}[t] + \text{Sin}[t]) + (3\,\text{Cos}[t] - \text{Sin}[t])\,\text{Sinh}[t])$$

$$\text{vy}[t\_] := \frac{1}{5}\,(-2\,\text{Cos}[t] + \text{Sin}[t] + \text{Cosh}[t]\,(-3\,\text{Cos}[t] + \text{Sin}[t]) + (3\,\text{Cos}[t] - \text{Sin}[t])\,\text{Sinh}[t])$$

ParametricPlot[{vx[t], vy[t]}, {t, 0.5, 3}]



6

```
Remove["Global`*"]
```

a

```
DSolve[{y'[t] == -y[t] t - t,
        y[0] == 1}, y, t]
```

$\left\{\left\{y \to \text{Function}\left[\{t\}, -e^{-\frac{t^2}{2}}\left(-2+e^{\frac{t^2}{2}}\right)\right]\right\}\right\}$

yExakt[t_] := $-e^{-\frac{t^2}{2}}\left(-2+e^{\frac{t^2}{2}}\right)$

```
yExakt[t] // Expand
```

$-1+2\,e^{-\frac{t^2}{2}}$

b

```
Solve[(yEu[k, Δt] - yEu[(k - 1) , Δt]) / Δt ==
  -yEu[(k - 1), Δt] (k - 1) Δt - (k - 1) Δt, {yEu[k , Δt]}]
```

$\{\{\text{yEu}[k, \Delta t] \to \Delta t^2 - k\,\Delta t^2 + \text{yEu}[-1+k, \Delta t] + \Delta t^2\,\text{yEu}[-1+k, \Delta t] - k\,\Delta t^2\,\text{yEu}[-1+k, \Delta t]\}\}$

yEu[k_, Δt_] := $\Delta t^2 - k\,\Delta t^2$ + yEu[-1 + k, Δt] + Δt^2 yEu[-1 + k, Δt] - k Δt^2 yEu[-1 + k, Δt];

```
Δt = 0.25; yEu[0, Δt] = 1;
```

```
Table[{k, k Δt, yEu[k, Δt]}, {k, 0, 4}] // TableForm
```

0	0.	1
1	0.25	1.
2	0.5	0.875
3	0.75	0.640625
4	1.	0.333008

```
yEu[4, Δt]
```

0.333008

```
yExakt[1] // N
```

0.213061

```
yEu[4, Δt] - yExakt[1] // N
```

0.119946

7

```
Remove["Global`*"]
```

a

```
DSolve[{y''[t] - 2 y'[t] + 1 / 2 y[t] == t - b, y[0] == 0, y'[0] == 0}, y, t]
```

$\left\{\left\{y \to \text{Function}\left[\{t\}, 8 - 2b - 4\,e^{\left(1-\frac{1}{\sqrt{2}}\right)t} - 3\sqrt{2}\,e^{\left(1-\frac{1}{\sqrt{2}}\right)t} + b\,e^{\left(1-\frac{1}{\sqrt{2}}\right)t} + \sqrt{2}\,b\,e^{\left(1-\frac{1}{\sqrt{2}}\right)t} - 4\,e^{\left(1+\frac{1}{\sqrt{2}}\right)t} + 3\sqrt{2}\,e^{\left(1+\frac{1}{\sqrt{2}}\right)t} + b\,e^{\left(1+\frac{1}{\sqrt{2}}\right)t} - \sqrt{2}\,b\,e^{\left(1+\frac{1}{\sqrt{2}}\right)t} + 2t\right]\right\}\right\}$

N[%]

$\{\{y \to \text{Function}[\{t\}, 8. - 2.\, b - 4. \times 2.71828^{0.292893\, t} - 3. \times 1.41421 \times 2.71828^{0.292893\, t} + b\, 2.71828^{0.292893\, t} + 1.41421\, b\, 2.71828^{0.292893\, t} - 4. \times 2.71828^{1.70711\, t} + 3. \times 1.41421 \times 2.71828^{1.70711\, t} + b\, 2.71828^{1.70711\, t} - 1. \times 1.41421\, b\, 2.71828^{1.70711\, t} + 2.\, t]\}\}$

y[t_, b_] := 8 - 2 b - 4 $e^{\left(1-\frac{1}{\sqrt{2}}\right) t} - 3\sqrt{2}\, e^{\left(1-\frac{1}{\sqrt{2}}\right) t} + b\, e^{\left(1-\frac{1}{\sqrt{2}}\right) t} + \sqrt{2}\, b\, e^{\left(1-\frac{1}{\sqrt{2}}\right) t} - 4\, e^{\left(1+\frac{1}{\sqrt{2}}\right) t} + 3\sqrt{2}\, e^{\left(1+\frac{1}{\sqrt{2}}\right) t} + b\, e^{\left(1+\frac{1}{\sqrt{2}}\right) t} - \sqrt{2}\, b\, e^{\left(1+\frac{1}{\sqrt{2}}\right) t} + 2 t$

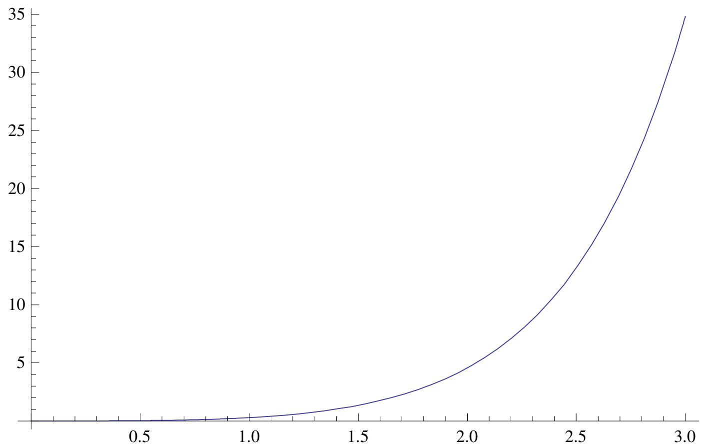
y[t, b]

$8 - 2 b - 4\, e^{\left(1-\frac{1}{\sqrt{2}}\right) t} - 3\sqrt{2}\, e^{\left(1-\frac{1}{\sqrt{2}}\right) t} + b\, e^{\left(1-\frac{1}{\sqrt{2}}\right) t} + \sqrt{2}\, b\, e^{\left(1-\frac{1}{\sqrt{2}}\right) t} - 4\, e^{\left(1+\frac{1}{\sqrt{2}}\right) t} + 3\sqrt{2}\, e^{\left(1+\frac{1}{\sqrt{2}}\right) t} + b\, e^{\left(1+\frac{1}{\sqrt{2}}\right) t} - \sqrt{2}\, b\, e^{\left(1+\frac{1}{\sqrt{2}}\right) t} + 2 t$

y[t, b] // ExpToTrig

$8 - 2 b + 2 t - 4 \text{Cosh}\left[\left(1-\frac{1}{\sqrt{2}}\right) t\right] - 3\sqrt{2}\, \text{Cosh}\left[\left(1-\frac{1}{\sqrt{2}}\right) t\right] + b\, \text{Cosh}\left[\left(1-\frac{1}{\sqrt{2}}\right) t\right] + \sqrt{2}\, b\, \text{Cosh}\left[\left(1-\frac{1}{\sqrt{2}}\right) t\right] - 4 \text{Cosh}\left[\left(1+\frac{1}{\sqrt{2}}\right) t\right] + 3\sqrt{2}\, \text{Cosh}\left[\left(1+\frac{1}{\sqrt{2}}\right) t\right] + b\, \text{Cosh}\left[\left(1+\frac{1}{\sqrt{2}}\right) t\right] - \sqrt{2}\, b\, \text{Cosh}\left[\left(1+\frac{1}{\sqrt{2}}\right) t\right] - 4 \text{Sinh}\left[\left(1-\frac{1}{\sqrt{2}}\right) t\right] - 3\sqrt{2}\, \text{Sinh}\left[\left(1-\frac{1}{\sqrt{2}}\right) t\right] + b\, \text{Sinh}\left[\left(1-\frac{1}{\sqrt{2}}\right) t\right] + \sqrt{2}\, b\, \text{Sinh}\left[\left(1-\frac{1}{\sqrt{2}}\right) t\right] - 4 \text{Sinh}\left[\left(1+\frac{1}{\sqrt{2}}\right) t\right] + 3\sqrt{2}\, \text{Sinh}\left[\left(1+\frac{1}{\sqrt{2}}\right) t\right] + b\, \text{Sinh}\left[\left(1+\frac{1}{\sqrt{2}}\right) t\right] - \sqrt{2}\, b\, \text{Sinh}\left[\left(1+\frac{1}{\sqrt{2}}\right) t\right]$

Plot[y[t, 0], {t, 0, 3}]



- **b**

y[1, 1]

$8 - 3\, e^{1-\frac{1}{\sqrt{2}}} - 2\sqrt{2}\, e^{1-\frac{1}{\sqrt{2}}} - 3\, e^{1+\frac{1}{\sqrt{2}}} + 2\sqrt{2}\, e^{1+\frac{1}{\sqrt{2}}}$

y[1, 1] // N

-0.757718

Lösungen 2

1

```
Remove["Global`*"]
```

▪ a

```
p1 = 0.005;  p2 = 1 - p1;
pa1 = 0.95; pa2 = 1 - pa1;
```

```
AA = p1 pa1
```

```
0.00475
```

```
AB = p1 pa2
```

```
0.00025
```

```
B = p2
```

```
0.995
```

```
AA + AB + B (* Test *)
```

```
1.
```

▪ b

```
1000 AB
```

```
0.25
```

```
2 × 1000 AB
```

```
0.5
```

```
(* Gleich oder mehr als 2000 *)
```

```
4 × 1000 AB
```

```
1.
```

2

▪ a

```
Remove["Global`*"]
```

```
n = 45;
xQuer = 772.0;
σQuadrat = 61^2;
σ = Sqrt[σQuadrat];
α = 1 - 0.95;
```

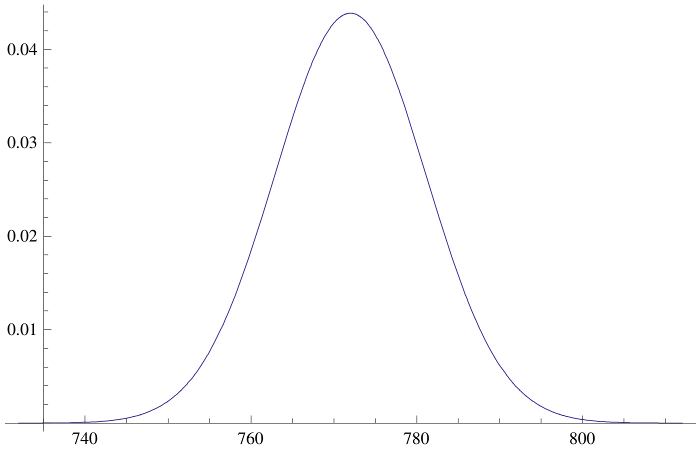
```
σ // N
```

```
61.
```

```
NV[x_, μ_, σ_, n_] := 1 / (σ / Sqrt[n] Sqrt[2 Pi]) E^(-1 / 2 ((x - μ) Sqrt[n] / σ)^2);
NV[μ, xQuer, σ, n]
```

$$\frac{3}{61} e^{-\frac{45(-772.+\mu)^2}{7442}} \sqrt{\frac{5}{2\pi}}$$

```
h = 40;  Plot[NV[μ, xQuer, σ, n], {μ, 772.0 - h, 772.0 + h}]
```




```
FindRoot[Integrate[NV[μ, xQuer, σ, n], {μ, 772.0 - hh, 772.0 + hh}] == 0.95, {hh, 35}]
```

{hh → 17.8226}

```
h = 17.82262499169023`;  Integrate[NV[μ, xQuer, σ, n], {μ, 772.0 - h, 772.0 + h}]
```

0.950000000000

Integriere nur über die halbe Normalverteilung:

```
Integrate[NV[μ, xQuer, σ, n], {μ, xQuer, x}] == (1 - α) / 2
```

$4.00880810182 \times 10^{-15} - 0.500000000000 \, \text{Erf}[60.0314 - 0.0777609 x] == 0.475$

```
rootOben = FindRoot[
  Evaluate[Integrate[NV[μ, xQuer, σ, n], {μ, xQuer, x}]] == (1 - α) / 2, {x, 772.0 + 35}]
```

{x → 789.823}

```
cOben = x /. rootOben
```

789.823

```
cUnten = xQuer - (cOben - xQuer)
```

754.177

```
Integrate[NV[μ, xQuer, σ, n], {μ, cUnten, cOben}]
```

0.950000000000

```
1 - Integrate[NV[μ, xQuer, σ, n], {μ, cUnten, cOben}]
```

0.050000000000

Das ist etwa α. Lösung: [cUnten, cOben] ist das Konfidenzintervall.

3

```
Remove["Global`*"]
```

a

```
M1 = {3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3, 2, 3, 8, 4, 6, 2, 6,
    4, 3, 3, 8, 3, 2, 7, 9, 5, 0, 2, 8, 8, 4, 1, 9, 7, 1, 6, 9, 3, 9, 9, 3, 7, 5};
M2 = {1, 0, 5, 8, 2, 0, 9, 7, 4, 9, 4, 4, 5, 9, 2, 3, 0, 7, 8, 1, 6, 4, 0, 6, 2,
    8, 6, 2, 0, 8, 9, 9, 8, 6, 2, 8, 0, 3, 4, 8, 2, 5, 3, 4, 2, 1, 1, 7, 0};
m1 = Mean[M1] // N
```

```
5.02041
```

```
m2 = Mean[M2] // N
```

```
4.32653
```

```
s1 = StandardDeviation[M1] // N
```

```
2.76503
```

```
s2 = StandardDeviation[M2] // N
```

```
3.08483
```

b

```
p = 1 / 10; M = Range[10]; Print[M];
EX = Sum[M[[k]] p, {k, 1, Length[M]}] // N
```

```
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

```
5.5
```

```
EquEXminEX = Sum[(M[[k]] - EX)^2 p, {k, 1, Length[M]}] // N
```

```
8.25
```

```
s = Sqrt[EquEXminEX]
```

```
2.87228
```

c Abwerichungen im %

```
(EX - m1) / EX 100
```

```
8.71985
```

```
(EX - m2) / EX 100
```

```
21.3358
```

```
(s - s1) / s 100
```

```
3.7339
```

```
(s - s2) / s 100
```

```
-7.39988
```

d

```
P1 = Table[{k, M1[[k]]}, {k, 1, Length[M1]}]
```

```
{{1, 3}, {2, 1}, {3, 4}, {4, 1}, {5, 5}, {6, 9}, {7, 2}, {8, 6}, {9, 5}, {10, 3}, {11, 5},
 {12, 8}, {13, 9}, {14, 7}, {15, 9}, {16, 3}, {17, 2}, {18, 3}, {19, 8}, {20, 4}, {21, 6},
 {22, 2}, {23, 6}, {24, 4}, {25, 3}, {26, 3}, {27, 8}, {28, 3}, {29, 2}, {30, 7}, {31, 9},
 {32, 5}, {33, 0}, {34, 2}, {35, 8}, {36, 8}, {37, 4}, {38, 1}, {39, 9}, {40, 7},
 {41, 1}, {42, 6}, {43, 9}, {44, 3}, {45, 9}, {46, 9}, {47, 3}, {48, 7}, {49, 5}}
```

```
P2 = Table[{k, M2[[k]]}, {k, 1, Length[M2]}]
```

```
{{1, 1}, {2, 0}, {3, 5}, {4, 8}, {5, 2}, {6, 0}, {7, 9}, {8, 7}, {9, 4}, {10, 9}, {11, 4},
 {12, 4}, {13, 5}, {14, 9}, {15, 2}, {16, 3}, {17, 0}, {18, 7}, {19, 8}, {20, 1}, {21, 6},
 {22, 4}, {23, 0}, {24, 6}, {25, 2}, {26, 8}, {27, 6}, {28, 2}, {29, 0}, {30, 8}, {31, 9},
 {32, 9}, {33, 8}, {34, 6}, {35, 2}, {36, 8}, {37, 0}, {38, 3}, {39, 4}, {40, 8},
 {41, 2}, {42, 5}, {43, 3}, {44, 4}, {45, 2}, {46, 1}, {47, 1}, {48, 7}, {49, 0}}
```

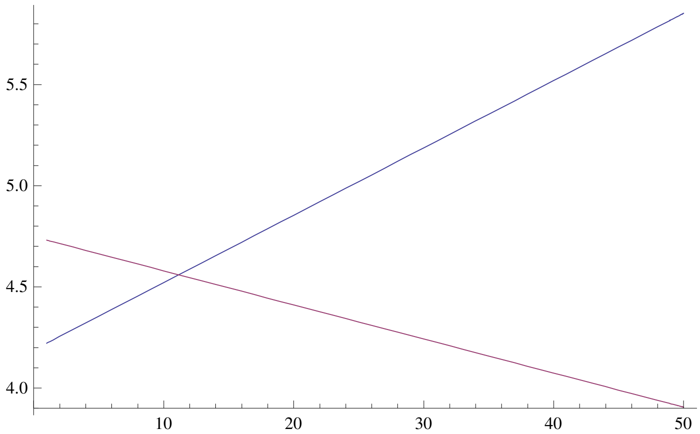
```
line1 = Fit[P1, {1, x}, x]
```

4.18878 + 0.0332653 x

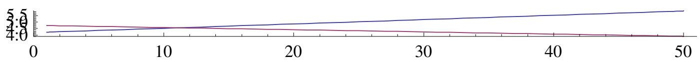
```
line2 = Fit[P2, {1, x}, x]
```

4.74745 - 0.0168367 x

```
Plot[{line1, line2}, {x, 1, 50}]
```




```
Plot[{line1, line2}, {x, 1, 50}, AspectRatio → Automatic]
```



4

```
Remove["Global`*"]
```

- **a**

```
parbildungen = 10 × 10
```

100

Punkte in einem Gitter 10 mal 10

- **b**

```
10!
```

3 628 800

- **c**

```
1 / 10! // N
```

2.75573×10^{-7}

- **d**

```
Binomial[10, 3]
```

120

- **e**

```
Binomial[10, 3] 3!
```

720

- **f**

```
gDurchm = 3 / 10 // N
```

0.3

5

```
Remove["Global`*"]
```

- **a**

```
<< StatisticalPlots`;
```

```
D1 = Table[(k / 10)^2, {k, 1, 10}]
```

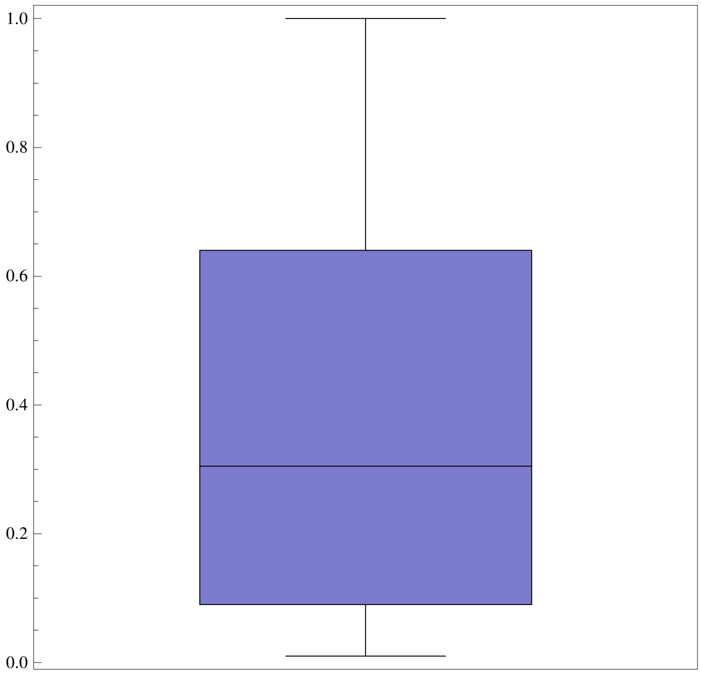
$\left\{\frac{1}{100}, \frac{1}{25}, \frac{9}{100}, \frac{4}{25}, \frac{1}{4}, \frac{9}{25}, \frac{49}{100}, \frac{16}{25}, \frac{81}{100}, 1\right\}$

```
D2 = Table[(k / 10)^3, {k, 1, 10}]
```

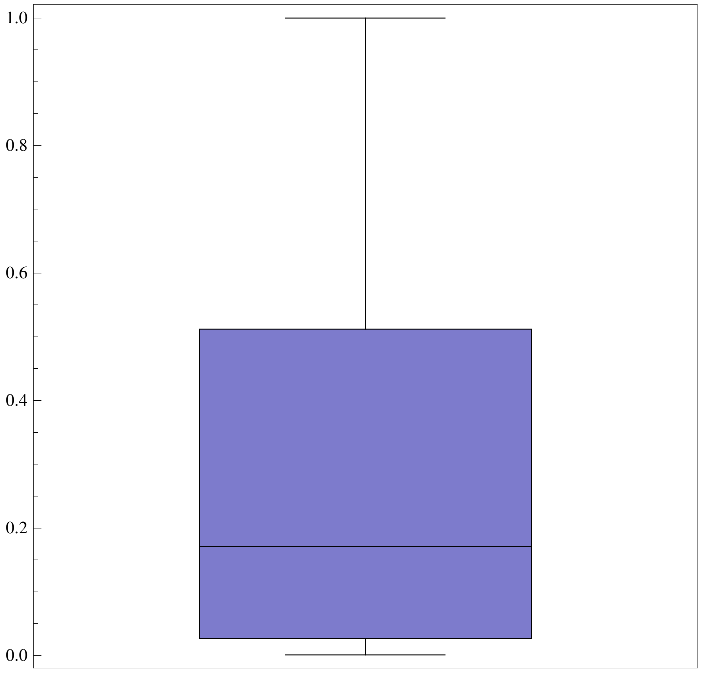
$\left\{\frac{1}{1000}, \frac{1}{125}, \frac{27}{1000}, \frac{8}{125}, \frac{1}{8}, \frac{27}{125}, \frac{343}{1000}, \frac{64}{125}, \frac{729}{1000}, 1\right\}$

```
BoxWhiskerPlot[D1]
```

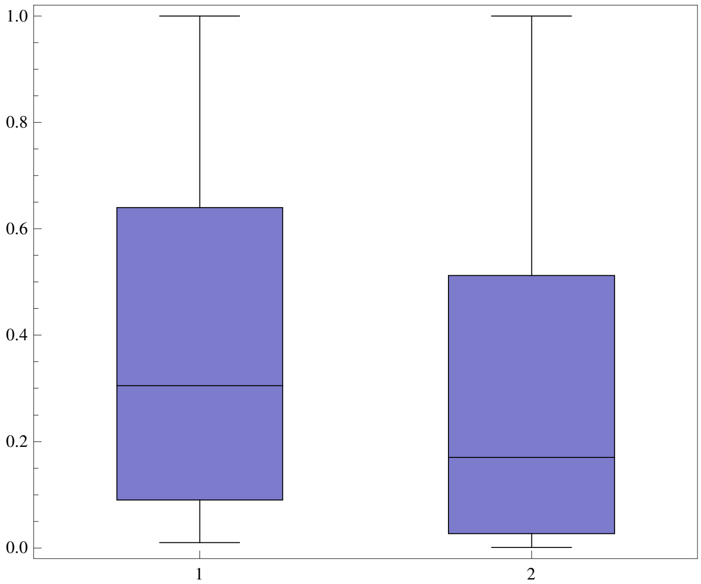
General::obsfun : The function BoxWhiskerPlot is now obsolete and has been superseded by BoxWhiskerChart.



```
BoxWhiskerPlot[D2]
```




```
BoxWhiskerPlot[D1, D2]
```



- **b Im 2. Satz systematisch Median und Quartile tiefer. Unwahrscheinlich dass Zufall.**

6.

```
Remove["Global`*"]
```

$n_0 = 35$
3 neutral
24 Differenzen kleiner 0 ==> Verfahren A besser
8 Differenzen grösser 0 ==> Verfahren B besser

H0 ==> beide Verfahren sind gleich
H1 ==> beide Verfahren sind nicht gleich, Verfahren verschieden

Frage: Wahrscheinlichkeit, dass unter Annahme von H0 trotzdem eine Abweichung zwischen A und B auftritt, dass also H0 falsch sein muss?
Sei x_i der i-te Wert der beim Verfahren (A), x_i' der i-te Wert beim Verfahren (B).
Sei $d_i = x_i - x_i'$. Dann ist unter Annahme von H0 das Auftreten einer negativen Differenz gleich wahrscheinlich wie das Auftreten einer positiven Differenz.
Wir nehmen an,dass die Verteilungen A und B angesichts der Messwerte x_i und x_i' stetig sind (keine Anzahlen scharf trennbar). Daher kann man angesichts der daraus resultierenden Wahrscheinlichkeitsdichten $P(X_i = X_i') = 0$ setzen. Die 3 neutralen werden daher aus der Betrachtung entfernt, da sie schleicht beurteilt sind und nichts zum Experiment beitragen.
Weiter sind die Zufallsgrössen für i=1,2,...,n unabhängig.
Damit wird neu n=24+8=32.
So ergibt sich z.B.für A neu eine Binomialverteilung mit $p=\frac{1}{2}$, $q = 1-\frac{1}{2}=\frac{1}{2}$ und n=32.
$P(X=24)=\binom{32}{24}\left(\frac{1}{2}\right)^{24}\left(\frac{1}{2}\right)^{(32-24)} = \binom{32}{24}\left(\frac{1}{2}\right)^{32}$, P(X=k) := f(k)

Um weiter zu kommen betrachten wir die neue Alternativhypothese H1, welche postuliert, dass die x_i-Werte im Durchschnitt wesentlich grösser sind als die x_i'-Werte.
H0 wird dann abgelehnt, wenn die Anzahl der positiven Differenzen k^+ einen kritischen Wert $k_{(1-\alpha)}$ überschreitet oder gleich ist.
Vorderung: $P(K^+ \geq k_{(1-\alpha)}) \leq \alpha$, α = Irrtumswahrscheinlichkeit.
Es gilt: $P(K^+ \geq k_{(1-\alpha)}) = \sum_{i=k\_(1-\alpha)}^{n} P(K^+ = i) \leq \alpha$ ==> $k_{(1-\alpha)}$ = ?

```
f[k_] := Binomial[32, k] (1 / 2) ^32;  f[k]
```

$$\frac{\text{Binomial}[32, k]}{4\,294\,967\,296}$$

```
f[32] // N
```

2.32831×10^{-10}

```
s[k_] := Sum[N[f[u]], {u, k, 32}];  s[k]
```

$1. + 2.32831 \times 10^{-10}$
(1. - 1. Binomial[32., -1. + k] Hypergeometric2F1[1., 1. - 1. k, 34. - 1. k, -1.])

s[k] ist die Wahrscheinlichkeit
P[K ≥ 24 (K grösser gleich 24)] = 1-P[K < 24 (K kleiner 24)]
P[K < 24(K kleiner 24)] = 1-P[K ≥ 24 (K grösser gleich 24)] = 1-s[k]

```
Prepend[Table[{k, s[k], 1 - s[k]}, {k, 0, 32}], {"n", "s[k]=α", "1-s[k]=1-α"}] // TableForm
```

n	s[k]=α	1-s[k]=1-α
0	1.	0.
1	1.	2.32831×10^{-10}
2	1.	7.68341×10^{-9}
3	1.	1.23167×10^{-7}
4	0.999999	1.27801×10^{-6}
5	0.99999	9.6506×10^{-6}
6	0.999943	0.0000565371
7	0.999732	0.000267526
8	0.998949	0.0010512
9	0.9965	0.00350018
10	0.989969	0.0100308
11	0.974949	0.0250512
12	0.944908	0.0550921
13	0.892336	0.107664
14	0.811457	0.188543
15	0.701693	0.298307
16	0.569975	0.430025
17	0.430025	0.569975
18	0.298307	0.701693
19	0.188543	0.811457
20	0.107664	0.892336
21	0.0550921	0.944908
22	0.0250512	0.974949
23	0.0100308	0.989969
24	0.00350018	0.9965
25	0.0010512	0.998949
26	0.000267526	0.999732
27	0.0000565371	0.999943
28	9.6506×10^{-6}	0.99999
29	1.27801×10^{-6}	0.999999
30	1.23167×10^{-7}	1.
31	7.68341×10^{-9}	1.
32	2.32831×10^{-10}	1.

Ein Wert k ≥ 24 (k grösser gleich 24) kommt daher mit der Wahrscheinlichkeit 0.00350018... kleiner vor, je nach k.
Für grössere α-Werte (alpha-Werte) müsste man k kleiner haben, damit P[K ≥ k, grösser gleich] ≤ α (kleiner gleich alpha) eintrifft. Für k ≥ 25 (k grösser gleich 25) hat daher die Alternative eine sehr kleine Wahrscheinlichkeit.
Da die eingetroffene reale Situation, die Alternative also mit k = 24 mit p= 0.00350018... so unwahrscheinlich aber dennoch real ist, müsste man H0 z.B. bei einem Signifikanzniveau von α = 0.05 oder bei α = 0.01ablehnen, denn es ist 0.00350018... < 0.01. Die Abweichung von einem erwarteten Resultat, dass die negativen und die positiven Differenzen etwa gleich oft vorkommen müsten, ist hier zu gross.

```
ListPlot[Table[{k, s[k]}, {k, 0, 32}]]
```