Lösungen

1

a

```
r = 10; ϕ = 2 Pi / 8; ϕ1 = ϕ / 2;  a = 2 r Sin[ϕ1]; a1 = a / 2;
OP1 = {r, a1, a1};  OP2 = {a1, r, a1};  OP3 = {a1, a1, r};

{ OP1,  OP2, OP3 }
```

$$\left\{\left\{10, 10\,\mathrm{Sin}\left[\frac{\pi}{8}\right], 10\,\mathrm{Sin}\left[\frac{\pi}{8}\right]\right\}, \left\{10\,\mathrm{Sin}\left[\frac{\pi}{8}\right], 10, 10\,\mathrm{Sin}\left[\frac{\pi}{8}\right]\right\}, \left\{10\,\mathrm{Sin}\left[\frac{\pi}{8}\right], 10\,\mathrm{Sin}\left[\frac{\pi}{8}\right], 10\right\}\right\}$$

```
{ OP1,  OP2, OP3 } // N
```

{{10., 3.82683, 3.82683}, {3.82683, 10., 3.82683}, {3.82683, 3.82683, 10.}}

b

```
InhaltDreieck = Norm[Cross[OP2 - OP1, OP3 - OP1]] / 2
```

$$\frac{1}{2}\sqrt{3}\left(100 - 200\,\mathrm{Sin}\left[\frac{\pi}{8}\right] + 100\,\mathrm{Sin}\left[\frac{\pi}{8}\right]^2\right)$$

```
% // N
```

33.0025

c

```
OS = Norm[(OP1 + OP2 + OP3) / 3]
```

$$\frac{10 + 20\,\mathrm{Sin}\left[\frac{\pi}{8}\right]}{\sqrt{3}}$$

```
% // N
```

10.1924

```
OS == r
```

False

d

```
InhaltQuadrat = a^2
```

$$400\,\mathrm{Sin}\left[\frac{\pi}{8}\right]^2$$

```
% // N
```

58.5786

```
Inhalt = 8 InhaltDreieck + 18 InhaltQuadrat
```

$$7200\,\mathrm{Sin}\left[\frac{\pi}{8}\right]^2 + 4\sqrt{3}\left(100 - 200\,\mathrm{Sin}\left[\frac{\pi}{8}\right] + 100\,\mathrm{Sin}\left[\frac{\pi}{8}\right]^2\right)$$

```
% // N
```

1318.44

```
Verhaeltlnis = Inhalt / (4 r^2 Pi)
```

$$\frac{7200\,\mathrm{Sin}\left[\frac{\pi}{8}\right]^2 + 4\sqrt{3}\left(100 - 200\,\mathrm{Sin}\left[\frac{\pi}{8}\right] + 100\,\mathrm{Sin}\left[\frac{\pi}{8}\right]^2\right)}{400\,\pi}$$

```
% // N
```

1.04918

2

```
Remove["Global`*"]
OP1 = {2, 3, 5}; OP2 = {1, 2, 6};  OQ1 = {-2, -3, -5}; OQ2[z_] := {-3, -1, z};
g[t] := OP1 + t (OP2 - OP1);  q[u_, z_] := OQ1 + u (OQ2[z] - OQ1);
```

■ a

```
OQ2f = OQ2[-2];
KuerzAbst[z_] :=
  Det[{(OP2 - OP1), (OQ2[z] - OQ1), OQ1 - OP1}] / Norm[Cross[(OP2 - OP1), (OQ2[z] - OQ1)]];
KuerzAbst[
 -2]
```

$\sqrt{38}$

```
% // N
```

6.16441

■ b

```
KuerzAbst[z] == 10
```

$$\frac{34 - 2\,z}{\sqrt{9 + \mathrm{Abs}[-7 - z]^2 + \mathrm{Abs}[4 + z]^2}} == 10$$

```
Solve[KuerzAbst[z] == 10] // Flatten
```

$$\left\{z \to \frac{1}{49}\left(-292 - 15\sqrt{39}\right),\ z \to \frac{1}{49}\left(-292 + 15\sqrt{39}\right)\right\}$$

```
% // N
```

{z → -7.87092, z → -4.04745}

■ c

2 Lösungen.

3

```
v1 = {-4, 0, -3, 5}; v2 = {1, 2, -4, 0} ;  v3 = {-3, 4, 0, 2};
v4 = {-1, 2, 3, 1}; X = {v1, v2, v3, v4} // Transpose; X // MatrixForm
```

$$\begin{pmatrix} -4 & 1 & -3 & -1 \\ 0 & 2 & 4 & 2 \\ -3 & -4 & 0 & 3 \\ 5 & 0 & 2 & 1 \end{pmatrix}$$

```
Dλ = {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -2, 0}, {0, 0, 0, 2}}; Dλ // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

- **a**

```
A = X.Dλ.Inverse[X]; A // MatrixForm
```

$$\begin{pmatrix} -\frac{187}{37} & \frac{14}{37} & -\frac{61}{74} & -\frac{395}{74} \\ \frac{214}{37} & -\frac{18}{37} & \frac{63}{37} & \frac{209}{37} \\ \frac{189}{37} & \frac{129}{74} & \frac{85}{74} & \frac{309}{74} \\ \frac{137}{37} & -\frac{31}{74} & \frac{53}{74} & \frac{325}{74} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} -5.05405 & 0.378378 & -0.824324 & -5.33784 \\ 5.78378 & -0.486486 & 1.7027 & 5.64865 \\ 5.10811 & 1.74324 & 1.14865 & 4.17568 \\ 3.7027 & -0.418919 & 0.716216 & 4.39189 \end{pmatrix}$$

```
74 A // MatrixForm
```

$$\begin{pmatrix} -374 & 28 & -61 & -395 \\ 428 & -36 & 126 & 418 \\ 378 & 129 & 85 & 309 \\ 274 & -31 & 53 & 325 \end{pmatrix}$$

- **b**

```
OQStrich = A.(v1 + v2 + v3 + v4)
```

{-1, -6, 7, 3}

- **c**

```
OQStrichStrich = (A + Transpose[A]).OQStrich
```

$$\left\{-\frac{66}{37}, \frac{1461}{37}, \frac{429}{74}, \frac{2283}{74}\right\}$$

```
N[%]
```

{-1.78378, 39.4865, 5.7973, 30.8514}

4

```
Remove["Global`*"]
```

```
EVNorm[Matrix_] :=
 Transpose[Table[N[Transpose[Matrix][[k]] / Norm[Transpose[Matrix][[k]]]],
   {k, 1, Length[Transpose[Matrix]]}]]
```

```
X = {{1, 2, 0, 1}, {0, 1, -2, 1}, {1, 1, 3, 0}, {-2, 0, 4, 1}};
Dl = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}};
M = X.Dl.Inverse[X];
MatrixForm[M]
```

$$\begin{pmatrix} 29 & -32 & -24 & 2 \\ 24 & -27 & -20 & 2 \\ 6 & -6 & -5 & 0 \\ 36 & -40 & -32 & 3 \end{pmatrix}$$

```
(* MatrixForm[M]//TeXForm *)
```

```
h1 = Transpose[X][[1]]; h2 = Transpose[X][[2]];
h3 = Transpose[X][[3]]; h4 = Transpose[X][[4]];
```

```
{h1, h2, h3, h4}
```

```
{{1, 0, 1, -2}, {2, 1, 1, 0}, {0, -2, 3, 4}, {1, 1, 0, 1}}
```

```
X // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & 1 & 3 & 0 \\ -2 & 0 & 4 & 1 \end{pmatrix}$$

```
Det[X]
```

```
-1
```

```
v1 = {-4, 0, -3, 5}; v2 = {-3, 4, 0, 2}; v3 = {1, 2, -4, 0};
v4 = {-1, 2, 3, 1}; Vmatr = {v1, v2, v3, v4};
```

```
Det[Vmatr]
```

```
148
```

■ a

```
Eigenvalues[M]
```

```
{-1, -1, 1, 1}
```

■ b Eigenvektoren zu doppelten Eigenwerten nicht eindeutig

```
Eigenvectors[M] // Transpose // MatrixForm
```

$$\begin{pmatrix} 1 & -4 & 1 & 2 \\ 1 & -6 & 1 & 1 \\ 0 & 3 & 0 & 1 \\ 1 & 0 & 2 & 0 \end{pmatrix}$$

```
EVNorm[Eigenvectors[M] // Transpose] // MatrixForm
```

$$\begin{pmatrix} 0.57735 & -0.512148 & 0.408248 & 0.816497 \\ 0.57735 & -0.768221 & 0.408248 & 0.408248 \\ 0. & 0.384111 & 0. & 0.408248 \\ 0.57735 & 0. & 0.816497 & 0. \end{pmatrix}$$

```
EVM = Eigenvectors[M]; ev1 = EVM[[1]]; ev2 = EVM[[2]]; ev3 = EVM[[3]]; ev4 = EVM[[4]];
{ev1, ev2, ev3, ev4}
```

```
{{1, 1, 0, 1}, {-4, -6, 3, 0}, {1, 1, 0, 2}, {2, 1, 1, 0}}
```

```
EAM = Eigenvalues[M]
```

```
{-1, -1, 1, 1}
```

```
M.ev1 == EAM[[1]] ev1
```

```
True
```

```
M.ev2 == EAM[[2]] ev2
```

```
True
```

```
M.ev3 == EAM[[3]] ev3
```

```
True
```

```
M.ev3 == EAM[[3]] ev3
```

```
M.ev4 == EAM[[4]] ev4
```

```
True
```

```
ev1.ev2
-10
ev1.ev3
4
ev1.ev4
3
ev2.ev3
-10
ev2.ev4
-11
ev3.ev4
3
```

Nie senkrecht.

```
Solve[{(α ev1 + β ev2) (γ ev3 + δ ev4) == 0, (α β)^2 == 1}, {α, β, γ, δ}]
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$\left\{\left\{\beta \to -\frac{1}{\alpha}, \gamma \to 0, \delta \to 0\right\}, \left\{\beta \to \frac{1}{\alpha}, \gamma \to 0, \delta \to 0\right\}, \left\{\alpha \to -2, \beta \to -\frac{1}{2}, \gamma \to 0, \delta \to 0\right\},\right.$
$\left\{\alpha \to -2 i, \beta \to -\frac{i}{2}, \gamma \to 0, \delta \to 0\right\}, \left\{\alpha \to 2 i, \beta \to \frac{i}{2}, \gamma \to 0, \delta \to 0\right\}, \left\{\alpha \to 2, \beta \to \frac{1}{2}, \gamma \to 0, \delta \to 0\right\},$
$\left\{\alpha \to -\sqrt{6}, \beta \to -\frac{1}{\sqrt{6}}, \gamma \to 0, \delta \to 0\right\}, \left\{\alpha \to -i\sqrt{6}, \beta \to -\frac{i}{\sqrt{6}}, \gamma \to 0, \delta \to 0\right\},$
$\left.\left\{\alpha \to i\sqrt{6}, \beta \to \frac{i}{\sqrt{6}}, \gamma \to 0, \delta \to 0\right\}, \left\{\alpha \to \sqrt{6}, \beta \to \frac{1}{\sqrt{6}}, \gamma \to 0, \delta \to 0\right\}\right\}$

Das System hat ausschliesslich nur Lösungen, bei denen der eine Faktor (und damit γ und δ) null ist, was keinen Eigenvektor ungleich null bedeutet.

==> Nie senkrecht.

- **c**

```
Inverse[M] // MatrixForm
```

$$\begin{pmatrix} 29 & -32 & -24 & 2 \\ 24 & -27 & -20 & 2 \\ 6 & -6 & -5 & 0 \\ 36 & -40 & -32 & 3 \end{pmatrix}$$

```
M == Inverse[M]
True
Eigenvalues[Inverse[M]]
{-1, -1, 1, 1}
```

- **d Eigenvektoren zu doppelten Eigenwerten nicht eindeutig**

```
Eigenvectors[Inverse[M]] // Transpose // MatrixForm
```

$$\begin{pmatrix} 1 & -4 & 1 & 2 \\ 1 & -6 & 1 & 1 \\ 0 & 3 & 0 & 1 \\ 1 & 0 & 2 & 0 \end{pmatrix}$$

```
EVNorm[Eigenvectors[Inverse[M]]] // Transpose // MatrixForm
```

$$\begin{pmatrix} 0.213201 & -0.852803 & 0.213201 & 0.426401 \\ 0.160128 & -0.960769 & 0.160128 & 0.160128 \\ 0. & 0.948683 & 0. & 0.316228 \\ 0.447214 & 0. & 0.894427 & 0. \end{pmatrix}$$

```
Eigenvectors[Inverse[M]] == Eigenvectors[M]
```

True

Die Eigenvektoren von M sind identisch mit denen von Inverse[M].

- **e**

```
TrM = Transpose[M]
```

{{29, 24, 6, 36}, {-32, -27, -6, -40}, {-24, -20, -5, -32}, {2, 2, 0, 3}}

```
Eigenvalues[TrM]
```

{-1, -1, 1, 1}

- **f**

```
Eigenvectors[TrM] // Transpose // MatrixForm
```

$$\begin{pmatrix} 2 & -1 & -3 & -3 \\ -4 & 1 & 2 & 3 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

```
EVNorm[Eigenvectors[TrM] // Transpose] // MatrixForm
```

$$\begin{pmatrix} 0.436436 & -0.57735 & -0.801784 & -0.639602 \\ -0.872872 & 0.57735 & 0.534522 & 0.639602 \\ 0. & 0.57735 & 0. & 0.426401 \\ 0.218218 & 0. & 0.267261 & 0. \end{pmatrix}$$

```
Eigenvectors[TrM] == Eigenvectors[M]
```

False

```
EVT = Eigenvectors[TrM]; eT1 = EVT[[1]];
eT2 = EVT[[2]]; eT3 = EVT[[3]]; eT4 = EVT[[4]];
{ev1, ev2, ev3, ev4}
```

{{1, 1, 0, 1}, {-4, -6, 3, 0}, {1, 1, 0, 2}, {2, 1, 1, 0}}

```
EATrM = Eigenvalues[TrM]
```

{-1, -1, 1, 1}

```
Solve[{(α ev1 + β ev2) (γ eT1 + δ eT2) == 0, (α β)^2 == 1}, {α, β, γ, δ}]
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$$\left\{\left\{\beta\to-\frac{1}{\alpha},\ \gamma\to0,\ \delta\to0\right\},\ \left\{\beta\to\frac{1}{\alpha},\ \gamma\to0,\ \delta\to0\right\},\ \left\{\alpha\to-2,\ \beta\to-\frac{1}{2},\ \gamma\to0,\ \delta\to0\right\},\right.$$
$$\left\{\alpha\to-2\,i,\ \beta\to-\frac{i}{2},\ \gamma\to0,\ \delta\to0\right\},\ \left\{\alpha\to2\,i,\ \beta\to\frac{i}{2},\ \gamma\to0,\ \delta\to0\right\},\ \left\{\alpha\to2,\ \beta\to\frac{1}{2},\ \gamma\to0,\ \delta\to0\right\},$$
$$\left\{\alpha\to-\sqrt{6},\ \beta\to-\frac{1}{\sqrt{6}},\ \gamma\to0,\ \delta\to0\right\},\ \left\{\alpha\to-i\sqrt{6},\ \beta\to-\frac{i}{\sqrt{6}},\ \gamma\to0,\ \delta\to0\right\},$$
$$\left.\left\{\alpha\to i\sqrt{6},\ \beta\to\frac{i}{\sqrt{6}},\ \gamma\to0,\ \delta\to0\right\},\ \left\{\alpha\to\sqrt{6},\ \beta\to\frac{1}{\sqrt{6}},\ \gamma\to0,\ \delta\to0\right\}\right\}$$

Das System hat ausschliesslich nur Lösungen, bei denen der eine Faktor (und damit γ und δ) null ist, was keinen Eigenvektor ungleich null bedeutet.

```
Solve[{(α ev1 + β ev2) (γ eT3 + δ eT4) == 0, (α β)^2 == 1}, {α, β, γ, δ}]
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$\left\{\left\{\beta\to-\frac{1}{\alpha},\gamma\to 0,\delta\to 0\right\},\left\{\beta\to\frac{1}{\alpha},\gamma\to 0,\delta\to 0\right\},\left\{\alpha\to -2,\beta\to-\frac{1}{2},\gamma\to 0,\delta\to 0\right\},\left\{\alpha\to -2\,i,\beta\to-\frac{i}{2},\gamma\to 0,\delta\to 0\right\},\left\{\alpha\to 2\,i,\beta\to\frac{i}{2},\gamma\to 0,\delta\to 0\right\},\left\{\alpha\to 2,\beta\to\frac{1}{2},\gamma\to 0,\delta\to 0\right\},\left\{\alpha\to-\sqrt{6},\beta\to-\frac{1}{\sqrt{6}},\gamma\to 0,\delta\to 0\right\},\left\{\alpha\to -i\sqrt{6},\beta\to-\frac{i}{\sqrt{6}},\gamma\to 0,\delta\to 0\right\},\left\{\alpha\to i\sqrt{6},\beta\to\frac{i}{\sqrt{6}},\gamma\to 0,\delta\to 0\right\},\left\{\alpha\to\sqrt{6},\beta\to\frac{1}{\sqrt{6}},\gamma\to 0,\delta\to 0\right\}\right\}$

Das System hat ausschliesslich nur Lösungen, bei denen der eine Faktor (und damit γ und δ) null ist, was keinen Eigenvektor ungleich null bedeutet.

```
Solve[{(α ev3 + β ev4) (γ eT1 + δ eT2) == 0, (α β)^2 == 1}, {α, β, γ, δ}]
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$\left\{\left\{\beta\to-\frac{1}{\alpha},\gamma\to 0,\delta\to 0\right\},\left\{\beta\to\frac{1}{\alpha},\gamma\to 0,\delta\to 0\right\},\{\alpha\to -1,\beta\to 1,\gamma\to 0,\delta\to 0\},\{\alpha\to -i,\beta\to i,\gamma\to 0,\delta\to 0\},\{\alpha\to i,\beta\to -i,\gamma\to 0,\delta\to 0\},\{\alpha\to 1,\beta\to -1,\gamma\to 0,\delta\to 0\},\left\{\alpha\to-\sqrt{2},\beta\to\frac{1}{\sqrt{2}},\gamma\to 0,\delta\to 0\right\},\left\{\alpha\to -i\sqrt{2},\beta\to\frac{i}{\sqrt{2}},\gamma\to 0,\delta\to 0\right\},\left\{\alpha\to i\sqrt{2},\beta\to-\frac{i}{\sqrt{2}},\gamma\to 0,\delta\to 0\right\},\left\{\alpha\to\sqrt{2},\beta\to-\frac{1}{\sqrt{2}},\gamma\to 0,\delta\to 0\right\}\right\}$

Das System hat ausschliesslich nur Lösungen, bei denen der eine Faktor (und damit γ und δ) null ist, was keinen Eigenvektor ungleich null bedeutet.

```
Solve[{(α ev3 + β ev4) (γ eT3 + δ eT4) == 0, (α β)^2 == 1}, {α, β, γ, δ}]
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$\left\{\left\{\beta\to-\frac{1}{\alpha},\gamma\to 0,\delta\to 0\right\},\left\{\beta\to\frac{1}{\alpha},\gamma\to 0,\delta\to 0\right\},\{\alpha\to -1,\beta\to 1,\gamma\to 0,\delta\to 0\},\{\alpha\to -i,\beta\to i,\gamma\to 0,\delta\to 0\},\{\alpha\to i,\beta\to -i,\gamma\to 0,\delta\to 0\},\{\alpha\to 1,\beta\to -1,\gamma\to 0,\delta\to 0\},\left\{\alpha\to-\sqrt{2},\beta\to\frac{1}{\sqrt{2}},\gamma\to 0,\delta\to 0\right\},\left\{\alpha\to -i\sqrt{2},\beta\to\frac{i}{\sqrt{2}},\gamma\to 0,\delta\to 0\right\},\left\{\alpha\to i\sqrt{2},\beta\to-\frac{i}{\sqrt{2}},\gamma\to 0,\delta\to 0\right\},\left\{\alpha\to\sqrt{2},\beta\to-\frac{1}{\sqrt{2}},\gamma\to 0,\delta\to 0\right\}\right\}$

Das System hat ausschliesslich nur Lösungen, bei denen der eine Faktor (und damit γ und δ) null ist, was keinen Eigenvektor ungleich null bedeutet.

==> Nie senkrecht.

- **g**

```
W = Transpose[Vmatr]; W // MatrixForm
```

$$\begin{pmatrix} -4 & -3 & 1 & -1 \\ 0 & 4 & 2 & 2 \\ -3 & 0 & -4 & 3 \\ 5 & 2 & 0 & 1 \end{pmatrix}$$

```
VolW = Det[W]
```

148

```
VolW = Det[M.W]
```

148

- ### h

```
Det[M]
```

1

```
VolMW = Det[M.W]
```

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```
VolMW / VolW
```

1

5

```
Remove["Global`*"]
```

```
a = {-2, 1, 2};  OQ = {-1, 0, 4};  ϕ = Pi / 6;
```

- ### a Drehung

```
mDrehung[ϕ_] := {{1, 0, 0}, {0, Cos[ϕ], -Sin[ϕ]}, {0, Sin[ϕ], Cos[ϕ]}};
mDrehung[Pi / 6] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

```
% // N // MatrixForm
```

$$\begin{pmatrix} 1. & 0. & 0. \\ 0. & 0.866025 & -0.5 \\ 0. & 0.5 & 0.866025 \end{pmatrix}$$

Matrixkonstruktion

Lokale Basis

```
(* Normiert einen Vektor *)
NVec[a_] := a / Norm[a];
(* Quadriert Komponenten eines Vektors *)
QVec[a_] := Table[a[[k]]^2, {k, 1, Length[a]}];
(* Numeriert Komponenten eines Vektors *)
QVecNr[a_] := Table[{k, a[[k]]^2}, {k, 1, Length[a]}];
(* Sucht die Nummer einer absolut maximal grossen Komponente *)
NrMaxQVec[a_] := Max[Table[If[a[[k]]^2 == Max[QVec[a]], k, 0], {k, 1, Length[a]}]];
(* Sucht die Nummer einer absolut minimal grossen Komponente *)
NrMinQVec[a_] :=
  Min[Table[If[a[[k]]^2 == Min[QVec[a]], k, Length[a] + 1], {k, 1, Length[a]}]];
b[a_, x_] := Table[ If[k == NrMaxQVec[a], 1, If[k == NrMinQVec[a], 0, x ] ]
     , {k, 1, Length[a]}];
solv = Solve[b[a, x].a == 0, {x}] // Flatten;
b[a_] := b[a, x] /. solv
e1 = {1, 0, 0}; e2 = {0, 1, 0}; e3 = {0, 0, 1};
 If[Element[NVec[a], Union[{e1, e2, e3}, -{e1, e2, e3}]],
  b[a_] := Cross[e1 + e2 + e3, NVec[a]], b[a] = b[a]];
basis[a_] := {NVec[a], NVec[b[a]], Cross[NVec[a], NVec[b[a]]]};
TrBasis[a_] := basis[a] // Transpose;
aVec1 = NVec[a]; aVec2 = NVec[b[a]];
aVec3 = Cross[NVec[a], NVec[b[a]]];
```

Kontrolle

```
basis[a]
```

$$\left\{\left\{-\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right\}, \left\{\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{3\sqrt{2}}, \frac{2\sqrt{2}}{3}, -\frac{1}{3\sqrt{2}}\right\}\right\}$$

```
Cross[a, basis[a][[1]]]
```

{0, 0, 0}

```
basis[a][[1]].basis[a][[2]]
```

0

```
basis[a][[1]].basis[a][[3]]
```

0

```
basis[a][[2]].basis[a][[3]]
```

0

```
basis[a][[1]] // Norm
```

1

```
basis[a][[2]] // Norm
```

1

```
basis[a][[3]] // Norm
```

1

```
TrBasis[a].e1 == aVec1
```

True

```
TrBasis[a].e2 == aVec2
```

True

```
TrBasis[a].e3 == aVec3
```

True

■ Matrixzusammensetzung

```
Print[Inverse[TrBasis[a]] // MatrixForm];
mDrehung[ϕ_] := {{1, 0, 0}, {0, Cos[ϕ], -Sin[ϕ]}, {0, Sin[ϕ], Cos[ϕ]}};
mDrehung[Pi / 6];
Print[mDrehung[Pi / 6] // MatrixForm];
matrix[ϕ_] := TrBasis[a].mDrehung[ϕ].Inverse[TrBasis[a]];
Print[matrix[Pi / 6] // MatrixForm];
Print[matrix[Pi / 6] // N // MatrixForm];
```

$$\begin{pmatrix} -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{3\sqrt{2}} & \frac{2\sqrt{2}}{3} & -\frac{1}{3\sqrt{2}} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

$$\begin{pmatrix} \frac{4}{9} + \frac{\frac{\sqrt{\frac{3}{2}}}{2} + \frac{1}{6\sqrt{2}}}{\sqrt{2}} + \frac{-\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{6}}}{3\sqrt{2}} & -\frac{2}{9} + \frac{2}{3}\sqrt{2}\left(-\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{6}}\right) & -\frac{4}{9} + \frac{\frac{\sqrt{\frac{3}{2}}}{2} + \frac{1}{6\sqrt{2}}}{\sqrt{2}} - \frac{-\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{6}}}{3\sqrt{2}} \\ \frac{1}{9} + \frac{1}{3\sqrt{3}} & \frac{1}{9} + \frac{4}{3\sqrt{3}} & \frac{5}{9} - \frac{1}{3\sqrt{3}} \\ -\frac{4}{9} + \frac{\frac{\sqrt{\frac{3}{2}}}{2} - \frac{1}{6\sqrt{2}}}{\sqrt{2}} + \frac{-\frac{1}{2\sqrt{2}} - \frac{1}{2\sqrt{6}}}{3\sqrt{2}} & \frac{2}{9} + \frac{2}{3}\sqrt{2}\left(-\frac{1}{2\sqrt{2}} - \frac{1}{2\sqrt{6}}\right) & \frac{4}{9} + \frac{\frac{\sqrt{\frac{3}{2}}}{2} - \frac{1}{6\sqrt{2}}}{\sqrt{2}} - \frac{-\frac{1}{2\sqrt{2}} - \frac{1}{2\sqrt{6}}}{3\sqrt{2}} \end{pmatrix}$$

$$\begin{pmatrix} 0.92557 & -0.363105 & 0.107122 \\ 0.303561 & 0.880911 & 0.363105 \\ -0.226211 & -0.303561 & 0.92557 \end{pmatrix}$$

■ Drehung

```
OQStrich = matrix[Pi / 6].OQ
```

$$\left\{-\frac{4}{9} - \frac{\frac{\sqrt{\frac{3}{2}}}{2} + \frac{1}{6\sqrt{2}}}{\sqrt{2}} - \frac{-\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{6}}}{3\sqrt{2}} + 4\left(-\frac{4}{9} + \frac{\frac{\sqrt{\frac{3}{2}}}{2} + \frac{1}{6\sqrt{2}}}{\sqrt{2}} - \frac{-\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{6}}}{3\sqrt{2}}\right),\right.$$

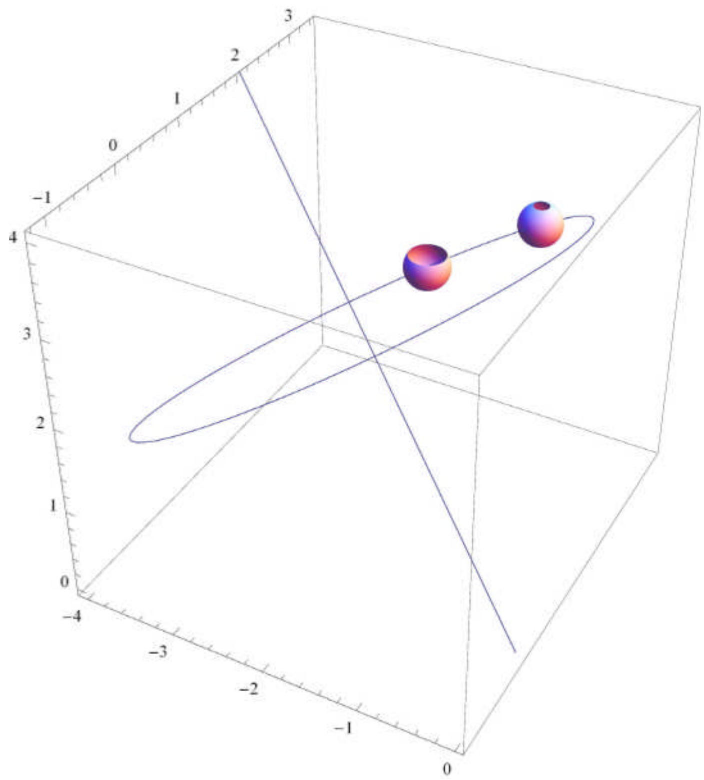
$$-\frac{1}{9} - \frac{1}{3\sqrt{3}} + 4\left(\frac{5}{9} - \frac{1}{3\sqrt{3}}\right),$$

$$\left.\frac{4}{9} - \frac{\frac{\sqrt{\frac{3}{2}}}{2} - \frac{1}{6\sqrt{2}}}{\sqrt{2}} - \frac{-\frac{1}{2\sqrt{2}} - \frac{1}{2\sqrt{6}}}{3\sqrt{2}} + 4\left(\frac{4}{9} + \frac{\frac{\sqrt{\frac{3}{2}}}{2} - \frac{1}{6\sqrt{2}}}{\sqrt{2}} - \frac{-\frac{1}{2\sqrt{2}} - \frac{1}{2\sqrt{6}}}{3\sqrt{2}}\right)\right\}$$

```
OQStrich // N
```

{-0.49708, 1.14886, 3.92849}

```
p1 = ParametricPlot3D[matrix[ϕ].OQ, {ϕ, 0, 2 Pi}];
p2 = ParametricPlot3D[ϕ aVec1, {ϕ, 0, 2 Pi}];
p3 = Graphics3D[{Sphere[OQ, 0.2], Sphere[OQStrich, 0.2]}];
Show[p1, p2, p3]
```



6

```
Remove["Global`*"]
```

■ a

```
OP1 = {0, 1, 1}; OP2 = {1, 0, -1}; OP3 = {1, 1, 1};
OP4 = {2, 6, 1}; OP5 = {-1, 5, 8}; OP6 = {-2, 12, 0};

G1 = {OP1, OP2, OP3} // Transpose;
G2 = {OP4, OP5, OP6} // Transpose;
Det[G1]
```

-1

```
Det[G2]
```

-290

```
G.G1 == G2
```

G.{{0, 1, 1}, {1, 0, 1}, {1, -1, 1}} == {{2, -1, -2}, {6, 5, 12}, {1, 8, 0}}

```
G = G2.Inverse[G1]; G // MatrixForm
```

$$\begin{pmatrix} -4 & 5 & -3 \\ 6 & 5 & 1 \\ -1 & 10 & -9 \end{pmatrix}$$

■ b

```
Dreh[ϕ_] := {{Cos[ϕ], -Sin[ϕ], 0}, {Sin[ϕ], Cos[ϕ], 0}, {0, 0, 1}}; Dreh[ϕ] // MatrixForm
```

$$\begin{pmatrix} \text{Cos}[\phi] & -\text{Sin}[\phi] & 0 \\ \text{Sin}[\phi] & \text{Cos}[\phi] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
Dreh[32 Degree] // MatrixForm
```

$$\begin{pmatrix} \text{Cos}[32\,°] & -\text{Sin}[32\,°] & 0 \\ \text{Sin}[32\,°] & \text{Cos}[32\,°] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
Dreh[32 Degree] // N // MatrixForm
```

$$\begin{pmatrix} 0.848048 & -0.529919 & 0. \\ 0.529919 & 0.848048 & 0. \\ 0. & 0. & 1. \end{pmatrix}$$

■ c

```
OP7 = G.Dreh[32 Degree].OP1
```

{-3 + 5 Cos[32 °] + 4 Sin[32 °], 1 + 5 Cos[32 °] - 6 Sin[32 °], -9 + 10 Cos[32 °] + Sin[32 °]}

```
OP7 // N
```

{3.35992, 2.06072, 0.0104002}

7

```
Remove["Global`*"]
```

■ a

```
a = 2 + 3 I; b = 3 - 2 I;
Solve[a Conjugate[z] == b, {z}]
```

{{z → i}}

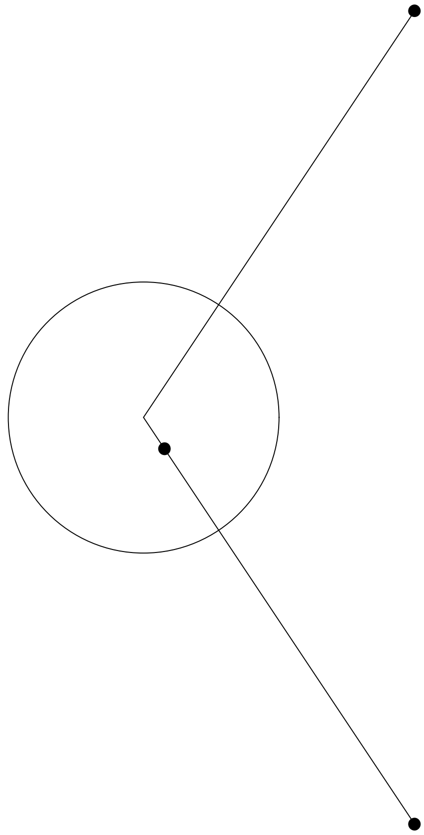
■ b

```
Arg[I]
```

$\frac{\pi}{2}$

- **c**

```
pt[z_] := Point[{Re[z], Im[z]}];
Show[Graphics[{Circle[{0, 0}, 1], PointSize[0.03], pt[a], pt[a],
   pt[Conjugate[a]], pt[1 / a], Line[{{Re[a], Im[a]}, {0, 0}, {Re[a], -Im[a]}}]}]]
```



- **d**

```
z2 = 1 / a
```

$\frac{2}{13} - \frac{3\,i}{13}$

```
solv = Solve[(z + b) ^4 == z2, {z}]
```

$\left\{\left\{z \to \left(\frac{2}{13} - \frac{3\,i}{13}\right)\left((-12 - 5\,i) + (2 + 3\,i)^{3/4}\right)\right\}, \left\{z \to \left(\frac{3}{13} + \frac{2\,i}{13}\right)\left((-5 + 12\,i) + (2 + 3\,i)^{3/4}\right)\right\}, \left\{z \to \left(-\frac{3}{13} - \frac{2\,i}{13}\right)\left((5 - 12\,i) + (2 + 3\,i)^{3/4}\right)\right\}, \left\{z \to \left(-\frac{2}{13} + \frac{3\,i}{13}\right)\left((12 + 5\,i) + (2 + 3\,i)^{3/4}\right)\right\}\right\}$

```
NTab = N[%]
```

{{z → -2.29609 + 1.82349 i}, {z → -2.82349 + 2.70391 i},
 {z → -3.17651 + 1.29609 i}, {z → -3.70391 + 2.17651 i}}

```
grTab = Table[Point[{Re[z], Im[z]}] /. NTab[[k]], {k, 1, Length[NTab]}]
```

{Point[{-2.29609, 1.82349}], Point[{-2.82349, 2.70391}],
 Point[{-3.17651, 1.29609}], Point[{-3.70391, 2.17651}]}

```
tab = Table[w[k] = z /. solv[[k]], {k, 1, Length[solv]}]
```

$\left\{\left(\frac{2}{13} - \frac{3\,i}{13}\right)\left((-12 - 5\,i) + (2 + 3\,i)^{3/4}\right), \left(\frac{3}{13} + \frac{2\,i}{13}\right)\left((-5 + 12\,i) + (2 + 3\,i)^{3/4}\right), \left(-\frac{3}{13} - \frac{2\,i}{13}\right)\left((5 - 12\,i) + (2 + 3\,i)^{3/4}\right), \left(-\frac{2}{13} + \frac{3\,i}{13}\right)\left((12 + 5\,i) + (2 + 3\,i)^{3/4}\right)\right\}$

```
Schwerpunkt = Apply[Plus, tab] / 4 // Simplify
```

```
-3 + 2 i
```

```
-b == Schwerpunkt
```

```
True
```

```
pt[z_] := Point[{Re[z], Im[z]}];
Show[Graphics[{PointSize[0.03],
   Point[{-2.2960945232139687`, 1.823485219886391`}],
   Point[{-2.8234852198863907`, 2.7039054767860318`}],
   Point[{-3.1765147801136093`, 1.2960945232139682`}],
   Point[{-3.7039054767860318`, 2.1765147801136093`}],
   pt[Schwerpunkt]}]]
```